PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

OF THE

UNITED STATES OF AMERICA

VOLUME 5, 1919

Publication Office: Williams & Wilkins Company, Baltimore
Editorial Office: Massachusetts Institute of Technology, Cambridge

Home Office of the Academy: Washington, D. C.

CONTENTS

NATIONAL RESEARCH COUNCIL

MATHEMATICS

ASTRONOMY

CHEMISTRY

GEOLOGY AND PALEONTOLOGY

BOTANY

ZOOLOGY

GENETICS

PHYSIOLOGY AND PATHOLOGY

ANTHROPOLOGY AND PSYCHOLOGY

OFFICERS AND MEMBERS OF THE ACADEMY
NOVEMBER 10, 1919

OFFICERS OF THE ACADEMY

CHARLES D. WALCOTT, *President*

A. A. MICHELSON, *Vice-President* — GEORGE E. HALE, *Foreign Secretary*

F. L. RANSOME, *Treasurer* — C. G. ABBOT, *Home Secretary*

Additional Members of the Council

E. G. CONKLIN	RAYMOND PEARL	J. J. CARTY
A. A. NOYES	W. H. HOWELL	H. H. DONALDSON

MEMBERS OF THE ACADEMY

ABBOT, CHARLES GREELEY. .... *Smithsonian Institution, Washington, D. C.*
ABBOT, HENRY L., U. S. A. .... *23 Berkeley St., Cambridge, Mass.*
ABEL, JOHN JACOB .... *Johns Hopkins University, Baltimore, Md.*
ADAMS, WALTER SYDNEY .... *Solar Observatory Office, Pasadena, Cal.*
AITKEN, ROBERT GRANT .... *Lick Observatory, Mt. Hamilton, Cal.*
ALLEN, J. ASAPH .... *American Museum of Natural History, N. Y. City*
AMES, JOSEPH S. .... *Johns Hopkins University, Baltimore, Md.*
BAILEY, LIBERTY HYDE .... *Ithaca, N. Y.*
BARNARD, E. E. .... *Yerkes Observatory, Williams Bay, Wis.*
BARUS, CARL .... *Brown University, Providence, R. I.*
BAXTER, GREGORY PAUL .... *T. J. Coolidge Jr. Mem. Lab., Cambridge, Mass.*
BELL, A. GRAHAM .... *1331 Connecticut Ave., Washington, D. C.*
BENEDICT, FRANCIS GANO .... *Nutrition Laboratory, Boston, Mass.*
BIRKHOFF, GEORGE DAVID .... *Harvard University, Cambridge, Mass.*
BLISS, GILBERT AMES .... *University of Chicago, Chicago, Ill.*
BOAS, FRANZ .... *Columbia University, New York City*
BOGERT, MARSTON TAYLOR .... *Columbia University, New York City*
BOLTWOOD, B. B. .... *Yale University, New Haven, Conn.*
BOLZA, OSKAR .... *Reichsgrafenstr, 10, Freiburg, Germany*
BRANNER, JOHN C. .... *Stanford University, California*
BRIDGMAN, PERCY WILLIAMS .... *Jefferson Phys. Lab., Cambridge, Mass.*
BRITTON, NATHANIEL LORD .... *New York Botanical Gardens, N. Y. City.*
BUMSTEAD, HENRY ANDREWS .... *Yale University, New Haven, Conn.*
CALKINS, GARY NATHAN .... *Columbia University, New York City*
CAMPBELL, D. H. .... *Stanford University, California*
CAMPBELL, WILLIAM W. .... *Lick Observatory, Mount Hamilton, California*
CANNON, WALTER BRADFORD .... *Harvard University, Cambridge, Mass.*
CARTY, JOHN J. .... *Am. Telegraph and Telephone Co., N. Y. City*
CASTLE, WILLIAM ERNEST .... *186 Payson Road, Belmont, Mass.*
CATTELL, JAMES McK. .... *Garrison, N. Y.*
CHAMBERLIN, THOMAS C. .... *University of Chicago, Chicago, Ill.*
CHANDLER, CHARLES F. .... *Columbia University, New York City*

CHITTENDEN, RUSSELL H. .... *Sheffield Scientific School, New Haven, Conn.*
CLARKE, F. W. .... *U. S. Geological Survey, Washington, D.C.*
CLARKE, J. M. .... *State Museum, Albany, N. Y.*
COMSTOCK, GEORGE C. .... *Washburn Observatory, Madison, Wis.*
CONKLIN, E. G. .... *Princeton, N. J.*
COULTER, J. M. .... *University of Chicago, Chicago, Ill.*
COUNCILMAN, WILLIAM T. .... *Harvard Medical School, Boston, Mass.*
CREW, HENRY .... *Northwestern University, Evanston, Ill.*
CROSS, WHITMAN .... *2138 Bancroft Place, Washington, D. C.*
CURTIS, HEBER DOUST .... *Lick Observatory, Mt. Hamilton, Cal.*
CUSHING, HARVEY .... *Harvard University, Cambridge, Mass.*
DALL, WILLIAM H. .... *Smithsonian Institution, Washington, D. C.*
DANA, EDWARD S. .... *Yale University, New Haven, Conn.*
DAVENPORT, CHARLES B. .... *Cold Spring Harbor, N. Y.*
DAVIS, WILLIAM MORRIS .... *31 Hawthorn St., Cambridge, Mass..*
DAY, ARTHUR L. .... *Corning Glass Works, Corning, N. Y.*
DEWEY, JOHN .... *Columbia University, New York City.*
DICKSON, LEONARD E. .... *University of Chicago, Chicago, Ill.*
DONALDSON, HENRY HERBERT .... *Wistar Institute of Anatomy, Phila., Pa.*
DUNN, GANO .... *43 Exchange Place, New York City*
DURAND, WILLIAM FREDERICK .... *Stanford University, Cal.*
ELKIN, WILLIAM L. .... *Yale University Observatory, New Haven, Conn.*
FEWKES, JESSE WALTER .... *Bureau of American Ethnology, Wash., D. C.*
FLEXNER, SIMON .... *Rockefeller Institute, New York City*
FOLIN, OTTO .... *Harvard Medical School, Boston, Mass.*
FORBES, STEPHEN ALFRED .... *Urbana, Ill.*
FRANKLIN, EDWARD CURTIS .... *Stanford University, California*
FREEMAN, JOHN RIPLEY .... *Providence, R. I.*
FROST, EDWIN B. .... *Yerkes Observatory, Williams Bay, Wis.*
GOMBERG, MOSES .... *University of Michigan, Ann Arbor, Mich.*
GOOCH, FRANK A. .... *291 Edwards St., New Haven, Conn.*
GOODALE, GEORGE L. .... *Harvard University, Cambridge, Mass.*
HALE, GEORGE E. .... *Solar Observatory Office, Pasadena, Cal.*
HALL, EDWIN H. .... *Harvard University, Cambridge, Mass.*
HALL, GRANVILLE STANLEY .... *Clark University, Worcester, Mass.*
HALSTED, WILLIAM STEWART .... *Johns Hopkins Med. School, Baltimore, Md.*
HARPER, R. A. .... *Columbia University, New York City*
HARRISON, ROSS G. .... *Yale University, New Haven, Conn.*
HASTINGS, CHARLES S. .... *Yale University, New Haven, Conn.*
HAYFORD, JOHN F. .... *Northwestern University, Evanston, Ill.*
HEKTOEN, LUDVIG .... *629 South Wood St., Chicago, Ill.*
HENDERSON, LAWRENCE JOSEPH .... *Harvard University, Cambridge, Mass.*
HERRICK, CHARLES JUDSON .... *University of Chicago, Chicago, Ill.*
HILLEBRAND, WILLIAM F. .... *Bureau of Standards, Washington, D. C.*
HOLMES, WILLIAM H. .... *U. S. National Museum, Washington, D. C.*
HOWARD, LELAND OSSIAN .... *U. S. Dept. of Agriculture, Washington, D. C.*
HOWE, HENRY MARION .... *Broad Brook Road, Bedford Hills, N. Y.*
HOWELL, WILLIAM H. .... *School of Hygiene and Public Health, Baltimore, Md.*
HUNT, REID .... *Harvard Med. Sch., Boston, Mass.*
IDDINGS, JOSEPH P. .... *U. S. Geological Survey, Washington, D. C.*
JACKSON, CHARLES L. .... *383 Beacon St., Boston, Mass.*
JENNINGS, HERBERT SPENCER .... *Johns Hopkins University, Baltimore, Md.*

JEWETT, FRANK BALDWIN........................*Western Electric Co., New York, N. Y.*
JOHNSON. TREAT BALDWIN...........................*Yale University, New Haven, Conn.*
JONES, WALTER.............................*Johns Hopkins University, Baltimore, Md.*
KASNER, EDWARD.................................*Columbia University, New York City*
KEMP, JAMES F...................................*Columbia University, New York City*
LANGMUIR, IRVING.............................*Gen. Electric Co., Schenectady, N. Y.*
LEUSCHNER, ARMIN O..........................*University of California, Berkeley, Cal.*
LEVENE, PHŒBUS AARON THEODOR..................*Rockefeller Institute, New York City*
LEWIS, GILBERT N.............................*University of California, Berkeley, Cal.*
LILLIE, FRANK RATTRAY..........................*University of Chicago, Chicago, Ill.*
LINDGREN, WALDEMAR.....................*Mass. Inst. of Technology, Cambridge, Mass.*
LOEB, JACQUES.....................................*Rockefeller Institute, New York City*
LUSK, GRAHAM.......................*Cornell University Medical College, New York City*
LYMAN, THEODORE.............................*Harvard University, Cambridge, Mass.*
MARK, EDWARD L...................................*109 Irving St., Cambridge, Mass.*
MAYOR, ALFRED GOLDSBOROUGH......................*276 Nassau St., Princeton, N. J.*
MELTZER, SAMUEL JAMES...........................*Rockefeller Institute, New York City*
MENDEL, LAFAYETTE B.............................*262 Conner St., New Haven, Conn.*
MENDENHALL, CHARLES ELWOOD................*University of Wisconsin, Madison, Wis.*
MENDENHALL, THOMAS C.....................*329 North Chestnut St., Ravenna, Ohio*
MERRIAM, C. HART..................................*1919 16th St., Washington, D. C.*
MERRIAM, JOHN CAMPBELL.....................*University of California, Berkeley, Cal.*
MERRITT, ERNEST...................................*Cornell University, Ithaca, N. Y.*
MICHAEL, ARTHUR...............................*219 Parker St., Newton Center, Mass.*
MICHELSON, ALBERT A...............................*University of Chicago, Chicago, Ill.*
MILLIKAN, ROBERT ANDREWS........................*University of Chicago, Chicago, Ill.*
MOORE, ELIAKIM H..................................*University of Chicago, Chicago, Ill.*
MORGAN, T. H.....................................*Columbia University, New York City*
MORLEY, EDWARD W...*West Hartford, Conn.*
MORSE, EDWARD S...*Salem, Mass.*
MORSE, HARMON N..........................*Johns Hopkins University, Baltimore, Md.*
MOULTON, F. R.....................................*University of Chicago, Chicago, Ill.*
NICHOLS, EDWARD L...................................*Cornell University, Ithaca, N. Y.*
NICHOLS, ERNEST F...............................*Yale University, New Haven, Conn.*
NOYES, ARTHUR A..........................*Mass. Inst. of Technology, Cambridge, Mass.*
NOYES, WILLIAM A..................................*University of Illinois, Urbana, Ill.*
OSBORN, H. F......................*American Museum of Natural History, New York City*
OSBORNE, T. B......................*Agricultural Experiment Station, New Haven, Conn.*
OSGOOD, WILLIAM FOGG..........................*Harvard University, Cambridge, Mass.*
OSTERHOUT, WINTHROP JOHN VANLEUVEN...........*Harvard University, Cambridge, Mass.*
PARKER, GEORGE H.................................*16 Berkeley St., Cambridge, Mass.*
PEARL, RAYMOND....................*School of Hygiene and Public Health, Baltimore, Md.*
PIRSSON, LOUIS V...................................*41 Trumbull St., New Haven, Conn.*
PRUDDEN, T. MITCHELL..............................*160 West 59th St., New York City*
PUMPELLY, RAPHAEL....................................*Gibbs Avenue, Newport, R. I.*
PUPIN, MICHAEL I..................................*Columbia University, New York City*
RANSOME, FREDERICK LESLIE..................*U. S. Geological Survey, Washington, D. C.*
REID, H. FIELDING..........................*Johns Hopkins University, Baltimore, Md.*
REMSEN, IRA...............................*Johns Hopkins University, Baltimore, Md.*
RICHARDS, THEODORE W....................*Wolcott Gibbs Mem. Lab., Cambridge, Mass.*
RIDGWAY, ROBERT..............................*1030 South Morgan St., Olney, Ill.*
ROSA, EDWARD B.............................*Bureau of Standards, Washington, D. C.*

Russell, Henry Norris.........................Princeton University, Princeton, N. J.
Sargent, Charles S.............................Arnold Arboretum, Jamaica Plain, Mass.
Schlesinger, Frank.............................Allegheny Observatory, Allegheny, Pa.
Schuchert, Charles.............................Yale University, New Haven, Conn.
Scott, William B................................Princeton University, Princeton, N. J.
Seares, Frederick Hanley.......................Mt. Wilson Obs., Pasadena, Cal.
Setchell, William Albert.......................University of California, Berkeley, Cal.
Smith, Alexander...............................Columbia University, New York City
Smith, Edgar F.................................University of Pennsylvania, Philadelphia, Pa.
Smith, Erwin F.................................Bureau of Plant Industry, Washington, D. C.
Smith, Theobald................................Princeton, N. J.
Squier, George Owen............................Chief Sig. Officer, U. S. Army, Washington, D. C.
Stieglitz, Julius...............................University of Chicago, Chicago, Ill.
Story, William E...............................Clark University, Worcester, Mass.
Stratton, Samuel Wesley........................Bureau of Standards, Washington, D. C.
Taylor, David Watson...........................Department of the Navy, Washington, D. C.
Thaxter, Roland................................Harvard University, Cambridge, Mass.
Thomson, Elihu.................................Swampscott, Mass.
Thorndike, Edward Lee..........................Columbia University, New York City
Trelease, William..............................University of Illinois, Urbana, Ill.
Trowbridge, Agustus............................Princeton University, Princeton, N. J.
Trowbridge, John...............................Harvard University, Cambridge, Mass.
Ulrich, Edward Oscar...........................U. S. Geological Survey, Washington, D. C.
Van Vleck, E. B................................University of Wisconsin, Madison, Wis.
Vaughan, Victor Clarence.......................University of Michigan, Ann Arbor, Mich.
Veblen, Oswald.................................Princeton University, Princeton, N. J.
Verrill, A. E...................................53 Rolston St., Whitneyville, Conn.
Walcott, Charles D.............................Smithsonian Institution, Washington, D. C.
Webster, Arthur G..............................Clark University, Worcester, Mass.
Welch, William H...............................807 St. Paul St., Baltimore, Md.
Wells, Horace L................................Yale University, New Haven, Conn.
Wheeler, William M.............................Harvard University, Cambridge, Mass.
White, David...................................U. S. Geological Survey, Washington, D. C.
White, Henry Seely.............................Vassar College, Poughkeepsie, N. Y.
Whitney, Willis Rodney.........................General Electric Co., Schenectady, N. Y.
Wilczynski, Ernest Julius......................University of Chicago, Chicago, Ill.
Wilson, Edmund B...............................Columbia University, New York City
Wilson, Edwin Bidwell..........................Mass. Inst. Techn., Cambridge, Mass.
Wood, Horatio C................................4107 Chester Avenue, Philadelphia, Pa.
Wood, Robert W.................................Johns Hopkins University, Baltimore, Md.
Woodward, Robert S.............................Carnegie Institution, Washington, D. C.

Smith, Sidney I................................Yale University, New Haven, Conn

FOREIGN ASSOCIATES

Arrhenius, S. A................................Nobelinstitute, Stockholm
Barrois, Charles...............................Université, Lille
Brøgger, W. C..................................Universitet, Christiania
Deslandres, Henri..............................Astrophysical Observatory, Meudon
Dewar, Sir James...............................University, Cambridge
Forsyth, A. R..................................Imperial College of Science and Technology, London

GEIKIE, SIR ARCHIBALD........*Haslemere, Surrey*
GROTH, PAUL VON........*Universität, Munich*
HEIM, ALBERT........*Zürich*
HILBERT, DAVID........*Universität, Göttingen*
KAPTEYN, JOHN C........*Rijks Universiteit, Groningen*
KLEIN, FELIX........*Universität, Göttingen*
KÖSSEL, ALBRECHT........*Heidelberg*
KUSTNER, KARL FRIEDRICH........*Bonn*
LANKESTER, SIR E. RAY........*South Kensington, London*
LARMOR, SIR JOSEPH........*St. Johns College, Cambridge*
LORENTZ, HENDRIK ANTON........*Rijks Universiteit, Leiden*
OSTWALD, WILHELM........*Grossbothen, bei Leipzig*
PAVLOV, IVAN PETROVITCH........*Inst. for Experimental Medicine, Petrograd*
PENCK, ALBRECHT........*Universität, Berlin*
PFEFFER, WILHELM........*Botanische Institut der Universität, Leipzig*
PICARD, CHARLES ÉMILE........*Université, Paris*
RUTHERFORD, SIR ERNEST........*University, Manchester*
SCHUSTER, ARTHUR........*Secretary of the Royal Society, London*
SEELIGER, HUGO RITTER VON........*Universität, Munich*
THOMSON, SIR JOSEPH........*University, Cambridge*
VOLTERRA, VITO........*Università, Rome*
VRIES, HUGO DE........*Universiteit, Amsterdam*
WAALS, JOHANNES D. VAN DER........*Amsterdam*
WALDEYER, WILHELM........*Universität, Berlin*
WOLF, MAX F. J. C........*Heidelberg*
WUNDT, WILHELM........*Universität, Leipzig*

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 5 JANUARY 15, 1919 Number 1

ON SOME FUNDAMENTALS OF PRE-CAMBRIAN PALEOGEOGRAPHY

By Rudolf Ruedemann

New York State Museum, Albany, N. Y.

Communicated by J. M. Clarke, November 26, 1918

It is an established fact that the Archean basement complex (representing the Archeozoic era) has undergone not only complete metamorphism but also a world-wide intense folding which undoubtedly is a true folding due to world-wide diastrophism. The later Pre-Cambrian rocks (classed formerly as Huronian, then as Algonkian and more recently as Algoman, Huronian and Algonkian rocks, representing Proterozoic time) have escaped metamorphism and folding in the interior of North America but elsewhere have undergone like folding as the Archean basement complex and with few exceptions in the same sense; for example, in the eastern Canadian shield, where both are folded from the southeast (Adams). This world-wide folding of the Pre-Cambrian rocks stands in striking contrast to the localized folding of the earth crust in all later time.

Starting from this fact of the complete folding of the exposed Archean areas and on the proper inference that the entire Archean basement complex, and with it the greatest part of the Proterozoic rocks of the Earth, is likewise folded, the writer has attempted a synthesis of the directive lines of arrangement of these Pre-Cambrian folds.

In such an attempt there are to be excluded:

1. All areas of metamorphic rocks which are either proven to be or suspected of being younger than Pre-Cambrian age; such as are found in Greece, Asia Minor, the Andaman and Antillean Islands (serpentines of Cuba, etc.), the gneisses of the Coast range, lower California, etc.

2. All rocks of Pre-Cambrian age involved in Post-Cambrian folding, as those of North Africa, Spain, France, western Germany, the Alps, eastern Australia, etc. But it is to be noted here that even in these cases, keen observers have often enough found that the Pre-Cambrian nuclei of mountain ranges retain an independent original direction of folding, and further that

in some cases the new folds have clearly followed old lines of folding (posthumous folds). There are further left, even in those regions that were overrun by the crustal waves of Post-Cambrian time, 'islands' or blocks that remained undisturbed and that give important information on the original direction of folding in the Pre-Cambrian basement complex. Such undisturbed blocks are found in the Rocky Mountains, in Bohemia, on Borneo, in Cambodia, etc.

The large areas that are then left and lend themselves to our inquiry, are the greater part of North America, small regions of South America, eastern Europe, northern and eastern Asia, all inner Africa and western Australia. The general facts gleaned from them are complemented by those obtained from the unfolded 'islands' or blocks.

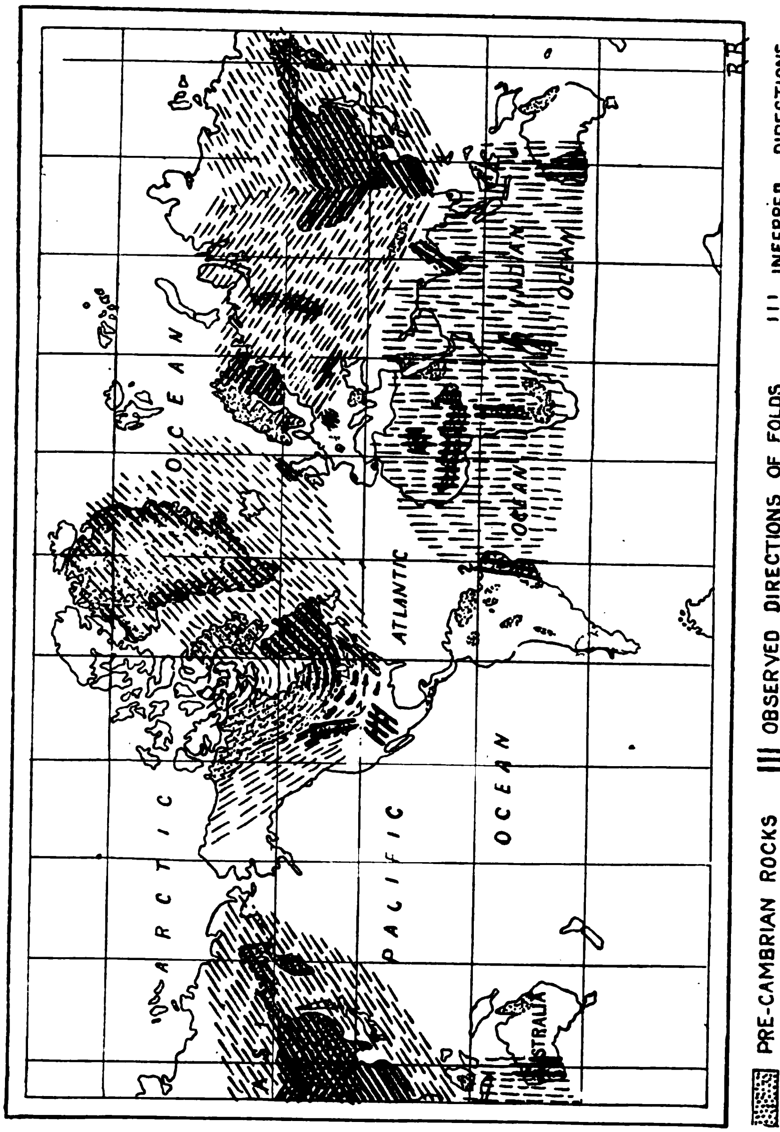
The most important of these areas are the so-called continental nuclei of North America (Canadian shield), Europe (Baltic shield) and Asia (Angara shield). An intended comparison of their principal directions of folding presupposes essential contemporaneity of the diastrophic revolutions they have undergone. This has been established by the correlations carried out by Adams, Sederholm and Willis. It is also to be noted that evidence of a like succession of eras and diastrophic events has been found in most of the other regions here mentioned as being amenable to our inquiry.

The source for this inquiry is chiefly Edward Suess' The Face of the Earth, where there is not only a large store of information as to the directions of Pre-Cambrian rock folding but where also the facts of the general directions of Pre-Cambrian folding in Asia and eastern Europe are clearly set forth. Nevertheless the assembling of the full data on Pre-Cambrian folding, which will be given in a later publication, requires laborious search through a widely extended literature.

With these introductory remarks we may briefly survey the preliminary results (which undoubtedly are still subject to important corrections) as to the arrangement of Pre-Cambrian folds.

In China, Richthofen, Pumpelly and Willis have found a uniform NE direction in the basement complex, which was completely folded and again abraded before the Cambrian submergence. This NE direction (the Sinian system of Richthofen and Pumpelly) continues through Korea, and Suess has shown its complete domination in all northeastern Asia east of Lake Baikal (his 'Baikal direction'), which is ENE-NE. This meets, in a meridional line, the 'Sajan direction' of Pre-Cambrian folds, which is WNW to NW and prevails west of Lake Baikal through all northern and middle Asia, Russia (Karpinski) and the Baltic shield, the latter being only a larger northern exposure of the Russian Pre-Cambrian plate. The latter extends through northern Europe much disturbed by later folding and probably abuts against the Laurentian plate in western Scotland.

In North America the Canadian shield exhibits in its eastern portion a uniform NE direction of the folds (Adams and Coleman). This direction


ARCTIC OCEAN
PACIFIC OCEAN
ATLANTIC
INDIAN OCEAN
AUSTRALIA
R.R.
PRE-CAMBRIAN ROCKS
OBSERVED DIRECTIONS OF FOLDS
INFERRED DIRECTIONS

continues through Greenland with some variation to NNW along the coast of Labrador through Tertiary crustal movements (Adams) and it is even recognizable in the western edge of Scotland. It is further observed in several ways, as in the interlocking NE-SW bands of different gneisses, showing ancient folding (Keith) in the southern extension of the Pre-Cambrian basement complex in the Appalachian and Piedmont systems. In the western part of the Canadian shield, north of Lake Superior, the Pre-Cambrian folds have assumed an E-W direction. Likewise the outcrops of Pre-Cambrian rocks south of Lake Superior in Wisconsin etc. show a predominant E-W direction. In the Ozark Mountain Pre-Cambrian nucleus however, a NW direction (strike N 50°W) is already recognizable; likewise in the Black Hills and farther south in the Arbuckle uplift in Indian territory (strike WNW) and in Texas, in Burnet county; where a monadnock of the ancient Laurentian land projects with a NW strike of the major folds in the Pre-Cambrian rocks. In the southern Rocky Mountains the Pre-Cambrian masses strike from S to N but exhibit a tendency to turn NNW to NW.

It thus appears that there can be recognized in the Pre-Cambrian base of North America a uniform structure consisting of a NE system of folds in the east, that however does not abut, as in Asia, against a NW system, but gradually swings into it.

In Africa large areas of the Pre-Cambrian basement complex are exposed in the Sahara Desert and south, throughout Central Africa to the Red Sea and across it into Arabia. In the Sahara and western Africa as far as the Kalahari Desert, the strike of the folds is reported to be uniformly N-S, as far as known today, with variations in the east to NNW etc. Likewise in West Australia the direction of the folded areas of metamorphosed Pre-Cambrian rocks is N-S with a slight trend to W; and, as far as we could find, the northerly direction is also present in Madagascar with variations to NE. It thus appears that there was a third immense tract of uniform Pre-Cambrian folding extending from West Africa to Australia.

There are a number of smaller areas of Pre-Cambrian rocks known that have not been folded since Pre-Cambrian time. These are in Asia, found in Cambodia, Borneo and India. The latter shows in the Arvali Mountains Pre-Cambrian folds striking NNE (N 36°E) and extending within 65 Km. of the Himalaya Mountains which from the north have overridden the Pre-Cambrian Mountain system. Farther south there is also a younger group (Algonkian?) of folds with E-W direction. These Indian Pre-Cambrian folds, especially as seen in the Arvali Mountains, are entirely distinct from the Asiatic system and probably referable to the African system. There is likewise in Bohemia a Pre-Cambrian complex, striking NE for the most part, that is independent of the Russian plate and the Baikal direction. In America the Colorado mass, exposed in the Grand Canyon and south, represents a body of Archean and Algonkian folds with a NE direction and distinctly outside of the Laurentian system of North America.

In South America, NW to W striking folds in rocks of Archean aspect north of the Amazonas and in middle and western Brazil are all of younger than Cambrian age. It is however possible that the N-S (NNE) direction of the folds of Archean rocks in the Sierra do Mar and S. de Mantigueira in eastern Brazil can be correlated with the N-S folds in Africa and Australia. In Argentina, south of Buenos Ayres, mountain ranges composed of granite and gneiss with a SE strike represent, according to Suess, a continuation of the Brazilian basement complex.

The Antarctic Continent has afforded rocks of Archean aspect and may represent another shield; so far, however, these rocks have been found only in loose blocks.

The most important fact standing out from these data is that of the existence of at least three vast areas of Pre-Cambrian rocks of a supercontinental order of size, with uniform structure. One of these comprises Eurasia, exclusive of India and parts of northern and middle Europe; another Africa, West Australia and possibly India and eastern Brazil; the third North America with Greenland and the North Atlantic region to the Shetland Islands, but probably exclusive of the Colorado nucleus.

It will be a matter of further inquiry whether these wide tracts of supercontinental size that responded as units to the diastrophic forces that mark the world-wide Archeozoic and Algoman revolutions are to be considered as the first expressions of the gigantic continents that we find in Paleozoic time in the sense that they are ancestral or arch-continents. Indeed they well correspond to the later continents, as seen in the north Atlantic extension of North America across Greenland towards Europe; the Afro-Australian continent (Gondwanaland) and the Eurasian continental mass of Paleozoic times (see paleogeographic maps by Lapparent, Frech, etc.); the oceans that now separate the continents having arisen by a gradual collapse of the earth crust, probably along great circles.

There is, however, this further possibility to be considered; that the NE and SE Pre-Cambrian fold systems of Eurasia and North America form one belt and the N-S fold system of the equatorial regions in Africa and Australia another one. In that case we should have, on the northern hemisphere, surrounding the North Pole, a continuous belt that consists of two pairs of directions, each composed of a NE and NW striking system of folds; the whole forming a zigzagged belt; and another belt of N-S folds approximately following the equator. Between the two belts lie the independent blocks of India, Bohemia, Colorado, etc., all, or most of them with a NE direction as if, in some way, leading from the equatorial belt up to the circumpolar belt. It will probably have to be left to future observations whether there are indications of a corresponding circumpolar belt on the southern hemisphere as slightly suggested by the occurrence in Argentina.

If these belts actually exist as sketched here (see chart), then the question arises whether they do not represent cosmic agencies that influenced the

whole earth crust. There come to mind the suggestions of George Darwin as to the possibility of tidal waves in the earth crust as orogenic forces that would cause an equatorial N-S direction in the folds and a NE direction in the northern hemisphere; and those of Douvillé and Prinz who would explain, one the prevailing E-W, the other the N-S directions of many of the mountain ranges of post-Proterozoic time by a former greater velocity of revolution (Suess). It would seem that these belts of Pre-Cambrian folds lend themselves still more readily to an explanation by one or the other of these factors than do the post-Proterozoic more local fold systems. Since the Pre-Cambrian folding both in America and Eurasia has a southerly component (thrust from SE and SW), a retardation of the revolution of the Earth and a resulting wandering of the crust towards the poles seems to be indicated.

On the other hand, it must also be asked whether the world-wide folding of the Archean basement complex could not be explained by simply terrestrial forces. In this connection the result of close mapping of the Pre-Cambrian folds, carried out in late years in Bohemia and Scandinavia is of great importance. It brings out closely compressed folds, whose strikes are tortuous and wavy curves and often subcircular and even angularly broken lines. This, it has been concluded, points to a tangential pressure, acting from all sides on an earth crust of fairly uniform composition (Uhlich), a pressure and a composition that could be found only in Pre-Cambrian or rather Archean time, and that means a uniform contraction of the entire earth crust such as could not be invoked for the post-Proterozoic mountain systems. If this view should supply a competent explanation for the world-wide Archean folding, it still leaves unaccounted for the presence of large systems of uniform folding, which, as we have already seen suggests the view that in Proterozoic and even in Archeozoic time the crust was separated into masses that correspond in position if not in area and configuration to the continents of Paleozoic and more recent time.

THE INORGANIC CONSTITUENTS OF LOBSTER SHELLS

By Frank Wigglesworth Clarke and George Steiger

U. S. Geological Survey, Washington

Read before the Academy, November 18, 1918

In the course of an extensive investigation relative to the inorganic constituents of marine invertebrates, by Clarke and Wheeler,[1] it was found that among the distinctly magnesian organisms the proportion of magnesia was dependent upon the temperature of the water in which the animals live. The cold water animals contained much less magnesium carbonate than those

from warm waters; a relation which was strikingly manifest in the analyses of echinoderms and alcyonarians and which has been amply verified by a considerable number of new analyses made since the original memoir was published. In other groups of organisms the same relation was suggested, but not actually proved to hold, for there were exceptions that needed explanation. In a series of eleven analyses of crustaceans (crabs, lobsters, shrimps etc.), the same variation in magnesia was strongly indicated, but with irregularities which appeared to require further investigation. It was conceivable that different parts of a shell or skeleton might differ in composition, or else that variations might be due to differences in age. It had already been found in the case of two sea urchins that the spines contained much less magnesia than the main body of the shells, but the question relative to age remained to be investigated.

Through the kindness of Dr. H. M. Smith, director of the U. S. Bureau of Fisheries, the large claws of two lobsters (*Homarus americanus*) from a single locality, Boothbay Harbor, Maine, were obtained, one from a small lobster, the other from a large specimen. The analyses gave the following results, after rejecting organic matter and water and recalculating to 100 %

	SMALL LOBSTER	LARGE LOBSTER
$SiO_2 + (Al, Fe)_2O_3$	0.19	0.81
$MgCO_3$	6.02	11.51
$CaCO_3$	80.52	64.37
$CaSO_4$	1.29	1.85
$Ca_3P_2O_8$	11.98	21.46
	100.00	100.00

The difference between these two analyses is very great, the large animal being much more highly magnesian and phosphatic than the small one. Unfortunately, however, the actual sizes of the two lobsters were not given, and more precise data were evidently desirable. Accordingly Dr. Smith had fragments from three lobsters sent to us, all from the same station as the others, with definite figures as to length and weight. The fragments, moreover, in each case represented both the large claw and the carapace, so that variations in the individual as well as variations in age could be determined. The analyses, six in number, were as follows:

1. Small lobster, length 8½ inches, weight 10 ounces.
2. Medium lobster, length 11½ inches, weight 2 pounds.
3. Large lobster, length 16½ inches, weight 5½ pounds. The claw is indicated by a, the carapace by b.

	1a	1b	2a	2b	3a	3b
$SiO_2+(Al,Fe)_2O_3$	0.33	0.35	0.36	0.66	0.31	0.57
$MgCO_3$	10.81	7.74	11.28	8.12	10.99	8.77
$CaCO_3$	72.41	78.98	55.46	70.58	56.89	65.14
$CaSO_4$	1.24	1.23	2.12	1.58	2.32	2.32
$Ca_3P_2O_8$	15.21	11.70	30.78	19.06	29.49	23.20
	100.00	100.00	100.00	100.00	100.00	100.00

In each case the claw is richer in magnesium carbonate and calcium phosphate than the carapace. The variations due to age appear more distinctly when the average of each pair of analyses is taken, as follows:

	1	2	3
$MgCO_3$	9.27	9.70	9.88
$CaCO_3$	75.69	68.02	61.01
$Ca_3P_2O_8$	13.45	24.92	26.35
$CaSO_4$	1.24	1.85	2.32

Here the progressive increase in magnesium carbonate and calcium phosphate is clearly shown; and it also appears in the percentages of calcium sulphate, although the last detail is less significant. The smallest lobster, moreover, differs in the composition of its inorganic portion from that of the two larger animals much more than they do from each other.

From the evidence now at hand it seems clear that some of the departures from regularity in the proportions of magnesium carbonate in the shells or skeletons of marine invertebrates are due to one or both of the two causes which were suggested at the beginning of this paper. It is desirable, therefore, in further investigations of this kind, that in the study of the more highly specialized organisms the analyses should represent the totality of the inorganic portions, and that animals of the same degree of maturity should be taken. With the lower classes of organisms the difficulties are not so great, and regularities are much more easily discovered.

(Published by permission of the Director of the U. S. Geological Survey.)

[1] Clarke and Wheeler, *Prof. Paper*, No. 102, *U. S. Geological Survey, Washington.*

FLAGELLATE AFFINITIES OF TRICHONYMPHA

By Charles Atwood Kofoid and Olive Swezy

Zoölogical Laboratory, University of California

Communicated by W. M. Wheeler, November 13, 1918

The methods of division among the Protozoa are of fundamental significance from an evolutionary standpoint. Unlike the Metazoa which present, as a whole, only minor variations in this process in the different taxonomic groups and in the many different types of cells in the body, the Protozoa have evolved many and widely diverse types of mitotic phenomena, which are characteristic of the groups into which the phylum is divided. Some striking confirmation of the value of this as a clue to relationships has been found in recent work along these lines. The genus *Trichonympha* has, since its discovery in 1877 by Leidy,[1] been placed, on the one hand, in the ciliates and, on the other, in the flagellates, and of late in an intermediate position between these two classes, by different investigators. Certain points in its structure would seem to justify each of these assignments. A more critical study of its morphology and especially of its methods of division, however, definitely place it in the flagellates near the Polymastigina.

At first glance *Trichonympha* would undoubtedly be called a ciliate. The body is covered for about two-thirds of its surface with a thick coating of cilia or flagella of varying lengths, which stream out behind the body. It also has a thick, highly differentiated ectoplasm which contains an alveolar layer as well as a complex system of myonemes. The nucleus, however, is that of a typical flagellate. The flagella may equally well be called cilia, since they are arranged in longitudinal rows on the surface of the body, each arising from a minute basal granule imbedded in the ectoplasmic layer. Each basal granule, however, is connected with a fine fibril arising from the myonemes in the ectoplasm. The myonemes form a closely anastomosing network over the body, taking their origin from a complex structure at the anterior end which we call the centroblepharoplast (fig. 1, *c*). This corresponds to the blepharoplast of *Trichomonas* (fig. 6, *c*). The entire group of flagella are thus bound together into one integrated unit, the basis of which is the centroblepharoplast, forming the neuromotor system.

This integrated organelle system is found in a simple form in *Trichomonas* (fig. 6) where it consists of a centrosome-blepharoplast (*c*) imbedded in the anterior end of the axostyle (*ax*). From it arise the flagella and the undulating membrane with its marginal filament and parabasal body.

A more complex form of the same system and one which in some features leads toward the stage attained by *Trichonympha* with its much greater multiplicity of flagella, is shown in the motor organelle complex of *Giardia*.[2]

Here we find eight distinct flagella arising from different points on the surface of the body, but connected internally by a number of fibers which all take their origin from the centroblepharoplast complex attached to the anterior ends of the axostyles. The condition in *Giardia* differs from that in *Trichonympha* mainly in the small number of parts of its motor organelle complex. Correlated with the very great increase in the number of flagella in the latter species, is the vast increase in the number of distinct myonemes which have been developed. This has also necessitated an enlargement and increased complexity in the centroblepharoplast complex at the anterior end of the body. These structures still remain, however, distinctly flagellate in their structural relationships, though superficially resembling the ciliary coating found in the holotrichous Ciliata.

The mode of division found in *Trichonympha* offers still more striking proof of its flagellate affinities, and also emphasizes the wide divergence existing between it and the ciliates. As the name signifies the centroblepharoplast complex takes the rôle of centrosome in mitosis. In the prophase this entire structure divides by longitudinal splitting, followed by a division of the ectoplasmic part of the body, including a separation of the myonemes and flagella into two parts (fig. 2). As the centroblepharoplast divides the two moieties spin out a darkly staining band between them, the paradesmose (*par*).

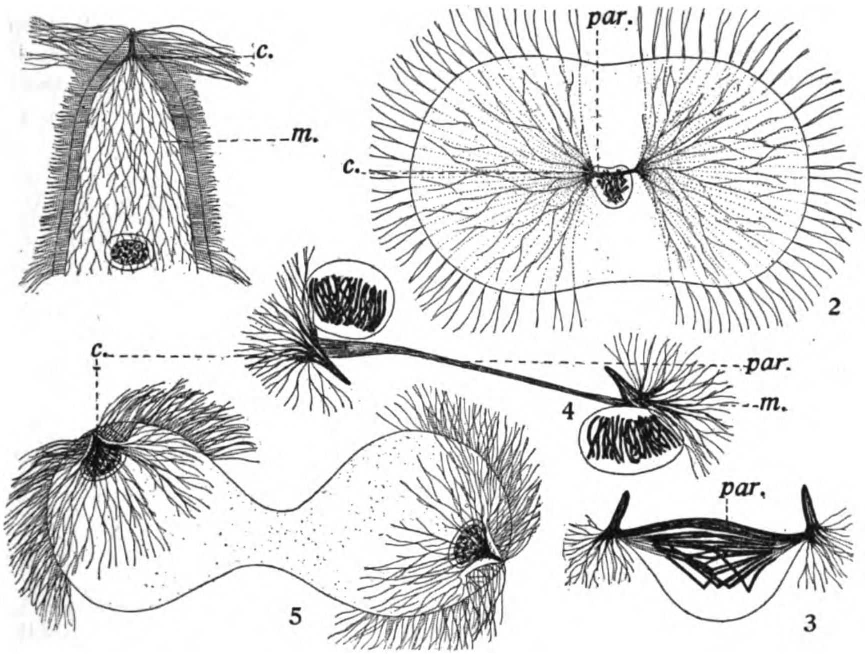
The nucleus migrates from its submedian position to the anterior part of the body. A precocious splitting of the chromosomes has taken place during the vegetative phase and the fifty-two chromosomes thus formed are reunited, forming twenty-six V-shaped chromosomes composed of distinct chromomeres (fig. 2). As the nucleus reaches the paradesmose it elongates until its length coincides with that of the paradesmose. The nuclear membrane remains intact throughout the entire process of division. Spindle fibers are formed from the ends of the paradesmose, or the centroblepharaplasts, pass through the nuclear membrane and become attached to the chromosomes, a fiber from one pole to one end while the other is connected with the opposite pole. With the shortening of the spindle fibers the chromosomes are straightened out in the equatorial plate stage (fig. 3).

In all these mitotic figures the divided centroblepharoplast functions as the centrosome, while the myonemes, surface ridges and flagella are found attached to each half and streaming out from it like the astral rays in mitosis, in a metazoan nucleus. The paradesmose lies above the nucleus, that is, towards the surface of the body, outside the nuclear membrane but partly enfolded within it.

In the telophase the constriction of the nuclear membrane takes place with the chromosomes still showing their attachment to the spindle fibers. The chromosomes are never drawn close to the poles as in mitosis in *Trichomonas*, but the fibers fade out before reconstruction begins in the nucleus

(fig. 4). As the daughter nuclei round up they become free from the paradesmose which still connects the two daughter blepharoplasts (fig. 4). Later the paradesmose loses its staining reactions and fades out or is absorbed, either by the cytoplasm or by the centroblepharoplasts. This is followed by division of the body and separation of the two daughter flagellates.

A comparison of various points of this process with certain ones in *Trichomonas* reveals a striking similarity in the two types. In the latter the

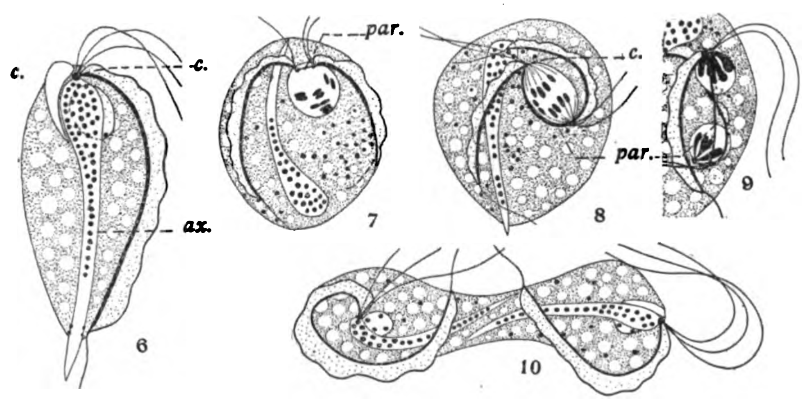


FIGS. 1–5. ***Trichonympha campanula*** **Kofoid and Swezy.** ***1.*** **Anterior portion of trophozoite showing three groups of flagella, centroblepharoplast, myonemes and uncleus. × 225.** ***2.*** **Late prophase of division. Ectoplasmic structures and flagella divided; paradesmose connecting the centroblepharoplasts; 26 split chromosomes. ×225.** ***3.*** **Metaphase of nucleus, × 600.** ***4.*** **Telophase. Centroblepharoplasts still connected by paradesmose. × 600.** ***5.*** **Mitosis completed, cytoplasmic division approaching. × 225.**

division of the centroblepharoplast also produces a connecting paradesmose (fig. 7, *par.*). Attached to each new centrosome or centroblepharoplast are the motor organelles, which are produced partly by division of the old structures and partly by new outgrowths. The condition here is similar to that in *Trichonympha* (fig. 2). A precocious splitting of the chromosomes produces ten moieties which become reunited at the time of spindle formation, giving five as the number of chromosomes going to each daughter cell. The same process also occurs in *Trichonympha*.

The spindle fibers are formed from the ends of the paradesmose or the centroblepharoplasts, pass through the nuclear membrane, which remains intact, and the chromosomes become arranged upon the spindle in the equatorial plate by a process analogous to that found in *Trichonympha*. The paradesmose is found outside the nuclear membrane (figs. 8, 9) and fades out in the late telophase precisely as does that of the other species.

It is thus evident that these two organisms, though differing widely in their general morphological characters, yet follow precisely similar modes of mitosis. These similarities consist in the following; Division in both is fundamentally longitudinal, including the splitting of the chromosomes, the centroblepharoplast and the entire body. The chromosomes are precocious in their splitting, doubling the number which later goes on the spindle. The



FIGS. 6–10. *Trichomonas angusta* Alexeieff, after Kofoid and Swezey.[4] × 1500. *6* Active motile form prior to division. *7.* Late prophase with five pairs of split chromosomes centroblepharoplasts connected by paradesmose. *8.* Late metaphase; paradesmose outside nuclear membrane. *9.* Telophase with centroblepharoplasts still connected by paradesmose. *10.* Mitosis completed, cytoplastic division approaching. *Ax.*, axostyle; *c.*, centroblepharoplasts; *m.*, myonemes; *par.*, paradesmose.

centroblepharoplast divides with a paradesmose formed to connect the two parts as they separate. The motor organelles are divided with new outgrowths to complete the full complement of each daughter cell. These remain attached to the centroblepharoplasts throughout the entire process of division. The nuclear membrane persists, with the spindle fibers passing through from the ends of the paradesmose, which remains outside the membrane. Pseudosynapsis occurs as part of the chromosome cycle in both forms. The paradesmose persists through the late telophase stage after the rounding up of the daughter nuclei, fading out before plasmotomy occurs.

Two phases of these processes deserve particular attention. These are the formation of the paradesmose and the attachment of the motor organelles to the centrosomes. So far as present records go both of these phenomena are

typically and solely flagellate in their occurrence. Nothing comparable is known to occur in the ciliates in which at mitosis the cilia bear no morphological relation to the centrosomes or to any other part of the mitotic figure.

In the group of flagellates comprised in the order Polymastigina, the paradesmose is a characteristic feature of mitosis in so far as these processes have been studied. In the trichomonad flagellates, as pointed out above, its occurrence is constant and characteristic. In the nearly related form, *Hexamitus*, the paradesmose is also present.[3] A peculiar development of this structure is found in the so-called sphere of *Noctiluca*. This is produced by the elongation and division of a mass of differentiated archoplasm outside the nucleus. The exact relation of this to the tentacle and flagellum of *Noctiluca* has not been determined, but its position would suggest that some close connection between them exists.

It has been pointed out in our earlier paper[4] on trichomonad flagellates that the paradesmose is not a precise homologue of the centrodesmose or central spindle of the metazoan type of mitosis. It is the result of two peculiar specializations in trichomonad division, the continuity of the nuclear membrane which excludes it from the typically axial position of the central spindle, and the connection of the centrosome-blepharoplast with the entire motor organelle complex. The latter feature is probably the most important one in its bearing on the development of the paradesmose, the strain on the spindle resulting from the constant activity of a group of flagella attached to each pole, requiring the development of a stronger support than is found in the mitotic structures where this condition does not prevail. As a result of this necessity we find a paradesmose developed in those flagellates where the flagella form part of the mitotic figure, but it is usually, if not always, absent where this does not occur.

In no other group of Protozoa do we find a structure approximating the paradesmose of the flagellates. Among the ciliates division of the nucleus occurs as distinct phenomena separated from, though synchronous with, the division of the motor organelles and other structures of the body.

Another point in the division process of *Trichonympha* which is equally important as those mentioned above, is the fact that division is longitudinal. This is shown in the longitudinal splitting of the centroblepharoplast and the ectoplasmic structures. This fact also serves to separate it from the ciliates where transverse division is general, and allies it with the flagellates.

The occurrence of these peculiar flagellate specializations in *Trichonympha* would therefore preclude the possibility of any ciliate affinities for that genus. This conclusion receives confirmation from a careful analysis of its morphological features which, though superficially ciliate in appearance, yet are fundamentally flagellate in their character and relationships.

The relations of other members of the group of curious and peculiar organisms which, in company with *Trichonympha*, are parasitic in the intestinal

tract of the termites, have also been obscured by the high degree of specialization shown in their motor organelles. In one of these, *Joenia*, Grassi and Foa,[5] have figured longitudinal division with the formation of a prominent paradesmose which persists until the daughter cells are ready for plasmotomy. The paradesmose here, as in other flagellates, is intimately connected with the flagella and their related neuromotor structures.

In *Lophomonas* the old motor organelles disappear, according to Janicki[6] who has described division in this form. An entire new motor organelle system is developed from the ends of the paradesmose after completion of division of the nucleus. The exact origin of the paradesmose in this case cannot be determined from the figures of Janicki, but it arises from the nuclear-neuromotor complex. It remains outside the nuclear membrane, hence is evidently a true paradesmose.

In the remainder of this group mitotic phenomena have not yet been described and it is in their morphology only that we must look for relationships. These are found in the relations of flagella, internal myonemes and centroblepharoplasts with their various modifications, the neuromotor system. In *Spirotrichonympha* Grassi[7] has figured, in these structures, relations which are comparable with those found in *Trichonympha*, although much simpler. The flagella are arranged in spiral courses around the body starting from the anterior tip. Each series is accompanied by a slender band or myoneme extending along the line of basal granules beneath the surface of the body. The number of lines of flagella and myonemes varies slightly in the different species. At the anterior tip of the body these are joined in a small granular mass, the centroblepharoplast.

Zulueta[8] has figured the same spiral myonemes in *Dinenympha* without the series of flagella outlining their course. A single flagellum arises from the end of each myoneme. At the time of division the small granular mass at the anterior tip of the body divides, each moiety taking four of the myonemes, and forming a paradesmose between them as they separate. This centroblepharoplast acts as the centrosome in the formation of the spindle for the division of the nucleus, with its attached myonemes taking the place of the astral rays, as in *Trichonympha* (fig. 2).

The same relations of motor organelles and internal myonemes, by means of which an integrated neuromotor system is formed, may be found throughout all the members of this group of organisms. The range in complexity extends from the trichomonad type of structure to that shown in *Trichonympha*, which exhibits a higher degree of specialization and development than do many of the lower Metazoa. This specialization is confined almost exclusively to the motor organelles and the accessory structures connected with them, the neuromotor system. At the time of division this acts as a unit, dividing and half going to each daughter cell. An apparent exception to this is found in *Lophomonas*, but here a part of the old neuromotor system

is evidently retained in the paradesmose and from it the new motor organelles of each daughter cell are formed.

In these structures and more particularly in the processes by means of which they are passed from one generation to the next, we have a clue to relationships that seems to be more fundamental and significant in its importance than are external features, which are constantly subject to modifications through environmental changes. The amount of specialization has in no way changed the basic facts of these relations and processes. Thus in the highly complex *Trichonympha* there are precise homologues with the simpler features in *Trichomonas* with its simple centroblepharoplast granule and few flagella.

The superficial resemblance between the trichonymph parasites of the termites and the ciliates is the result of the high degree of specialization and evolutionary development to which the former have attained, and is not indicative of a derivative relationship of even the most remote kind between the two. The Trichonymphidae are fundamentally and characteristically flagellate in their type of structure as well as in their methods of division. We may therefore dismiss completely the early conception of Leidy, Kent and their followers that the Trichonymphs are ciliates and revise our wide-spread conception that flagella are universally or even characteristically few in number. These protoplasmic processes are flagella primarily because of their relation to the nucleus and the mitotic figure. Flagella are attached directly or indirectly to the centrosome and share in mitotic processes. Cilia are not thus attached and have no correlated part in mitosis. Numbers contribute no necessary part of this definition of flagella, they apparently do of cilia.

We may also dismiss the later conception of Hartmann[9] that the Trichonymphs are an intermediate group between flagellates and ciliates. In the first place a transition type between these primary groups can not be expected as parasites of a highly organized group of social insects. The appearance of transition is illusory, depending on superficial structures and numbers merely, while the deeply significant mitotic process and its structures are unequivocally flagellate in nature. We therefore reject Hartmann's[9] transitional conception and with it his Trichonymphida and retain Grassi's Hypermastigina as the fitting as well as the legitimate designation for this most highly specialized group of the flagellates.

[1] Leidy, J., *Proc. Acad. Nat. Sci., Philadelphia*, (Ser. 2), **7**, 1877, (146–149).

[2] Kofoid, C. A., and Christiansen, E. B., *Univ. Calif. Publ. Zool., Berkeley*, **16**, 1915, (30–54), pls. 5–8, 1 fig. in text.

[3] Swezy, O., *Ibid.*, **16**, 1915, (71–88), pls. 9–11.

[4] Kofoid, C. A., and Swezy, O., *Proc. Amer. Acad. Arts Sci., Boston*, **51**, 1915, (289–374), pls. 1–8, 7 figs. in text.

[5] Grassi, B., and Foa, A., *R. C. R. Acad. Lincei, Rome*, (See. 5), 13, 1904, (241–253), 17 figs. in text.
[6] Janicki, C., *Zs. wiss. Zool., Leipzig*, 95, 1910 (243–315), pls. 6–9.
[7] Grassi, B., and Sandias, A., *Galatola, Catania*, 1893, (1–151), pls. 1–5.
[8] Zulueta, A. de., *Trab. Nac. Cienc. Nat., Madrid*, 23, 1915, 25 pp., 1 pl. 6 figs. in text.
[9] Hartmann, M. *Hertwig's Festsch.*, 1, 1910 (349–392), pls. 27–30, 3 figs, in text.

THE TERNARY SYSTEM $CaO-MgO-SiO_2$

By J. B. Ferguson and H. E. Merwin

Geophysical Laboratory, Carnegie Institution of Washington

Communicated by A. L. Day, November 25, 1918

A number of investigations dealing with one or more of the four oxides, lime, alumina, magnesia and silica, have in recent years been carried out at the Geophysical Laboratory of the Carnegie Institution of Washington, as a preliminary step in the study of the rocks and minerals of the earth's crust. These studies[1] include all of the possible binary systems and three of the four possible ternary systems which may be constructed from these oxides. The fourth ternary system has been studied only in part. We desire in this paper to present a summary of the results we have obtained in a study of the remainder of this fourth ternary system, and also to correlate these results with the results previously obtained.

The experimental methods employed are similar to the methods used in the previous investigations at this laboratory. Samples of known compositions were prepared by fusing together in platinum crucibles weighed amounts of chemically pure calcium carbonate, magnesia and silica and subsequently reducing the samples to fine powders. The production of homogeneous samples usually necessitated the repeating of this process once or twice. The investigation of a sample was conducted as follows: a small quantity of the sample was wrapped in a piece of platinum foil which was about 1 sq. cm. in area and this charge was tied by a fine platinum wire to a small ring of marquardt porcelain. The porcelain ring was hung on a fine platinum wire, the ends of which were connected to two stout platinum leads. A marquardt porcelain tube carried these leads and also a platinum-platinrhodium thermoelement the junction of which was not more than a few millimeters from the charge. The charge, thermoelement, etc., were inserted into a hot platinum resistance furnace and the furnace temperature was regulated in such a manner as to maintain the charge at a predetermined temperature for a sufficient time to allow an equilibrium condition to be attained (usually from fifteen to thirty minutes but sometimes as long as forty-eight hours). The wire supporting the ring and the charge was then fused by passing an electric cur-

rent through it, and the charge fell into a mercury bath which instantly chilled it. When cold the charge was opened and the content was examined under a petrographic microscope by means of which the phases present could be identified.

The results of the hundreds of heat treatments which were carried out during this study cannot be presented in detail in a paper as brief as this one must be, but we hope in a general way to indicate their character.

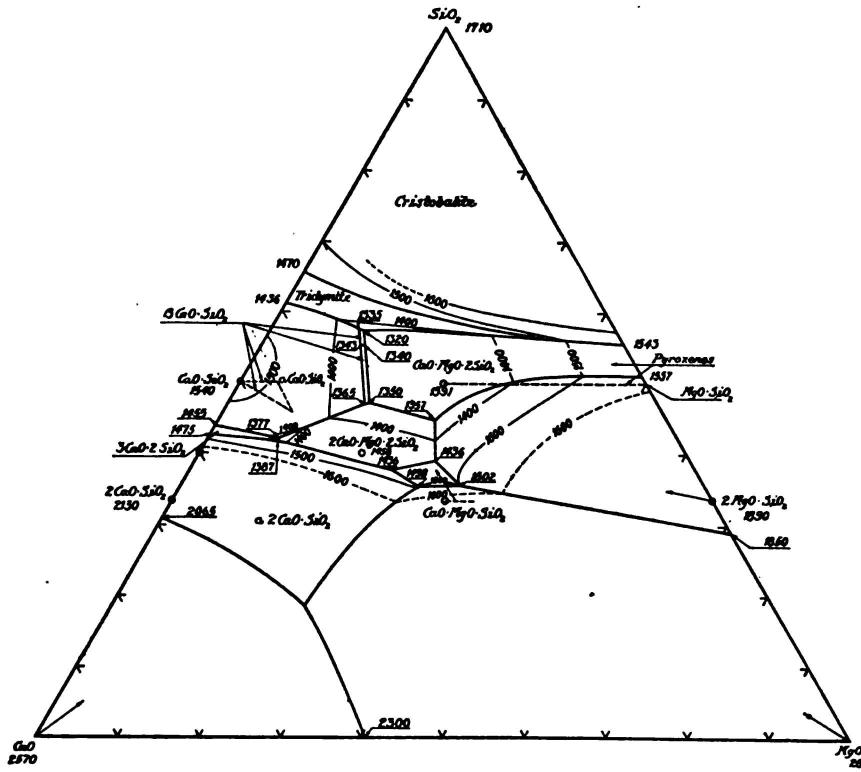


FIG. 1. THE TRIANGULAR CONCENTRATION DIAGRAM IN MOL-PER CENT

The heavy lines are the boundaries of the fields, the solid light lines are the isotherms which are readily measured, the lighter dotted lines are the high temperature isotherms which are not so accurately known and the heavier dotted lines are limits of the solid solutions. The solid solutions may also be distinguished from the isotherms in that they have arrows which indicate the fields which correspond to the respective solid solutions. The temperatures given correspond to (1) the quintuple points, (2) the quadruple points, (3) the melting points of the compounds and (4) the melting points of the component oxides.

The phases which occur in the three main binary systems and which also have fields in the ternary system are: (1) Cristobalite, SiO_2, (2) Tridymite, SiO_2, (3) Pseudowollastonite, α $CaO.SiO_2$, (4) Tricalcium di-silicate, 3CaO. $2SiO_2$, (5) α Calcium orthosilicate, α $2CaO.SiO_2$, (6) Lime, CaO, (7) Periclase, MgO, (8) Forsterite, $2MgO.SiO_2$.

The binary compounds which do not have fields in the ternary system are: (1) Tricalcium silicate, $3CaO.SiO_2$, (2) Clino-enstatite, $MgO.SiO_2$.

The ternary compounds which have fields in the ternary system are: (1) Diopside, $CaO.MgO.2SiO_2$, (2) $2CaO.MgO.2SiO_2$.

The additional phases which have fields in the ternary system are not of constant composition but are ternary in character. They are:

1. The wollastonite ($\beta CaO.SiO_2$) solid solutions. These consist of either two series of solid solutions or an area of solid solution. The one series may contain up to 18 mols. per cent of diopside, $CaO.MgO.2SiO_2$ and the other up to 44 mols. per cent of the compound $2CaO.MgO.2SiO_2$. If an area of solid solution exists, it will probably extend from the one series to the other forming a triangular area with the apex at the $CaO.SiO_2$ composition. The wollastonite-pseudowollastonite inversion, $\beta CaO.SiO_2 \rightleftarrows \alpha CaO.SiO_2$ normally occurs at 1200°C. but this inversion temperature is raised by the addition of the dissolved substances reaching a maximum of 1343°C. with the diopside series and of 1365°C. with the $2CaO.MgO.2SiO_2$ series. Only those solid solutions which invert at the higher temperatures occur as primary phases since the liquidus at no point falls below 1320°C.

2. The pyroxene solid solutions which form a continuous series with end members, diopside, $CaO.MgO.2SiO_2$ and clino-enstatite, $MgO.SiO_2$, all occur as primary phases.

3. The monticellite ($CaO.MgO.SiO_2$) solid solutions which may contain up to 11% of forsterite, $2MgO.SiO_2$, are partly represented as primary phases. The pure compound monticellite probably does not occur, as a phase, stable in contact with a liquid.

Of the phases which have just been enumerated the $\beta CaO.SiO_2-2CaO.MgO.2SiO_2$ solid solutions, and the compound $2CaO.MgO.2SiO_2$ have not been previously noted. This latter compound has practically identical optical properties with the mineral åkermanite for which the formula $8CaO.4MgO.9SiO_2$ has been proposed, by Schaller.[2]

The summarized temperature and concentration relations are shown in figure 1. The compositions are herein represented on an equilateral triangle in mol-per cent, and the diagram includes (1) the location of the fields, with their boundary curves, (2) the location of the invariant points, (3) the temperatures which correspond to the fixed points, and (4) the isotherms which indicate the slopes which the various fields have on a solid model upon which the temperatures of complete melting are represented as vertical distances above a triangular concentration diagram similar to the one shown.

[1] Rankin, G. A., and Wright, F. E., *Amer. J. Sci., New Haven*, **39**, 1915, (1). Rankin, G. A., and Merwin, H. E., *J. Amer. Chem. Soc., Easton, Pa.*, **38**, 1916, (568). Rankin, G. A., and Merwin, H. E., *Amer. J. Sci.*, **45**, 1918, (301). Bowen, N. L., *Ibid.*, **38**, 1914, (207). Allen, E. T., White, W. P., Wright, F. E., Larsen, E. S., *Ibid.*, **27**, 1909, (1).

[2] Schaller, W. T., *Bull. U. S. Geol. Survey, Washington*, No. 610, 1916.

QUANTITATIVE RELATIONS BETWEEN CHROMATIN AND CYTOPLASM IN THE GENUS ARCELLA, WITH THEIR RELATIONS TO EXTERNAL CHARACTERS

By R. W. Hegner

School of Hygiene and Public Health, Johns Hopkins University

Read before the Academy, November 18, 1918. Communicated by H. S. Jennings.

A problem that has attracted considerable attention during the past twenty years is that of the quantitative relations between the nucleus and the cytoplasm of animal cells.

According to Richard Hertwig, who has been the foremost advocate of the nucleo-cytoplasmic-relation theory, a balance between nuclear and cytoplasmic masses exists in the normal cell, this balance being due to the interchange of materials between nucleus and cytoplasm. This state of equilibrium may be disturbed by such factors as changes in temperature, over-feeding or starvation. The result of these disturbing agents is an excess of nuclear over cytoplasmic materials. This excess of nuclear material leads to the depression of the cell, which finally ends in death unless normal mass relations are re-established in some way. The restoration of the normal equilibrium may be attained by the giving up of nuclear material to the cytoplasm, by ordinary cell division, or by the addition of a foreign element through conjugation. Hertwig and others have attempted to account for many of the complicated stages in the life cycles of Protozoa with this theory, but while this hypothesis is apparently applicable to many phenomena it will not bear close analysis, and a review of the extensive literature on this subject reveals a fatal lack of data on which to base the theory.

The material that I have used in my investigations consisted of several species of shelled fresh-water Protozoa of the genus Arcella. Arcella dentata has a shell with tooth-like projections extending out from the periphery. This shell averages about 130 microns in diameter and 50 microns in thickness.

There is a circular mouth opening in the ventral wall of the shell, through which protoplasmic extensions are pushed out that serve as locomotor organs and for capturing food. The cytoplasmic body within the shell contains two nuclei situated on opposite sides of the mouth opening. These nuclei are of the vesicular type, with the chromatin aggregated into a spherical mass in the center. Since the shell is almost transparent, especially in young specimens, it is easy to measure both the entire nucleus and the chromatin-mass within it, in the living animal. This makes Arcella peculiarly favorable for nucleo-cytoplasmic studies. The usual method of reproduction is binary division. When a certain amount of protoplasm has been built up within the shell and environmental conditions are favorable, the cytoplasm protrudes from the mouth of the shell and secretes a new shell; then, approximately one-

half of the protoplasm passes back into the old shell, and the two shells separate. It is thus possible to distinguish between parent and offspring.

The experiments performed on pure lines of Arcella dentata prove that a definite relation exists between nuclear number and cell size within each pure line.

One of the members of line 150 was cut in two so that each half contained a single nucleus. Both halves continued to live and reproduce; the offspring were slightly irregular in shape, but the spines could be counted easily and the diameter measured. These offspring were smaller than the original parent and possessed fewer spines. They were each provided with only one nucleus. From these two uninucleate half-specimens were derived 209 uninucleate descendants which had a mean spine number of 11 and a mean diameter of 116 microns.

After a number of generations that differed in different cases, all of the uninucleate specimens produced empty shells and became binucleate again. Apparently during this process the single nucleus divided into two, as in ordinary division, but the new shell that was formed, was cast off empty, and all of the cytoplasm and both nuclei were retained in the parent shell. The binucleate specimens thus formed, gave rise at once to binucleate offspring. These offspring were larger than the parent; and their offspring were still larger. In this way a gradual increase in diameter and in spine number took place generation after generation, until at the end of the third or fourth generation, an equilibrium was established and a mean diameter and spine number were regained characteristic of the line before the experiments were begun. Thus in line 150, the size of the organisms and the characteristics of the shell depend upon the number of nuclei, and each nucleus is accompanied by a rather definite quantity of cytoplasm.

Other lines of Arcella dentata were subjected to this and other kinds of operations and the data obtained fully confirm the conclusion just stated.

Arcella polypora differs from Arcella dentata in the absence of spines and in the greater number of nuclei. In this species the nuclei are distributed at approximately equal distances from one another. It was found that within a line derived by fission from a single specimen, the number of nuclei varied. In line 5 they varied from 3 to 7. When the diameters of these specimens were compared with the number of nuclei they contained, a remarkably high correlation was revealed. The mean diameter of specimens with 3 nuclei was 109 microns; of those with 4 nuclei, 113; of those with five nuclei, 120; of those with six nuclei, 127; and of those with seven nuclei, 130 microns. It is evident that as the number of nuclei increases the size of the organism increases, and that in this species as in Arcella dentata, a rather definite amount of cytoplasm accompanies each nucleus. No specimens with less than 3 nuclei appeared in the cultures, so operations were resorted to, in order to obtain individuals with one and two nuclei. Specimens were cut into

pieces, and these pieces continued to live and reproduce. From these pieces a few offspring with one and 2 nuclei were obtained. Specimens with one nucleus averaged 82 microns in diameter; and those with two nuclei, 86 microns. Here, as in normal specimens, size and nuclear number are closely correlated.

Other lines of Arcella polypora behaved in similar fashion, but the data show that the ratio between nuclear number and cell size differs markedly in the different lines. Thus the number of nuclei in line 34 ranged from 5 to 10, but the specimens were smaller than were those with a lesser number of nuclei in line 5. The specimens in line 5 with 4 nuclei were similar in size to those in line 34 with 8 nuclei; those in line 5 with 5 nuclei were about as large as those in line 34 with 9 nuclei; and those in line 5 with 6 nuclei, approximated in diameter those in line 34 with 10 nuclei.

This condition suggested the possibility that size in these organisms is controlled not by the number of nuclei, but by the *amount of chromatin* within the nuclei. Accordingly, measurements were made of the chromatin masses in specimen from a number of lines, and this hypothesis was confirmed in a remarkable manner.

Measurements were first obtained of specimens of Arcella dentata. It soon became evident that in this species, the larger the specimen the greater the amount of chromatin within the nuclei. A chromatin-cytoplasmic mass relation was thus established. Measurements were then made of the chromatin masses in specimens from lines 5 and 34 of Arcella polypora. From these measurements, the volumes of the chromatin masses were computed, and they were found to be greater in line 5 than in line 34. For example, the total volume of the 6 chromatin masses in an average specimen of line 5 proved to be approximately equal to the total volume of the 10 chromatin masses of an average specimen of the same size in line 34. Similar conditions were found to exist when the total volume of chromatin within other specimens of line 5 was compared with the total volume of chromatin within specimens of the same size in line 34. These data prove that the quantity of chromatin contained in specimens of the same size within the two lines, 5 and 34, was very nearly the same, regardless of the variations in the number of nuclei. Thus, in Arcella polypora, as in Arcella dentata, there is a certain amount of chromatin associated with a certain amount of cytoplasm, and a definite chromatin-cytoplasmic-mass-relation is shown to exist.

The data presented above not only show a mass relation between chromatin and cytoplasm, but also a relation between chromatin mass and external measurable characters. For example, in Arcella dentata, both the diameter of the shell and the number of spines are correlated with the chromatin mass; and in Arcella polypora, the diameter of the shell also depends upon the total volume of chromatin.

A summary of some selection work that I carried on last year with Arcella dentata was published in the October number of the PROCEEDINGS of this

Academy. I concluded from this work on selection, that from the descendants of a single specimen produced by binary division, lines could be distinguished that were hereditarily diverse as regards spine number and diameter. The present study indicates that these heritable diversities may have been due to changes in the volume of the chromatin. These chromatin-cytoplasmic studies also have a bearing on the selection work carried on by various investigators, notably by Jennings with Difflugia and by Root with Centropyxis. Arcella, Difflugia, and Centropyxis all belong to the same group of Protozoa, the fresh-water Rhizopods; but the nuclei can not be seen in either Difflugia or Centropyxis; and hence at least some of the results obtained by Jennings and by Root may have been due to changes in nuclear number and consequently in chromatin mass, rather than an hereditary change in the chromatin itself as seemed probable. An increase or decrease in nuclear number, however, does not account for simultaneous and independent diversities such as Jennings found in Difflugia with respect to shell diameter and length of spines, unless the assumption is made that certain nuclei exert an influence upon certain shell characters and other nuclei upon other shell characters. Evidence was obtained from my studies of Arcella polypora that hereditarily diverse strains with respect to nuclear number and shell diameter could be distinguished within a single line. More data regarding this and other related problems are very desirable.

A THEOREM ON POWER SERIES, WITH AN APPLICATION TO CONFORMAL MAPPING

By T. H. Gronwall

Range Firing Section, Aberdeen Proving Ground

Communicated by E. H. Moore, December 2, 1918

Note I on Conformal Mapping under aid of Grant No. 207 from the Bache Fund.

Theorem: Given a power series $w(z) = \sum_0^\infty a_n z^n$ *convergent for* $|z| < 1$ *and such that* $\sum_0^\infty n|a_n|^2$ *converges, and writing*

$$z = 1 - \rho e^{\theta i}, \ z' = 1 - \rho e^{\theta' i},$$

where $\rho > 0$ *and* θ *and* θ' *vary with* ρ *subject only to the conditions*

$$-\frac{\pi}{2} + \epsilon \leqq \theta \leqq \frac{\pi}{2} - \epsilon, \quad -\frac{\pi}{2} + \epsilon \leqq \theta' \leqq \frac{\pi}{2} - \epsilon,$$

ϵ *being positive but arbitrarily small, then*

$$w(z) - w(z') \rightarrow 0 \text{ as } \rho \rightarrow 0$$

uniformly in respect to θ *and* θ'.

It should be noted that no assumption is made regarding the convergence of the power series at $z = 1$. For the proof, we write $w(z) = w_1(z) + w_2(z)$, where

$$w_1(z) = \sum_0^{N-1} a_n z^n, \quad w_2(z) = \sum_N^{\infty} a_n z^n.$$

First, we have

$$w_1(z) - w_1(z') = \sum_1^{N-1} a_n (z^n - z'^n)$$
$$= (z - z') \sum_1^{N-1} a_n \, (z^{n-1} + z^{n-2} z' + \cdots + z'^{n-1})$$

and since $|z - z'| = \rho \, |e^{\theta i} - e^{\theta' i}| \leqq 2\rho$, $|z| < 1$ and $|z'| < 1$, it follows that

$$|w_1(z) - w_1(z')| \leqq 2\rho \sum_1^{N-1} n \, |a_n|. \tag{1}$$

In the second place,

$$|w_2(z) - w_2(z')| \leqq \sum_N^{\infty} |a_n| \, |z^n - z'^n| = \sum_N^{\infty} \sqrt{n} \, |a_n| \, . \, \frac{|z^n - z'^n|}{\sqrt{n}},$$

and applying Lagrange's inequality

$$|w_2(z) - w_2(z')|^2 \leqq \sum_N^{\infty} n \, |a_n|^2 \, . \, \sum_N^{\infty} \frac{|z^n - z'^n|^2}{n}$$
$$\leqq \sum_N^{\infty} n \, |a_n|^2 \, . \, \sum_1^{\infty} \frac{|z^n - z'^n|^2}{n}.$$

Denoting by the $\bar{z}$ the conjugate of z, we have

$$|z^n - z'^n|^2 = (z^n - z'^n) \, (\bar{z}^n - \bar{z}'^n) = (z \, \bar{z})^n + (z' \, \bar{z}')^n - (z \, \bar{z}')^n - (z' \, \bar{z})^n,$$

and consequently, since $|z| < 1$, $|z'| < 1$,

$$\sum_1^{\infty} \frac{|z^n - z'^n|^2}{n} = \log \frac{(1 - z \, \bar{z}') \, (1 - z' \, \bar{z})}{(1 - z \, \bar{z}) \, (1 - z' \, \bar{z}')}.$$

Introducing $z = 1 - \rho \, e^{\theta i}$, $z' = 1 - \rho \, e^{\theta' i}$, a simple calculation shows that for $\rho < \sin \epsilon$ and observing the limitations governing θ and θ',

$$\frac{(1 - z \, \bar{z}') \, (1 - z' \, \bar{z})}{(1 - z \, \bar{z}) \, (1 - z' \, \bar{z}')} = \frac{2 + 2 \cos (\theta + \theta') - 2 \, \rho \, (\cos \theta + \cos \theta') + \rho^2}{(2 \cos \theta - \rho) \quad (2 \cos \theta' - \rho)}$$
$$< \frac{2 + 2 + 2 \, (1 + 1) + 1}{(2 \sin \epsilon - \sin \epsilon) \, (2 \sin \epsilon - \sin \epsilon)} = \frac{9}{\sin^2 \epsilon},$$

so that finally

$$|w_2(z) - w_2(z')| \leqq \left(\sum_N^{\infty} n \, |a_n|^2 \right)^{\frac{1}{2}} \left(\log \frac{9}{\sin^2 \epsilon} \right)^{\frac{1}{2}} \tag{2}$$

for $\rho < \sin \epsilon$.

Since $\Sigma n \,|a_n|^2$ converges, the right hand member in (2) may be made less than an arbitrarily small δ for $\rho < \sin \epsilon$ by taking N sufficiently large, and having fixed N, the right hand member in (1) may be made less than δ by taking ρ sufficiently small. Thus, for ρ sufficiently small, $|w(z) - w(z')| < 2\,\delta$ independently of θ and θ', which proves the theorem.

Let us now assume in particular that $w = w(z)$ maps the circle $|z| < 1$ conformally on a simple (i.e. simply connected and nowhere overlapping) region D in the w-plane, and that all points of D are within a circle of radius R (this latter condition can always be brought about by a linear transformation on w and the extraction of a square root[1]); then $\sum_1^\infty n\,|a_n|^2$ converges[2] (and is less than R^2). Finally, suppose that $w(z')$ approaches a limit w_0 when z' approaches unity on the real axis; our theorem then shows that $w(z)$ approaches the same point w_0 on the boundary of D when z tends toward unity along any curve interior to both the unit circle and an angle less than π formed by two straight lines through $z = 1$ and symmetrical in respect to the real axis. This proposition is usually derived, somewhat less directly, from the distortion theorem.[3]

[1] Koebe, *J. Math., Berlin*, **145**, 1915.

[2] Fejér, *Festschrift . . . H. A. Schwarz*, Berlin, 1914.

[3] Koebe, l. c., and Study, *Konforme Abbildung einfach zusammenhängender Bereiche*, Leipzig, 1913.

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 5 FEBRUARY 15, 1919 Number 2

IS THE ARRANGEMENT OF THE GENES IN THE CHROMOSOME LINEAR?

By W. E. Castle
Bussey Institution, Harvard University

Read before the Academy, November 18, 1918

Every biologist is familiar with the remarkable discoveries of Morgan and his associates concerning the germ-cells of Drosophila. One of the most important of these discoveries is concerned with the phenomenon of linked inheritance. This kind of inheritance, while entirely conformable with Mendel's law, forms a very distinct and important class of cases whose existence has been brought to light since the rediscovery of the general law in 1900. Under the general law it is found that characters which behave as distinct units in heredity assort quite independently of each other. Thus if parents are crossed one of which possesses two characters, A and B, while the other lacks them, then the offspring of this cross will transmit A and B sometimes associated in the same gamete, sometimes in different gametes, the two events being under the laws of chance equally probable.

But in linked inheritance, a phenomenon first made known to us through the work of Bateson and his associates in England, later more fully explored and explained by Morgan, A and B are not wholly independent of each other in transmission. If they enter a cross together, they have a tendency to stay together in later generations; and if they enter a cross separately, they have a tendency to remain apart in later generations. Morgan has suggested that what binds or links two characters together is the fact that their genes lie in the same body within the cell-nucleus. Such bodies he supposes are the chromosomes. The evidence for this conclusion is very strong. Morgan and his associates have demonstrated the existence in Drosophila of four groups of linked genes corresponding with the four pairs of chromosomes which the cell-nucleus of Drosophila contains. Morgan has further suggested (and has beyond doubt established the fact) that the genes within a linkage system have a very definite and constant relation to each other. He supposes their

arrangement to be linear and in the group of genes most exhaustively studied, that of the 'sex chromosome' has represented them in a 'chromosome map, as shown in Diagram I.

That the arrangement of the genes within a linkage system is strictly linear seems for a variety of reasons doubtful. It is doubtful, for example, whether an elaborate organic molecule ever has a simple string-like form. Let us, therefore, examine briefly the evidence for or against the idea of linear arrangement of the genes. It is supposed by Morgan that two genes lying in the same chromosome show close linkage if they lie close together, but less linkage if they lie farther apart, and that the farther apart they are the less will be their linkage. As a measure of the distance apart of two genes he takes the percentage of cross-overs between them. This term requires a word of explanation. If two genes, A and B, enter a cross in the same gamete and emerge from it in different gametes, one of them has evidently *crossed-over* from the chromosome in which it lay to the mate of that chromosome (all chromosomes being paired prior to the formation of gametes). Likewise if the two genes, A and B, having entered a cross separately (being contributed by different parents), later emerge from the cross together, it is evident that one of them has again *crossed-over* so as to lie in the same chromosome as the other. The readiness with which cross-overs occur between two genes will on Morgan's hypothesis depend on their distance apart and the percentage of cross-overs between genes will be proportional to the distances between them. These assumptions have abundantly proved their utility as a working hypothesis, for it has been found possible, knowing what certain cross-over values are, to predict others with a fairly good degree of accuracy.

If the arrangement of the genes is strictly linear, so that A, B, C, etc., lie in a straight line, then it should be possible to infer the distance AC, if the distances AB and BC are known, since $AC = AB + BC$. But if the distance AC is less than the sum of AB and BC, then the arrangement can not be linear, since B will lie out of line with A and C. In reality it has been found that the distances experimentally determined between genes remote from each other are in general less than the distances calculated by summation of supposedly intermediate distances, and the discrepancy increases with increase in the number of known intermediate genes. To account for this discrepancy Morgan has adopted certain subsidiary hypotheses, of 'interference,' 'double crossing over,' etc., in accordance with which it is supposed that cross-overs between nearer genes interfere with or lessen the apparent amount of crossing-over with genes more remote. He therefore bases his chromosome map on summation of the shorter distances. This, however, leads to results which can be shown to be impossible.

Morgan's map of the sex-chromosomes places five out of twenty-nine genes at distances between 55 and 66 from the zero end of the chromosome, where yellow is located. A moment's reflection will show these to be impossible re-

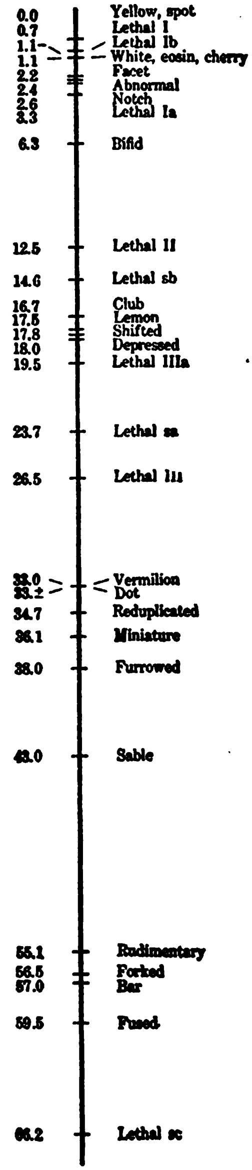


DIAGRAM I.

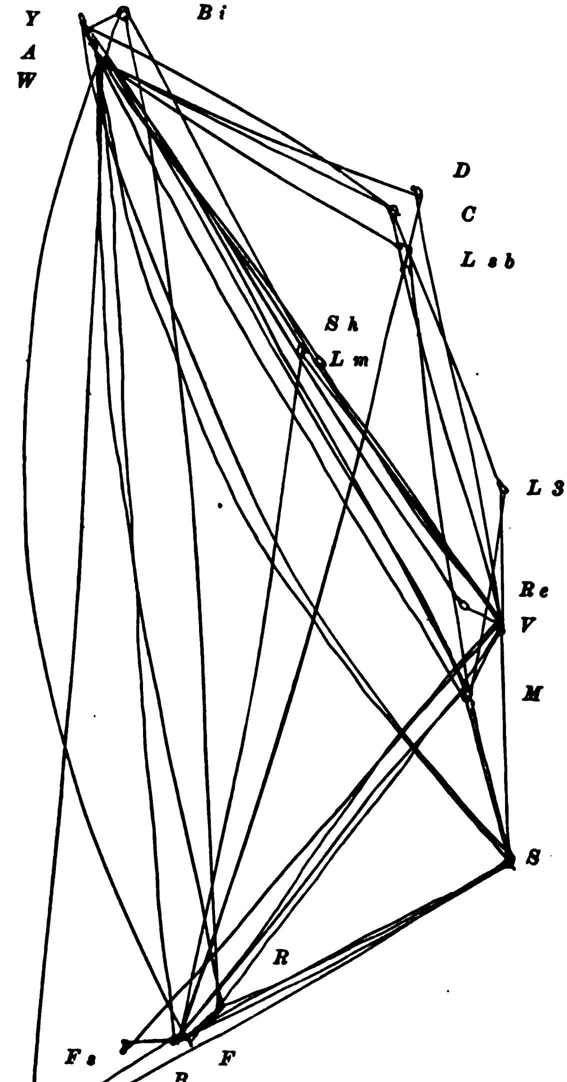
"Diagram I shows the relative positions of the genes of the sex-linked characters of *Drosophila*. One unit of distance corresponds to 1% of crossing-over."

Morgan and Bridges, p. 23.

lations, for a cross-over percentage greater than fifty is absurd. If A and B assort wholly independently, without any linkage whatever, just as they would in ordinary Mendelian inheritance where no linkage exists, cross-overs and non-cross-overs will be equal, 50% each. Any cross-over value consistently less than 50% shows linkage. *A cross-over value greater than fifty cannot exist.* For there must be either linkage or no-linkage. But no-linkage means 50% cross-overs, and linkage means less than 50% cross-overs. Hence a value greater than 50% cannot occur.[1] As an alternative to the hypothesis of linear arrangement it is possible to suppose that the arrangement of the genes is not linear, that the nearer genes are not directly in line with the more remote ones.

If the arrangement of the genes is not linear, what then is its character? This query led me to attempt graphic presentation of the relationships indicated by the data of Morgan and Bridges[2] but finding this method unsatisfactory I resorted to reconstruction in three dimensions, which has proved very satisfactory. The data used consist of the experimentally determined cross-over percentages between twenty genes of the sex-chromosome of Drosophila, as given in Table 65 of Morgan and Bridges. The only hypothesis involved in the reconstruction is Morgan's fundamental one that distances between genes are proportional to cross-over percentages. The secondary hypothesis, that distant genes are really farther apart than indicated by the experimental data, is rejected for the reason already explained, that impossible relationship are thereby entailed. Taking the data, then, exactly as they stand, we find it possible to make a very complete and on the whole self-consistent reconstruction of the architecture of the sex-chromosome from the linkage relations of its genes. A small ring of wire is taken to represent the locus of a gene. Two genes are placed as far apart (in inches) as there are units in the cross-over percentage between them. Thus between yellow-body and white-eye there is shown by Morgan's data to be a cross-over value of 1.1%. Consequently the rings which represent these genes in the reconstruction are joined by a wire 1.1 inches long. Between white and vermilion the cross-over percentage is 30.5. Accordingly the connecting wire in this case is made 30.5 inches long. Proceeding in this way the reconstruction shown in figures 1 and 2 is obtained. It indicates an arrangement of the genes not by any means linear, but rather in the form of a roughly crescentic plate longer than it is wide, and wider than it is thick. It is shown in side view in figure 1, and in edge view in figure 2.

That the arrangement of the genes can not by any possibility be linear is shown by two critical cases, the loci for *bifid* and *abnormal*. Bifid (*Bi*, figs. 1 and 2) is shown by a series of over 3,600 observations to be at a distance 5.5 from yellow. It is almost the same distance from white, viz., 5.3, as shown by 23,595 observations. Yet white and yellow are distant from each other only 1.1 units, as shown by 81,299 observations. Therefore bifid can lie neither above nor below yellow and white, in the line which joins them, but



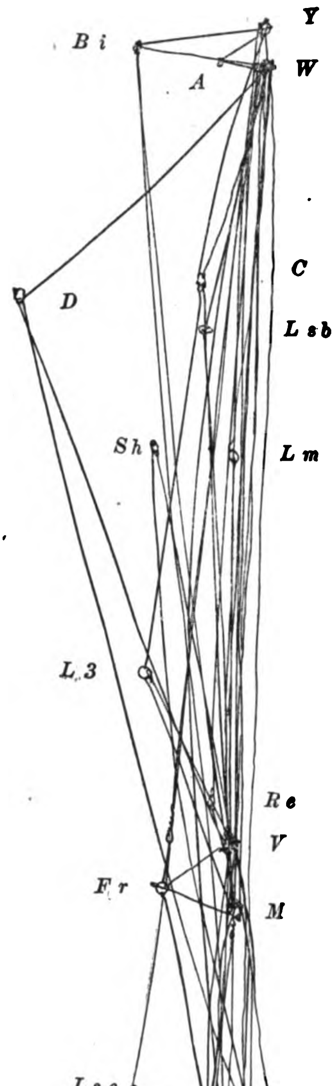


FIG. 1. SIDE VIEW OF MODEL

FIG. 2. EDGE VIEW OF MODEL

Seen at right angles to figure 1.

Showing relative positions of genes of 20 sex-linked characters of Drosophila, linear arrangement not being assumed. For significance of letters, compare Diagram I.

must lie laterally about equidistant from both. Other linkage relationships of bifid show that it lies as far as possible removed from the plane in which most of the genes lie. The other case mentioned is that of abnormal abdomen (*A*, figs. 1 and 2). Its linkage relations are known with only two other genes, yellow and white. But the relation of these two with each other is one of the best determined and the linkage of abnormal with each of them rests on more than 15,000 observations in each case. The yellow-white linkage is 1.1, as already stated; abnormal-yellow is 2.0, and abnormal-white 1.7. These relations make it impossible for abnormal to lie in line with yellow and white. Until a third linkage relation of abnormal is determined, it may swing freely round the line which joins yellow with white but can never come into line with them. A third linkage relation having once been determined for abnormal, its linkage with any other gene in the sex-chromosome could be readily predicted from direct measurement of the reconstructed figure. The actual test of the utility of this method of portraying linkage relationships could easily be made by first forecasting by measurement what undetermined linkage values are likely to be and then actually determining them by experiment. Such predictions could not fail to come nearer the truth than predictions based on a linear map, if as I have suggested the arrangement is really not linear.

What, it might be asked, does this reconstruction signify? Does it show the actual shape of the chromosome, or at any rate of that part of it in which the observed genetic variations lie? Or is it only a symbolical representation of molecular forces? These questions we can not at present answer. A first step toward answering them will be the construction of a model which will give us reliable information as to undetermined genetic relationships. A model which will answer questions truthfully must be a truthful presentation of actual relationships even though we do not know whether they are spatial or dynamic.

If the arrangement of the genes in the chromosome is not linear, Morgan's theory of linkage must be somewhat modified. (1) The fundamental assumption that the genes lie in the chromosomes and have a definite orderly arrangement there is not disturbed. (2) The further assumption that the respective distances between the genes determine their closeness of linkage one with another may also stand unchallenged. (3) But the assumption that the arrangement of the genes within the chromosome is linear cannot be accepted without proof, which at present is lacking. This assumption has made necessary other secondary assumptions, likewise unproved, which are superfluous if this one is abandoned.

Such an unproved secondary hypothesis is that of double-crossing-over. The experimental data show that double-crossing-over *must occur*, if the arrangement of the genes is linear. For if three genes, A, B, C, are linear in their arrangement in the order named, and all lie in the same gamete, and if subsequently A and C are found together in one gamete and B in another,

it is evident that this rearrangement can have come about only as a result of two breaks in the linkage chain, viz., one between A and B, and another between B and C. But if the arrangement is *not* linear, double-crossing-over need not be assumed as an explanation of the observed regroupings. For if A, B, and C are linked in a triangle, not in a straight line, then B may be freed from its connections with A and C without necessarily disturbing the connection of A and C with each other. Freeing of B will involve no greater number of breaks than the freeing of either A or C. It will still be true, however, as indicated by the experimental data, that certain groupings of three particular genes are easier to obtain than others. Thus in the case of the three genes white, bifid and vermilion, it is hardest to obtain the regrouping which involves detaching bifid from the other two. Morgan assumes that this is because bifid lies *between* the other two in a single linkage chain and so could be detached only by two breaks; it is possible, however, that the reason may be that bifid lies peripherally in the linkage system and could be detached only by an oblique longitudinal break, whereas either of the others could be detached by a simple transverse break. Similarly in the trio, white-vermilion-sable, it is vermilion which is difficult to detach; and in the group, vermilion-sable-bar, it is sable. Always it is the *middle one* considered with reference to the long axis of the system. This may be because, as Morgan supposes, only transverse breaks occur, of which two taking place simultaneously are required to produce the difficult regrouping, or it may be because transverse breaks are more frequent than oblique longitudinal ones, of which a single one would suffice to accomplish the regrouping, if the genes are not strictly linear in arrangement.

The phenomenon of 'coincidence' as described by Weinstein[8] is this. If crossing-over occurs toward one end of a chromosome, it is less likely to occur simultaneously elsewhere in the same chromosome. Crossing-over in one part of a chromosome is thus supposed to 'interfere with' crossing-over elsewhere in the same chromosome. If we adopt the hypothesis of linear arrangement, interference must be assumed to occur. Observed facts require this. But if we do not adopt this hypothesis but suppose that what have been called 'double cross-overs' are really the result of single oblique or single longitudinal breaks, then the supposed phenomenon of interference may mean only this, that transverse breaks are more likely to occur than longitudinal ones.

Finally, if the genes are not arranged in a single linear chain, the chiasmatype theory will need to be reëxamined. Such a purely mechanical theory seems inadequate to account for interchange of equivalent parts between twin organic molecules, such as the duplex linkage systems of a germ-cell at the reduction division must be. It seems more probable that preceding the reduction division a period of instability within the chromosome molecule comes on. Twin molecules are now closely approximated and parts of one may leave their former connections and acquire new connections with the

corresponding parts of the other twin. It is evident from the experimental data, notably that of Muller,[4] that new connections are not formed with any torn fragment of chromosome which happens to come into the proper position, but that connections are always formed at exactly corresponding points with homologous systems of genes.[5] It is like the replacement of one chemical radicle with another within a complex organic molecule and it seems highly probable that such is its real nature.

[1] The distances shown in Morgan's chromosome map in excess of 50 (admittedly not obtained experimentally but only by summation) are therefore too large. Accordingly, if one clings to the assumption that the arrangement of the genes is linear, it must be, not that the longer distances are too short, as Morgan has assumed, but that the short distances are too long. Therefore, any hypotheses framed to account for an apparent shortening of the long distances are superfluous. The long distances given by direct experiment are long enough; they approach the limit of the possible, viz., 50%. Thus in table 65 of Morgan and Bridges, we find the following high cross-over percentages given by direct experiment:—yellow-bar, 47.9; white forked, 45.7; and white-lethal *sc*, 46.0. What is needed therefore, if the linear arrangement hypothesis is retained, is a secondary hypothesis to explain why the short distances given by experiment are too long.

But if we abandon the hypothesis of linear arrangement, all secondary hypotheses are unnecessary. The experimentally obtained cross-over percentages may be accepted at their face value, which in every case fall within the limits of the possible, 0 and 50.

[2] Morgan, T. H., and Bridges, C. B., Sex-linked inheritance in Drosophila. *Carnegie Inst. Washington, Publ.*, No. 237, 1916, (88 pp., 2 pl.).

[3] Weinstein, A., *Genetics*, 3, 1918, (135–172).

[4] Muller, H. J., *Amer. Nat., Lancaster, Pa.*, 50, 1916.

[5] The case of 'deficiency' studied by Bridges (*Genetics*, 2, pp. 445–460, Sept. 1917) forms an apparent exception to the rule. Here a certain segment of the linkage system was as regularly wanting as it is commonly present. The regularity of the process, however, shows that the principle of union at particular points still holds. In the deficiency race, a new and simplified linkage system had been established and this persisted.

THE LINKAGE SYSTEM OF EIGHT SEX-LINKED CHARACTERS OF DROSOPHILA VIRILIS (DATA OF METZ[1])

BY W. E. CASTLE

BUSSEY INSTITUTION, HARVARD UNIVERSITY

Communicated December 9, 1918

In an earlier paper it has been shown that the arrangement of the genes in the sex-chromosome of *Drosophila ampelophila* is probably not linear, and a method has been developed for constructing a model of the experimentally determined linkage relationships. From such a model one may by direct measurement ascertain what other undetermined linkage values are likely to be. In order to test the utility of this method, it is desirable that it be tried out as widely as possible and the results for different cases compared with each other. For such use, suitable material is found in a paper by Metz[1] dealing

with the linkage relations of eight sex-linked characters in *Drosophila virilis*, a species distinct from *D. ampelophila*, which has been so exhaustively studied by Morgan and his pupils. Of the eight characters studied by Metz, two agree morphologically and in their linkage relations with each other, with similar characters of *D. ampelophila*. The six others have no exact counterpart among the known mutations of *D. ampelophila*. The two characters in question are yellow body and forked bristles. Yellow body, in both species, lies at the extreme, 'zero' end of the linkage system. Forked, in both species lies at a distance of 40 or over from yellow. In *D. virilis* the distance is exactly 40, according to the observations of Metz; but in *D. ampelophila*, according to Morgan and Bridges, the distance is about 6 or 7 units greater. But inspection of figure 1 (p. 29) shows that this estimate is probably too

TABLE 1

GENES	TOTALS	CROSS-OVERS	PERCENTAGE CROSS-OVERS
Yellow-frayed	308	4	1.3
Yellow-vesiculated	3,548	621	17.4
Yellow-magenta	3,049	1,157	38.0
Yellow-hairy	162	65	40.1
Yellow-forked	3,781	1,510	40.0
Yellow-glazed	1,328	591	44.5
Yellow-rugose	1,060	502	47.3
Frayed-vesiculated	296	55	18.6
Vesiculated-magenta	2,799	944	33.7
Vesiculated-forked	3,761	1,395	37.1
Vesiculated-glazed	877	359	42.1
Vesiculated-rugose	1,696	729	42.9
Magenta-forked	2,529	95	3.7
Magenta-rugose	824	197	23.9
Hairy-forked	162	5	3.1
Forked-glazed	749	193	25.7
Forked-rugose	1,158	321	27.7

high, since the wire joining white (*W*) with forked (*F*) in the model is too long to harmonize fully with other linkage-values given by Morgan and Bridges, the wire being curved. The distance yellow-forked is not given by Morgan and Bridges but it evidently should be about one unit greater than the distance, white-forked, which is given as 45.7. If this estimate of the distance is too high, as figure 1 indicates, then the distance yellow-forked is probably not very different in the two species of Drosophila and will be found to be not far from 40 in both.

The linkage values found by Metz for the eight sex-linked genes of *D. virilis* have been gathered from his several tables, averaged and brought together in table 1 herewith. They form the basis of the reconstruction shown in figures 3 and 4.

Metz, adopting Morgan's system of linear grouping, shows the eight genes in a linear chain more than 80 units long, although the greatest distance experimentally found between any two genes is 47.3 (yellow-rugose). This discrepancy, like the similar ones observed in *D. ampelophila*, shows the inadequacy of the hypothesis of linear arrangement. For the maximum possible

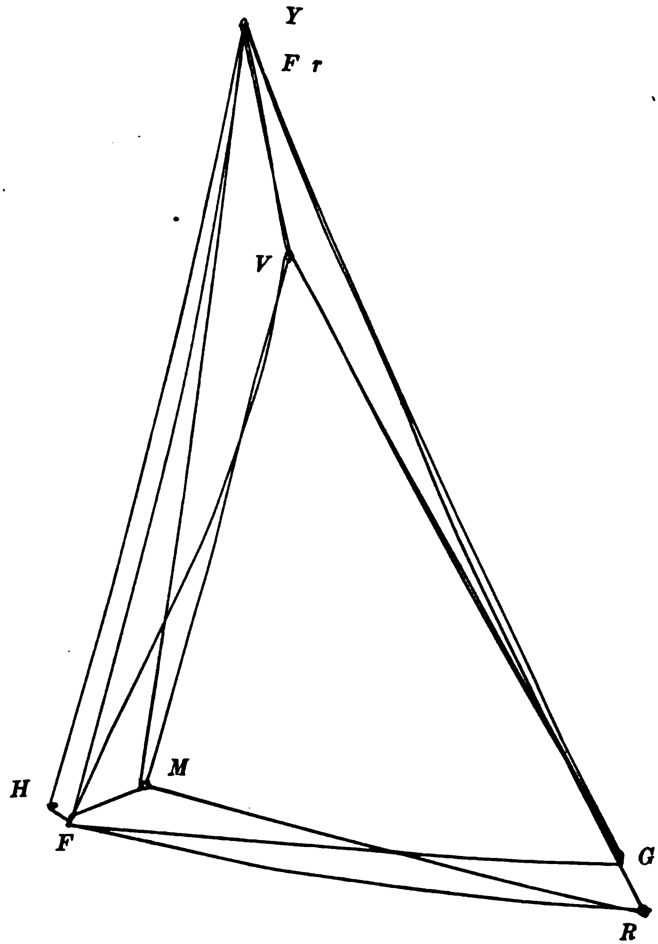


FIG. 3. MODEL SHOWING RELATIVE POSITIONS OF GENES OF 8 SEX-LINKED CHARACTERS OF DROSOPHILA VIRILIS

cross-over percentage is 50 and this is in no case exceeded by data given by direct experiment.

Reconstruction in three dimensions (figs. 3 and 4) shows even more clearly in this case than in that of *D. ampelophila*, that a linear arrangement is out of the question. The reconstructed figure is roughly in the form of a tetrahedron. Figures 3 and 4 are views taken at right angles to each other corre-

sponding with the two views given for *D. ampelophila* in figures 1 and 2. The eight genes lie in four groups at the four apices of the figure, in groups of 2, 3, 2, and 1 respectively. The figure has a very definite and rigid form, for geometrical reasons. Three additional linkage relations should be known to determine fully the position of certain of the genes within their respective

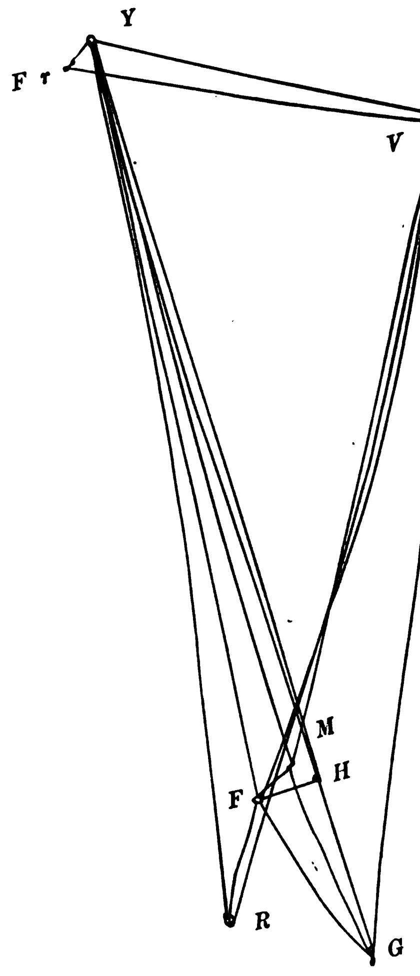


FIG. 4. MODEL SEEN AT RIGHT ANGLES TO FIGURE 3

groups. These are magenta-hairy, glazed-rugose, and frayed-forked (or frayed-glazed), where the figures show connecting wires to be wanting.[2] Given these missing connections, the form of the model would be very fully determined in every detail. Any newly discovered sex-linked gene of *D. virilis* having been located with reference to three of these eight genes, its rela-

tions to all the others could be predicted by direct measurement from the model.

[1] Metz, C. W., *Genetics*, 3, 1918, (107–134).

[2] From measurement of the model, it may be predicted that the cross-over percentage between magenta and hairy will be about 4 or 5, and the glazed-rugose percentage about the same, probably a little greater. The position of frayed in the system is not fully determined, as only two linkage relations of frayed are known. But it may be predicted, from measurement, that the frayed-forked cross-over percentage will lie between 39 and 41, and that frayed-glazed will lie between 43 and 46, provided of course that the relations given in table 1 have been determined with sufficient accuracy.

THE CAUSE OF PROLIFERATION IN BEGONIA PHYLLOMANIACA

By Erwin F. Smith

Laboratory of Plant Pathology, United States Department of Agriculture

Read before the Academy, November 18, 1918

The cause of the excessive production of adventive shoots on the leaves and internodes of this plant (a very strange phenomenon) is attributed to excessive loss of water, due to woundings or other causes. Usually in regeneration the response is not far from the place of injury, here it may be at a long distance from the wounded part, e.g., roots wounded and response in the top of the plant, although a direct response from the injured part can also be obtained. The paper will be published in full in *The Journal of Agricultural Research.* The following is a synopsis:

1. Ordinary begonia leaves when detached from the plant and pegged down on moist sand develop roots and shoots from cut places and this method is used by gardeners for the propagation of begonias. Many other plants are propagated in this way, e.g., the hyacinth from bulb scales.

2. But the leaves and shoots of this begonia *proliferate while still attached to the plant.*

3. They will proliferate on the plant very freely when wounded, making small forests of shoots on the thickened red lips of the wound if the wounds are made in quite young tissues, but not otherwise (young leaf blades were used).

4. They will frequently proliferate in the top parts of cuttings (on leaves and internodes) especially if the cuttings are dried for a day or two before planting.

5. They will proliferate most astonishingly at the top of the plant (both from leaves and internodes) if the roots are wounded, but here again only quited young tissues can be shocked into the production of such shoots. This is the most striking fact I have discovered, viz., that the prolification may occur at a long distance from the place of wounding and must be from young

tissues. So far as known to the writer, it is the first example of response of this sort at a distance from the point of injury.

6. I have also some evidence that leaves will proliferate locally under colonies of sucking insects (mealy bug, white fly), also that withholding water from the plant for a few days will cause it to proliferate.

7. The nature of the shock appears to lie in the sudden interruption of the water current which is conceived to cause cell-precipitates or plasmolysis of young totipotent cells which begin to grow when they have recovered from the shock.

8. The prolification at times is so much like a forest that one must assume that the whole surface (epidermis) of immature shoots is full of cells capable of growing into new plants if properly shocked but that as the tissue matures these cells either lose their power of response, or become more perfectly protected.

9. These adventive shoots, for the most part, perish quickly and cannot be regarded as branches, since they have no initial connection with the ordinary cambium, or xylem-phloem of the mother plant. They are rather to be classed with filial teratomas. Later, a small proportion of them establish connections with the conductive tissues of the mother and persist, i.e., become abnormally situated branches.

10. My observations contradict those of Prillieux and confirm those of Verlot and of Caruel that buds may arise from the ordinary trichomes. They may develop either from the base or the middle of acicular hairs. Such hairs arise from a red tissue, the other parts of the epidermis being green. I have also seen them developing from the base of glandular hairs which are abundant on the young internodes, but they are not restricted to these pairs.

THE PERCENTAGE NUMBER OF METEORITE FALLS AND FINDS CONSIDERED WITH REFERENCE TO THEIR VARYING BASICITY

By George P. Merrill

Department of Geology, United States National Museum, Washington, D. C.

Communicated by C. G. Abbot, January 9, 1919

Various compilations relating to time and distribution of meteorite falls have been made with a view of correlating them with periodic showers, but with, thus far, the only result of showing that there is no apparent connection between them.[1] Viewing the subject from a geological standpoint, that is, from the standpoint of an earth made up by the gradual accumulation of meteoric materials, and considering also the apparent more basic nature of the earth's interior as compared with the outer portion, I have thought it possible some light might be thrown upon it through a consideration of the

percentages of actually observed falls and the relative basicity of their materials. The results of such consideration are given below:[2]

Of the total 367 known meteoric irons there were seen to fall but 17, or less than 5%. These are essentially metallic; ultra basic.

Of the 31 known stony irons variously classed as Lodhranites, Pallasites and Mesosiderites, carrying at times as high as 50% metal, there were seen to fall but 5, or in round numbers 16%.

Of the 370 known stones composed mainly of silicate minerals, with chondritic structure, carrying from 5 to 25% metal (Howarditic chondrites to Ureilites inclusive), there were seen to fall 322, or 87%.

Of the 21 calcium-aluminum-rich stones, carrying less than 1% metal, free of chondrules, and variously classed as Angrites, Eukrites, Shergottites and Howardites, there were seen to fall 20, or 95%.

Of the 12 magnesia rich stones essentially free from metal without chondrules and classed as Bustites, Chassignites, Chladnites and Amphoterites, the most acidic types known, there were seen to fall 12, or 100%.

As there is little reason for supposing that falls of one kind are not as conspicuous as those of another, it would seem a fair assumption that those of which the seen percentage was the smallest were the earliest, perhaps largely prehistoric.[3] Hence arises the thought of a gradual decreasing basicity or what is the same thing, increasing acidity of accumulated materials, as time goes on. While it would seem absurd to claim that such a change could manifest itself perceptibly during the few years of observation, it is nevertheless worthy of note that, however much uncertainty is attached to the period of fall of upwards of 95% of the known meteoric irons, the stones of the last two classes mentioned, which are of the most acidic type and with one or two exceptions are iron free, have fallen within a period of a little more than one hundred years.

Following out the same line of thought, it would seem possible that the thousands of meteors which are known to enter our atmosphere daily and yet leave no record of their fall, might be products of a still further differentiation of cosmic matter (or perhaps derived from an entirely different source) and of such eminently combustible material as to be largely consumed in their flight.

Additional interest is attached to this suggestion from the fact that there are known but eight carbonaceous meteorites, i.e., eight stones in which an uncombined carbon (or possibly hydrocarbon) occurs in such quantities as to give them a distinctive character and which, therefore, might be considered liable to destruction by heat while passing through our atmosphere. All of these eight were seen to fall, the first, that of Alais, France, in 1834 and the latest, that of Indarch, India, in 1891.

It is possible to account for some of the facts here given on the assumption that many meteorites are of an extremely perishable nature, and unless seen to fall and sought for immediately, likely to become destroyed through dis-

integration. Further than this, a meteoric stone would be less likely to attract the attention and curiosity of the ordinary individual than would an iron. So far as the first possibility is concerned, I think that all who have had to do with meteorite collections will agree that as a general rule the irons, through their susceptibility to a damp atmosphere and consequent rusting, require much more attention for their preservation than do the stones. The second possibility is, however, one that must be given consideration.

[1] See Chapter IV of Farrington's *Meteorites*, Chicago, 1915.

[2] The figures here given relative to number of falls are believed to be substantially correct up to 1916. Accurate statistics since that date are not available.

[3] It would be a natural supposition that the fall of an iron would be less noticeable than that of a stone since the former would be less liable to break up—explode—in its passage through the atmosphere. Unfortunately, the literature is not sufficiently explicit on this point to bear out the supposition. Hidden, to be sure, states that the fall of the Mazapil iron was accompanied only by a loud sizzing sound, there being no explosion or loud detonation. On the other hand, Kunz states that the fall of the Cabin Creek iron was "accompanied by a very loud report which caused the dishes to rattle," and the fall of the Nedagolla iron is also stated to have been accompanied by an explosion. Accounts of other falls are either noncommittal on this point or equally contradictory, and it is evident accurate information is lacking.

NOTE ON A CONTACT LEVER, USING ACHROMATIC DISPLACEMENT FINGERS

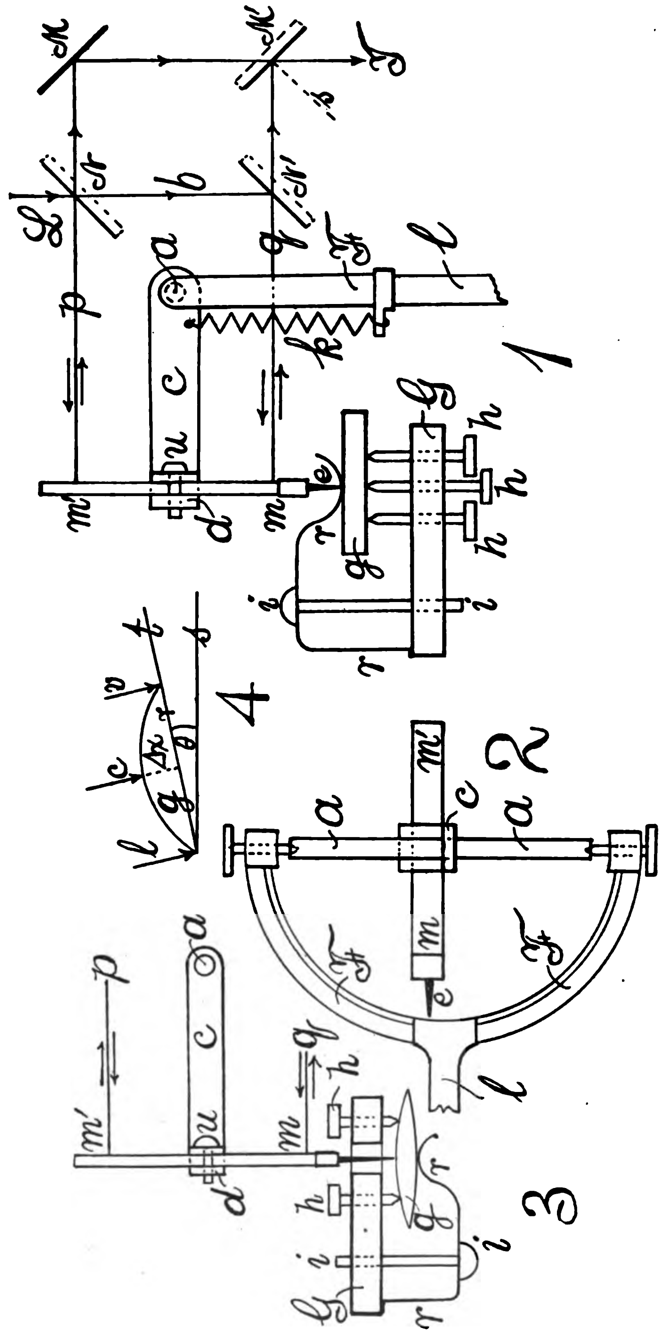
BY CARL BARUS

Communicated, December 27, 1918

1. Apparatus.—The method heretofore described for the measurement of small angles by the aid of the rectangular interferometer, lends itself conveniently for the construction of apparatus like the contact lever, or the spherometer. Having work needing such instruments in view, I designed the following simple apparatus for the purpose.

Figure 1 is a plan of the design; figure 2 an elevation of the fork and appurtenances; figure 3 (plan) finally shows the same apparatus adapted for use as a spherometer. The interferometer receives the white light from a collimator at L. After the reflections and transmissions controlled by the mirrors M, M', N, N', and the auxiliary mirror mm', as indicated in the figure, the light is conveyed into the telescope at T for observation of the interferences. The mirror M', is on a micrometer with the screw s normal to its face.

It is through the mirror mm' that the small angles are to be measured and this is therefore mounted at one end of the lever dc, capable of rotating around the long vertical axle aa, in the circular fork FF. The latter is rigidly mounted on the bed of the apparatus by aid of the stem l in the rear. The lever c is bent upward at right angles at d, and it is here that the mirror mm' is firmly



secured by bolts etc., as at *u*. The spring *k* draws the lever toward the front of the diagram, so that the blunt metal pin *e* suitably attached to the end of *mm'* may be kept in contact with the glass plate *g* to be tested.

The plate *g*, in order to be examined as to its degree of plane parallelism, must be capable of sliding up and down or right and left under standard conditions. To obtain these the stout bar *G* rigidly attached like *l* to the base of the apparatus, has been provided, carrying three set screws *h, h, h,* the points of which lie in the same circumference about 120° apart. They therefore constitute a kind of tripod against which the plate *g* is firmly pressed by the flat spring or clip *rr* and screw *i*. This method of mounting may be appropriately varied in accordance with the tests to be made on the plate *g*, its shape, etc. Similarly the set screws *h, h, h,* may be placed nearer together or further apart in appropriate screw sockets, and finally the lever *c* may be lengthened or shortened at pleasure. The pin *e* remains in permanent contact with the plate *g* in consequence of a wide circular hole in the clip *rr*; or *e* may clear *rr*, above or below it.

If but one face of the plate *g* is to be tested, the system *Ghrg* must slide as a whole right and left, nearly parallel to the rays *pq*. In such a case everything will depend on the excellence of the slide carrying the system. I did not attempt to make such arrangements, as I had no need of data of this kind; but parts *MM'*, *NN'*, *Fcmm'* and *Grg* were nevertheless mounted on heavy slides (lathe-bed fashion) for convenience in securing a variety of adjustments.

In figure 3 the bar *G* has been reversed in position and the contact pin *e* now passes through a circular hole in *G* to be in contact with a lens *g*, for instance, kept pressed to the tripod screws *h, h, h,* in the same way as before. The latter should in general be much closer together than the figure shows. The instrument is now a spherometer.

The experiments indicated that the mounting of the contact pin *e* to the extremity of the mirror *mm'* may be the occasion of annoyances. For on sliding *g* right and left or even up and down, the mirror *mm'* is liable to be flexed. In such a case the achromatic fringes rapidly lose sharpness, not to speak of the errors involved. I endeavored to avoid this by keeping the pin *e* out of contact with the plate *g* by a special lever (not shown), while *g* was being displaced and to test a number of successive contacts thereafter; but it is best to mount *e* on a *separate* rigid cross-piece parallel to *mm'* and firmly attached to *c*. In such a case no flexure of *mm'* can occur and the contacts may also be repeated at pleasure. Before each reading the bar *G* should be gently tapped.

The achromatic fringes can be found only through the spectrum fringes. This is not usually difficult remembering that not only must the slit images in the spectrum be in contact throughout, but the two beams must be locally in contact on the mirror *M'*. Moreover the mirrors *M'* and *N'* must be equally thick and the silvered faces all turned towards the auxiliary mirror *mm'*.

In the figures the contact lever *mme* is horizontal. It may also be mounted vertically but in such a case it is less easy to mount the mirrors M, M', N, N', when the system is to be exposed to tremors.

2. *Equations.*—If the mirrors M, M', etc., are set at an angle i, if the deflection of the auxiliary mirror is θ, and if the breadth of the ray parallelogram MM' or NN' is b, we may write

$$b\,\Delta\,\theta = \Delta\,N\cos i \tag{1}$$

where $\Delta\,N$ is the displacement at the micrometer at M'. If r is the length of the lever c figure 1 and $\Delta\,x$ the displacement of the pin e

$$r\,\Delta\,\theta = \Delta\,x \tag{2}$$

Hence

$$\Delta\,x = (r\cos i\,/\,b)\,\Delta\,N \tag{3}$$

Thus the apparatus is more sensitive as r is smaller and b is larger. In the instrument used (adapted from an earlier apparatus)

$$r = 11 \text{ cm.};\ b = 10 \text{ cm.};\ i = 45°$$

so that

$$\Delta x = .778\Delta N \tag{4}$$

But the main condition of sensitiveness is contained in the size of the fringes, and these may be made indefinitely large by suitable rotation of the mirrors M and M', for instance, in like direction on a horizontal axis (local coincidence of rays on M'). Since

$$2\Delta N\cos i = n\lambda$$

in case of the passage of n fringes, equation (3) becomes

$$\Delta x = nr\lambda/2b \tag{5}$$

Hence the limiting sensitiveness ($n = 10$) would be (with the above data)

$$\Delta x = 33 \times 10^{-6} \text{ cm.} \tag{6}$$

for a single fringe, a few tenths of which may be registered with certainty. When the achromatic fringes are used, it is however usually more convenient to *standardize* the ocular plate micrometer in the telescope, directly by aid of the screw micrometer at M', figure 1. If the ocular plate is divided in tenth millimeters along a centimeter of length and the fringes are of moderate size, one may estimate that about 40 scale parts correspond to $\Delta N = 10^{-3}$ cm. so that a single scale part of displacement of the achromatics is equivalent to $\Delta N = 25 \times 10^{-6}$ cm., while a few tenths of a scale part may here also be estimated.

If the apparatus (fig. 3) is to be used as a spherometer, the ordinary method of measuring from a plate of glass is at once available. If r is the radius of the circle of the tripod and Δx the height of the central foot, we obtain as usual for the radius R required

$$R = r^2/2\Delta x \tag{7}$$

This method gives good results for lenses of all curvatures, however strong, as the tests made indicated. But it is not necessary to use the plate to obtain a fiducial reading, provided the system *Gg* carrying the lens *g*, is on good right and left slides. For in figure 4, let θ be the angle between the plane of the tripod and the slides, and let three readings of ΔN be taken for three preferably equidistant points *l*, *c*, *v*, of the lens, by sliding *Gg* over equal distances, *r*. Let the reading be

$$y = N, \quad y' = N + r \tan\theta + \Delta N, \quad y'' = N + 2r \tan\theta \tag{8}$$

where ΔN corresponds to Δx in figure 4. Hence

$$2y' - (y + y'') = 2\Delta N$$

and equations (4) and (7) apply as before. This method also gives good results even for short distances, *r*.

3. Observations.—The use of the apparatus figure 1, with the strip of glass *g* to be tested sliding up or down, did not at first give satisfactory results, because the mirror *mm'* was too thin (2 mm. thick). It was found however, that on breaking contact at *c* during the sliding of *g* between successive positions or by gently tapping the bar or standard *G*, very fair results were obtainable. There would have been no difficulty in using a thick glass mirror *mm'* ($\frac{1}{4}$ inch or more) in which case the annoyance of flexure would have been negligible. But all difficulties were eliminated by using an independent arm to carry *e*, as stated above. Tapping at *G* before each observation is essential. The observations themselves must be omitted here. Tests of the degree of wedge-shape of long strips of glass from centimeter to centimeter of length, were made in detail, using both the screw and the ocular micrometer. Similarly, lenses of all focal lengths from a few centimeters to 100 cm. either convex or concave, were examined by the apparatus figure 3, with surprising ease and accuracy. The parts of the surfaces of such lenses may be explored to the fraction of a wave length, for successive circular patches of a cm. of radius, or less.

Finally the spherometer method of figure 4 and equation (8) gave entirely satisfactory results. An application for the measurement of (elastic) micrometric displacements will be discussed in a subsequent paper.

INTERFERENTIAL CONTACT LEVER EXPERIMENTS RELATING TO THE ELASTICS OF SMALL BODIES[1]

By Carl Barus

Department of Physics, Brown University

Communicated, December 27, 1918

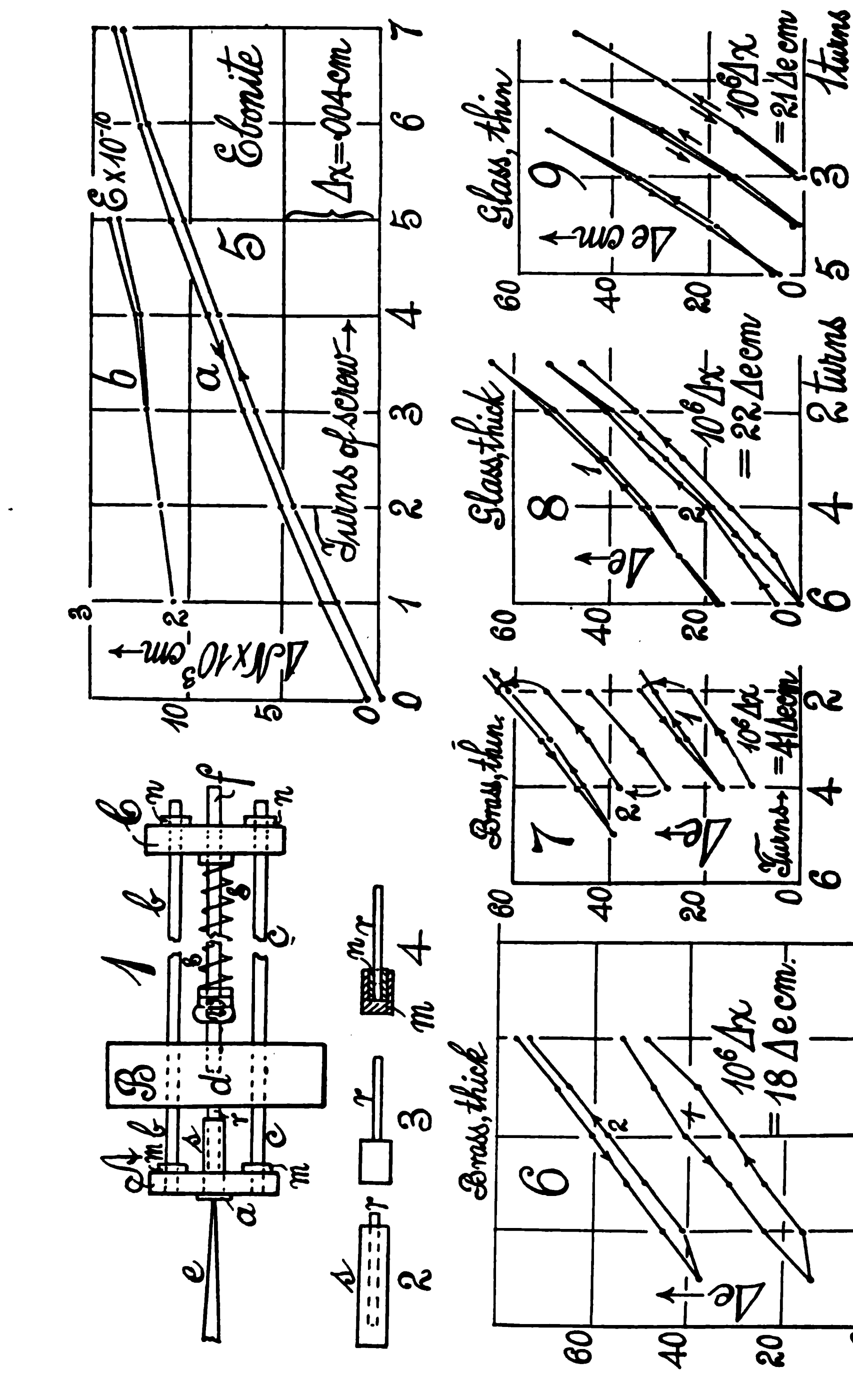
1. Introductory.—In a preceding paper I communicated a series of experiments on the traction modulus of small bodies, using an interferometer design, which worked admirably so far as the optic measurements were concerned. The mechanical part of the contrivance showed an apparent yield, the nature of which I was unable to detect, but which seemed in some way, to be associated with the flexure of parts of this massive apparatus. In fact pulleys and weights were used for imparting stress. It may be argued that any contrivance of this kind, however convenient in other respects, is dangerous because of the force couples introduced, even when the rigid parts of the apparatus are nearly 2 inches thick, as in the case in question.

In the present apparatus all this is completely avoided by the use of pushing springs to impart stress, and the interferential contact lever to measure strain. True, friction enters into functioning of such an apparatus to a menacing degree. It thus becomes an experimental question to determine in how far it can also be eliminated by judicious tapping, etc. Cf. § 6.

2. Apparatus.—The simplest of the apparatus designed is shown in figure 1. The rod to be tested, 1 to 3 cm. long, is at *r* held in a brass sheath *s*, loosely fitting it. See figure 2. This is screwed into the middle of the massive brass cross piece *A*. A little disc of glass has been attached at *a*, and the end, *e*, of the *contact lever* touches it to indicate the small elongations. The longitudinal displacements Δx of the pin *e* are observed by the interferometer, as explained in the preceding paper.[2]

B is a cast iron brick, about 10 inches high, 2 inches thick, and 3.5 inches broad, provided with 2 horizontal ½ inch perforations, parallel to each other and normal to the large face. Through these pass the ½ inch brass rods *bb* and *cc* loosely, rigidly connecting the cross piece *A* with the similarly massive cross piece *C* (screws and nuts *m*, *n*). The rectangle *AC* is thus free to slide in *B*, except so far as it is limited by the contact of the rod *r* with the smooth face of the brick *B*.

To apply stress, the system *d*, *w*, *S*, *f*, has been provided, consisting of the stiff open spring *S* encircling the brass rod *df*, firmly screwed into the brick *B* and *d*, but passing loosely through a perforation in the middle of *C*. The end near *d* of the rod *df* is threaded (20 threads to the inch), so as to admit of the compression of the spring *S*, by aid of the thumb nut *w*. *S* was a precision spring, taken from an indicator apparatus and provided as usual with two end brass collars. It is essential that the sliding parts of the apparatus


Ebonite
Turns of screw
$\Delta x = .004$ cm
Brass, thick
$10^6 \Delta x = 18 \Delta e$ cm.
Brass, thin.
$10^6 \Delta x = 41 \Delta e$ cm
Glass, thick
$10^6 \Delta x = 22 \Delta e$ cm
Glass, thin
$10^6 \Delta x = 21 \Delta e$ cm

work smoothly and with a minimum of friction. Such as exists may be eliminated by tapping b and c before each observation. Thrusts up to 15 to 20 kg. may be easily applied by the thumb nut w. These stresses act in the direction ar and fd, colinearly and there are no couples endangering the accuracy of the elastic displacements of r. The stress is standardized in terms of the observed rotation of the thumb nut w.

Figures 2, 3, 4 are details, showing the methods of clutching the rod r, sheathed in figure 2, shouldered in figure 3, soldered in a small cap m with fusible metal n, in figure 4. The thick ends are threaded to be received by A figure 1.

3. *Observations. Hard rubber.*—As in the preceding paper (l. c.) if Δx is the longitudinal compression of the rod r in s,

$$\Delta x = (r \cos i / b) \Delta N \tag{1}$$

where ΔN is the displacement of the micrometer, at $i = 45°$ to the rays, b the breadth of the ray parallelogram, and r the effective length of the contact lever. Furthermore since the modulus E for the length of rod L and section A is $E = (F / A) / (\Delta x / L)$, F being the thrust,

$$E = F L b / A r \cos i. \Delta N \tag{2}$$

The ocular micrometer if used is to be standardized in terms of ΔN by direct comparison; i.e., if the former datum is Δe arbitrary scale parts, $\Delta e / \Delta N$ must be known.

To graduate the spring S, the apparatus ABC, figure 1, was detached from the interferometer and the brick B fastened near the edge of a strong flat table, with its large face toward A lowermost and horizontal. The rectangle AC was thus vertical, A below C, just clearing the edge of the table. Weights from 1 to 9 kg. were now hung from A, compressing the spring S by measureable amounts. In this way it was found that the stretch 0.7 mm. corresponded to 1 kg. Since the threads of w were 1.275 mm. apart, it follows that 1 rotation of the thumb screw w corresponds to 1.82 kg. or to 1.78×10^6 dynes. In the interferometer, $b = 9.3$ cm., $r = 11.0$ cm. were directly measured.

The test rod r was here of hard rubber of length $L = 2.47$ cm., diameter $= 0.377$ cm., and area $A = 0.112$ cm.^{2} Hence for n turns of the screw w, (equation (2), ΔN in cm.)

$$E = 4.69 \times 10^7 n / \Delta N; \tag{3}$$

or if we express ΔN in 10^{-3} cm.,

$$E = 4.69 \times 10^{10} \times n / \Delta N \tag{4}$$

The fringes, found without difficulty, were small though here more than adequate for the purpose. Measurements were made in *cycles*, care being

taken to repeatedly tap the movable parts of the apparatus before each reading, and these came out remarkably smoothly at once.

An example of experiments made after some improvements of apparatus and carried out with the rod specified through a range of about 15 kg. is given in figure 5, *a*. The rod stood the stress well except at the end where it showed slow viscous shrinkage. The data (ordinates) contain the running displacement ΔN of the micrometer screw in terms of the successive turns of the forcing screw, *w*, figure 1. Stress increasing or decreasing is indicated by arrows.

The values of E were computed from 3 turns (5.46 kg.) of the forcing screw. For loads up to 5 additional turns (total 11 kg.), the data for E are practically identical, both in the outgoing and return series. See figure 5, *b*. At 6 turns (total 13 kg.) the rod apparently yields; but at 7 turns it again stiffens in both cases. As a whole the data are quite as good as the reading of the micrometer screw admits. To interpret this apparent increase of E it would be necessary to use a thinner rod, as the following experiments with brass and glass suggest. Again, only in case of more rigid rods, where ΔN fails, is it necessary to use the ocular micrometer (Δe).

4. The same. Brass.—By way of contrast, a thick solid brass rod, L = 2.34 cm. long, 0.376 cm. in diameter, area A = .111 cm.^{2} was now put into the sheath, *s*, figure 1, and tested, the aim being to redetermine the limit of measurement. Here $n / \Delta e$ or the ocular micrometer is the essential datum and $\Delta e / \Delta N$ must be known.

The interferometer was modified to guard against displacements due to tremor, large fringes were installed and readings were made several times before and after tapping. There was but little difference. An example of such results is given in figure 6, (1) and (2), where $\Delta e / \Delta N \times 10^3 = 43$, and therefore per turn of screw ($n = 1$), $E = 10^{11} \times 19.3 \Delta e$.

In the first series Δe per turn was 8.0 and hence $E \times 10^{-11} = 2.4$; in the second $\Delta e = 7.8$ and $E \times 10^{-11} = 2.5$. Seeing that a scale part of Δe in figure 6 is but 23×10^{-6} cm., these results are experimentally very good; but their absolute values, as given by E, is nevertheless very low. The rates for the outgoing and return series are identical. The backlash, as it were, on passing from one to the other is probably in the apparatus.

In triplet observations, naturally, higher values of E will be found; for instance, between 3 and 4 turns of the screw, $\Delta e = 5.5$ per turn, appeared in successive independent experiments. Thus $E \times 10^{-11} = 3.5$.

An example may now be adduced of experiments made with a brass rod, thin and shouldered as indicated in figure 3, the large end ($\frac{1}{4}$ inch in diameter) being threaded and screwed into the cross-piece A. The dimensions of the thin part were, length L = 1.8 cm., diameter = .22 cm., A = .038 cm.^{2} The fringe factor was $\Delta e / \Delta N \times 10^3 = 29$. The mean rate per turn was found to be $\Delta e = 4.9$ and $10 \times E^{-11} = 29.2 / 4.9 = 6.0$.

It seemed therefore, worth while to further decrease the section. This was done, the dimensions being, length L = 1.8 cm., diameter 0.175 cm., A = 0.0199 cm.^{2} The results gave $\Delta e / \Delta N \times 10^8 = 26.0$ and therefore $E = 5 \times 10^{11} / \Delta e$. The rates per turn lie between $\Delta e = 5.6$ (returning) and $\Delta e = 6.1$ (outgoing), so that $E \times 10^{-11}$ is between 8.9 and 8.2, respectively. This is so near the normal value for brass that a further decrease of section of the rod figure 3, was undertaken. The final dimensions were L = 1.8, diameter = 0.138, A = 0.015 cm.^{2} The results are given in figure 7, care having been taken not to overstrain the thin rod. Here $\Delta e / \Delta N \times 10^8 = 25.8$, $E \times 10^{-11} = 66.4 / \Delta e$ and Δe per turn lies between 6.7 (outgoing) and 8.1 (returning). Hence $E \times 10^{-11}$ = 9.9 and 8.2, respectively, so that the normal modulus of brass has actually been reached. In fig. 7 the cycles have been spaced as shown by the arrows, to show the separate observations.

5. *Glass*.—The glass rod first tested was L = 2.33 cm. long, 0.37 cm. in diameter, so that A = 0.107 cm.^{2}

With a robust interferometer the outgoing and return data were nearly coincident; but the graphs were not as a rule straight. The mean rate per turn was found to be $\Delta e = 8.6$. The fringes were of moderate size ($\Delta e / \Delta N \times 10^8 = 27.5$), so that $10^{-11} E = 12.75 / \Delta e = 1.5$, a very low result. Larger fringes were now installed, giving $\Delta e / \Delta N \times 10^8 = 34.8$. The results after regrinding the contact face are shown in figure 8 and are again nearly coincident, but lie on curved loci. In the first series the larger rate is $\Delta e = 8.4$ per turn; in the second series $\Delta e = 10.0$ per turn. Hence

$$10^{-11} E = 16.1 / \Delta e = 1.9 \text{ and } 1.6, \text{ respectively,}$$

larger than the preceding; but this is still only about one-third of the normal modulus of glass.

The endeavor was now made to proceed as in the case of brass above, with a shouldered rod and thinner sections. With this, in view, the glass rod was fixed in a small hollow cup, figure 4, with fusible metal. The cup being threaded was thereupon screwed into the cross-piece A, figure 1. With a glass rod L = 1.9 cm., A = 0.28 cm.,^{2} moduli as high as $E \times 10^{-11}$ = 3 to 4 were obtained. On taking the rod out however, it became clear that there had been gradual yielding of the fusible metal clutch. Hence I returned finally to the sheath method (fig. 2) using a thin glass rod, L = 2.54, cm. long, 0.185 cm. in diameter, A = 0.026 cm.^{2} The results are given in figure 9. The graphs are nearly coincident but curved. The mean rates for the higher loads are per turn of the screw, $\Delta e = 10.4$ (incoming) and $\Delta e = 8.6$ (outgoing). $\Delta e / \Delta N \times 10^8 = 28.8$, being the fringe factor, $E \times 10^{-11} = 57.86 / \Delta e = 5.5$ and 6.8, respectively. Hence here also, as in the case of the brass rod above, the normal value of the modulus has been reached; i.e., one may expect the data for E to be correct on their absolute values, if the ratio of length of rod to diameter is of the order of 10 to 1.

6. Conclusion.—The present experiments made with a totally dissimilar apparatus and in a different manner, are nevertheless (notwithstanding the relative simplicity of the present design) not markedly superior to the earlier experiments (l. c.), as a whole. The misgiving which I felt (see § 1) regarding the force couples entering into the earlier method was not therefore justified. Both apparatus function admirably so far as the optics of the method are concerned. This is particularly noteworthy when one considers the admissibility of the rather rough treatment needed in work of the present kind. Both apparatus are liable to give misleading results from the same cause; i.e., from an insufficiently uniform and continuous contact of the two ends of the rod with the abutments. From this results appreciably unequal distribution of stress in the sections of the rod and possibly flexure. There seems to have been no serious yield in the abutments, etc., of either apparatus.

The values of the modules E as a consequence come out too small. There can therefore (tapping admitted) have been no serious discrepancy from friction in the application of stress; for this would have made E too large. Moreover all slight dislocations within the interferometer as the result of any reasonable jar were finally eliminated, so that the cycles practically closed or merely gave evidence of a difference of slope in the outgoing and return series. Such an effect would be expected from viscosity and hysteresis.

I was at first inclined to regard the small values of the modulus E as an actual or trustworthy result, in keeping with the peculiar crushing stress applied. But inasmuch as E may be increased to the normal value by successively decreasing the diameter of the rod, in the case of glass and even of brass, the small values of E must be associated with the lack of contact at the abutments of the rod. Rods about 1 to 2 cm. in length should not be thicker than 1 or 2 millimeters (ratio about 10 to 1), if results are to be assured in their absolute values. And here again a thin rod, r, with *two* thick ends, if both ends are firmly clutched without strain, is the ultimate desideratum. Figures 3, 4, 2 (sheath, s), are admissible expedients, the latter being particularly convenient. The relative results are almost always smooth and admirable to a fraction of a wave length; but for relatively large sections they can not be interpreted owing to the sectional discrepancy in question. This also is relative in its character; at least for moduli markedly above 10^{10}. Thus it is as difficult to obtain the true modulus for a glass rod as for a brass rod, although the latter body is far more rigid.

It is not easy to interpret the apparant hysteresis in many of the above graphs; for this is always associated with possible changes in a complicated train of apparatus. Similarly the different rates in the outgoing and the return series may be variously explained. If the measurements are made in triplets between definite steps of pressure, this difference soon vanishes. Hence this procedure is to be preferred.

[1] Abridged from a forthcoming report to the Carnegie Institution of Washington, D. C.
[2] These PROCEEDINGS, 3, 1917, (693–696).

NATIONAL RESEARCH COUNCIL

MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

AT THE NATIONAL RESEARCH COUNCIL BUILDING,

NOVEMBER 12, 1918, at 9.30 A. M.

Present: Messrs. Cross, Howe, Johnston, Mendenhall, Merriam, Millikan, Pupin, Rous, Washburn, Woods and Woodward.

In the absence of the Chairman of the Board Mr. Woodward, upon motion, served as temporary Chairman.

The minutes of the preceding meeting of October 8, and of the intervening meetings of the Interim Committee, having been circulated by mail, were approved.

The following letter from the President of the National Academy of Sciences, Mr. Walcott, was read for the information of the Executive Board:

> It gives me pleasure to inform you that the plan for International Organization of Research presented by Dr. Hale to the Council of the National Academy of Sciences, and approved by that body at the Annual Meeting of the Academy in April, 1918, has been unanimously adopted by the Inter-Allied Conference, which was held under the auspices of the Royal Society of London, on October 9, 1918.
>
> May I request that you bring this matter to the attention of the members of the Executive Board of the Research Council at its next meeting?

The Acting Chairman of the Council presented the question of the status of the war organization of the Council in view of the fact that the war has been brought practically to an end, and Mr. Pupin

Moved: That a Committee of five be appointed to consider the problems of reorganization of the Council under present conditions, and to make recommendations. *Adopted.*

The Acting Chairman of the Council appointed the following Committee: Messrs. Merriam (Chairman), Howe, Johnston, Millikan, Pupin.

Engineering Division.—Moved: That Mr. George H. Clevenger, of the Bureau of Mines, be appointed Chairman of the Section on Metallurgy of the Division of Engineering, at a salary at the rate of $5,000 per year, the appointment to be during the present fiscal year ending June 30, 1919.

Adopted.

The meeting adjourned at 11.00 a.m.

PAUL BROCKETT, *Assistant Secretary.*

General statements as to recent activities of some of the Divisions of the Council are appended.

Division of Engineering.—The tests of tractors and of special balloons have been very successful, and have established definitely the value of these two devices; and important progress has been made in the experiments on one of the special guns and one type of tank. In addition, a new hydraulic power transmission system, and a new navigation apparatus for aircraft have been taken in hand.

Various committees of the *Section on Metallurgy* have been active and the work of the *Helmet Committee* has been hastened by increasing the experiments in pressing refractory steels into helmet shape. Important progress has been made in the electric welding of ships. An additional committee to study the metallurgical aspects of this welding has been formed.

Division of Chemistry.—The principle item of interest to be reported for this month concerns the investigations in progress at various universities under the auspices of the Division. Fifty Research Problems are now being investigated; three of these have been satisfactorily completed and the data obtained transmitted to the Government agencies needing them.

Division of Geology and Geography.—The text books for use in S. A. T. C. courses, entitled "Military Geology and Topography" and "Introductory Meteorology" appeared toward the close of the month. Several conferences were held under the auspices of the Committee on Education and Special Training on the question of providing maps for use in the S. A. T. C. courses in colleges; as a result of this consultation a special committee has been appointed, of which P. S. Smith is Chairman, to make recommendations.

The Division has taken up the study of the problems of reconstruction in the courses of instruction and research in geology and geography. This subject was referred to the Geology Committee of which J. M. Clarke is Chairman, and to the Committee on Geography of which W. M. Davis is Chairman. Each committee is to consider its particular phase of the problem, including the institution of closer relations between scientific and engineering courses and of adequate provision for pure research.

Major Douglas W. Johnson, a member of the Executive Committee of the Division, has returned from Europe after several months' absence, during which he had exceptional opportunity to study the utilization of geological and geographical information in war. Through his observations and his very clear and concise reports to the Division he has been able to supply information of great value.

Division of Medicine.—During the month of October the Medical Division of the National Research Council received for use in research from Dr. W. W. Keen an additional gift of $101.74 representing further proceeds from the sale of his book entitled "The Treatment of War Wounds." Special disbursements were made in support of work (a) on the cause of influenza, and (b) the employment of "immunized" skin grafts in the healing of infected wounds. Six new research projects were authorized.

Division of Agriculture.—Considerable time has been given to the preparation of courses for the Students' Army Training Corps, in Biology, Animals in Relation to Disease, Protozoa, and Economic Botany, and of a Wood Inspectors' course with a view to the preparation of men competent to inspect wood for various uses.

The work of the *Fertilizer Committee* is progressing satisfactorily. Plans have been made with the coöperation of the U. S. Department of Agriculture to enlarge the project to include several additional states in the Mississippi Valley. Preliminary steps have also been taken to organize group projects in the eastern and southern states with reference to the use of fertilizers.

A *Salt Nutrition* project, which has for its object the determination of the absolute salt requirements, with special reference to potash, of important crops, has been formulated and coöperative arrangements completed for the inauguration of the work.

Considerable attention has been given by the *Committee on Botanical Raw Products* to sources of fiber supply, rubber and various valuable gums and resins, and reports regarding these projects have been furnished to branches of the Government where work seemed to be most closely related.

The work of the *War Emergency Board of Plant Pathologists* has progressed very satisfactorily and several important circulars have been issued by the Board for the information of plant pathologists in this and foreign countries. The losses to our food crops through plant diseases are enormous and work of this kind offers one of the best means for increasing the food supply. The Chairman of the Division is in Europe on special business for the Government and while there will confer with the foreign representatives of the Council and with representatives of some of the European Governments regarding coöperative work in the control of plant diseases. This is especially important with reference to potato diseases as this crop must be depended upon for supplying a large part of the starchy food for Europe as well as for our own army.

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 MARCH 15, 1919 Number 3

NOTE ON THE SELF-ADJUSTING INTERFEROMETER IN RELATION TO THE ACHROMATIC FRINGES

BY CARL BARUS

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Communicated, January 3, 1919

1. *Introductory.*—This apparatus was first used for the case of coincident ray systems by Michelson and Morley[1] in their famous experiments on the Fresnel-Fizeau coefficient and has since been similarly employed by Zeeman.[2] It has so many practical advantages that a special reference here is justified. It is shown in a modified form[3] in figure 1, where L is a pencil of white light preferably from a collimator. It is separated by the half silvered plate N into the two beams $L12345T$ and $L16785T$, both of which are recombined at 5 and then enter the telescope at T. It is merely necessary to rotate any of the mirrors, say N, around a vertical axis until the two vertical white wide slit images coincide in the telescope, when brilliant fringes will be at once obtained on the coincident white fields. The central fringe is achromatic, for the system is self-compensating, or the glass paths are rigorously equal. The fringes may be enlarged to infinite size and then reduced in size again, the phenomenon being symmetrical, by rotating any mirror, say N, about a horizontal axis.

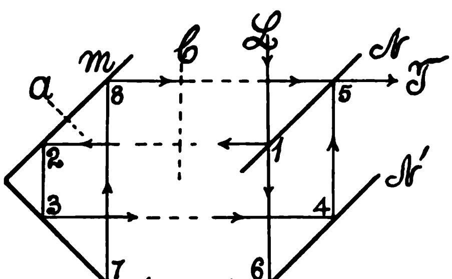
The mirrors N and N' are rigidly fixed to a carriage capable of sliding right and left parallel to the lines 85, etc. Hence the rays 85 and 21 and the rays 34 and 76 may be brought to coincidence or separated in any degree, at pleasure. If the slide were perfect the fringes would not be disturbed by this process; but few slides are perfect to this degree, and the fringes will change size somewhat, since there is rotation. Practically this is of no consequence.

The fringes, which when sharp are necessarily horizontal, may also be changed in size by inserting a plate glass compensator about 5 mm. thick in the paired beams 85, 21, or 34, 76. When this is rotated on a horizontal axis, the fringes pass through infinite size, and this arrangement is particularly adapted for the detection of the character of the fringes and will be so used presently.

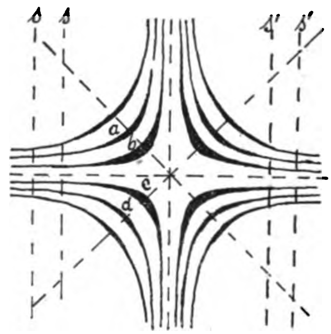
If a direct vision prism or grating is placed in front of the telescope, the spectrum is seen to be crossed by intense black lines, very nearly parallel and horizontal, but actually diverging from blue to red symmetrically up and down from the horizontal central black line. It is not necessary here, that the slit be fine. In fact it may be several millimeters broad without destroying these spectrum fringes, if essentially horizontal.

If the linear phenomenon of reversed spectra, coinciding on a certain line of the spectrum is wanted, a prism may be inserted between *m* and *m'* suitably adjusted. These fringes then also appear at once and may be put in any color, at pleasure, on rotating any mirror, say *N*, on a vertical axis. Rotation on a horizontal axis enlarges.

Finally if separate plate glass compensators are placed in the paired beams 85 and 21, for instance, and rotated around a horizontal axis independently, the fringes may be displaced up or down the slit image for the purpose of measurement. A double offset air compensator consisting of 3 right-angled



1



2

V-mirrors with their corresponding faces parallel, the central V-mirror movable on a micrometer (described in my paper on gravitation) is particularly available. Such a compensator would be placed normally to the rays 85 and 21, for instance, to give them lateral path length. In these cases the spectro-telescope may also be used, where the strong bands register the displacement in any wave length. Since the slit may be broad, there is here also a great abundance of light.

The rays 85, 21, may be made of almost any reasonable length, and distance apart, if the mirrors *N*, *N'*, *m*, *m'*, are broad. To secure greater length, the mirrors *mm'* (rigidly connected) may be moved at pleasure in the direction 58, without disturbing the fringes, good slides presupposed. The rays 85 and 21, 34 and 76 may be separated, if x is the available breadth of mirror to an extent $x \cos 45°$, by moving the rigid system *NN'* in the direction 85.

If either of the mirrors *m* or *m'* is separated (as for instance at *a* in figure 6), the part may be placed on a micrometer; but the apparatus would not then be selfadjusting; for the parts will not in general be coplanar.

To secure the best conditions for sharp strong fringes the two slit images seen in the telescope must be of equal intensity, and this depends on the half-silver N. On testing a number of plates it is usually easy to find one which fulfills this condition nearly enough. The fringes are still good, even when the intensity of images is noticeably different. The secondary images due to the reflection from glass faces are either weak or (if thick plates are used) these reflections may be blotted out by small opaque screens suitably fixed to the mirrors m and N. It is interesting to observe that with proper adjustment these secondary reflections carry their own fringes, some of which are modified in like conditions more rapidly than the main set. Intersecting fringes producing a beaded structure and fringes moving in opposite directions are also observed.

2. *Character of the achromatic fringes.*—Since the achromatic fringes are quite symmetric, consisting of a central white or fringe, flanked on either side by 3 or 4 colored fringes rapidly decreasing in intensity, it is obvious that (practically) they must consist of superposed monochromatic confocal hyperbolae. This may be well shown in the present apparatus, where the fringes are stationary and are displayed relative to horizontal and vertical lines of symmetry. To carry out the experiment, it is best to insert a single plate compensator (say C, fig. 6, 5 mm. thick) normally into the rays 85 and 21, preferably in the same vertical plane. When the plate is not perfect, it may be necessary to adjust for coincidence of slit images. If now this plate is rotated about a horizontal axis (normal to the lines 85, 21) the fringes walk laterally through the broad coincident slit images, in such a way as to clearly outline a moving design of the form given in figure 2. In other words, as a first approximation (for the case is, of course, essentially more complicated) the achromatic fringes may be assumed to be a family of confocal equilateral hyperbolas, referred to given horizontal and vertical axes. When the rays 85, 12 are at the same level, the broad slit image is in a position of symmetry relating to the hyperbolae, figure 2. When this is not the case, the image is at ss or $s's'$, with the fringes very rapidly becoming horizontal. Since this design is similarly carried out with decreasing coarseness from red to violet, it is clear that a single characteristic central achromatic fringe results, invaluable for purposes of displacement interferometry, from its smallness and since from the breadth of slit it can be made so intensely luminous. When the path difference of the rays 85, 12, in figure 1 is changed by the micrometer or by independent compensators, the figure 2 shifts bodily up or down the slit image. It is also obvious that when the fringes with white light are horizontal, they must appear as horizontal black bands in the spectro-telescope, regardless of the width of slit used; and hence these fringes also are excessively luminous, while their displacement may be referred to a definite wave length. If the interferometer is not selfadjusting, the axes of figure 2 are as a rule inclined, and fringes are obtained in all angles of altitude needing special adjustment. The spectrum fringes then demand a fine slit but are also hori-

zontal. The shift of achromatic fringes due to micrometer displacement may therefore be at once expressed in terms of the spectrum fringes rigorously in a given wave length.

A very interesting transformation of the design, figure 2, will be noticed, if by rotating any of the mirrors, *N*, *m*, figure 1 for instance, the two white slit images seen in the telescope, are passed horizontally through each other. During this motion the originally vertical, nearly linear achromatic fringe, passes through the form of the area between the hyperbolae *a* and *b*, figure 2; next through the area between *b* and *c* (coincidence of slit images); then into the area between *c* and *d*; finally again into a vertical hair line, always retaining its individuality among the surrounding colored fringes of similar shapes. The whole is particularly vivid if the fringes are observed with the ocular drawn well forward, quite out of focus. The same may be done by adding a diopter spectacle lens (convex or concave), to the objective. The bearing of this will be seen presently.

3. Curved compensators.—If the rays 85, 21 in figure 1 are brought to coincidence, it is obvious that a lens, either convex or concave, may be inserted between the mirrors *m* and *m'* and normal to the rays, without destroying the interferences, though they must be greatly modified in form. If the lens is symmetrically inserted, the two broad slit images will be equally wide; so that coincidence is perfect. The fringes so obtained are usually large, brilliantly colored circles. In case of imperfect plate they become oval and coarse. The large central disc is achromatic. To center the fringes the mirror *m'* may be rotated on a vertical and horizontal axis, until the symmetrical circular figure is obtained. Here again the individuality and even the approximate position of the achromatic fringe is retained on passing the broad slit images through each other; but the sequences of types of fringe is peculiar. As the slit images separate toward the right or the left, because of the corresponding rotation of *m* on a vertical axis, the originally colorless disc of the circular fringes not only moves to the right or left, but at the same time becomes very vividly colored. The coarse fringes now show considerable resemblance to the coronas of cloudy condensation, in which there is also a colored disc. When the slit images have been markedly separated, the disc vanishes and thinner lines appear, at first as complete circles surrounding the fading disc, but rapidly losing curvature to become vertical. Throughout the whole transformation there has been a grouping of symmetrically concentric colored circles, on both sides of the achromatic circle. To state this differently: each originally linear fringe, in turn, on expanding (slit images approaching coincidence), contracts vertically and broadens horizontally into a disc, which retains the color of the fringe out of which it originates. The same result may be obtained by moving the lens inserted between *m* and *m'* into both rays, fore and aft (direction 8, 5). Similarly the corresponding sequence between horizontal fringes appears, on moving the lens up and down. If the *m*, *m'*, mirrors are moved bodily fore and aft, however, the circular

fringes merely pass horizontally through the field without appreciably changing form.

It makes little difference whether concave or convex lenses are inserted between m and m', except that the objective of the telescope will have to be armed with a convex or a concave lens (of the same strength) respectively, to assist in focussing. But here again the most vivid effects are obtained with the ocular drawn out of focus. I examined lenses of 1, 2, and 3 diopters of focal power, concave and convex. There would be nothing against the treatment of stronger lenses, only the secondary adjustments become increasingly difficult, unless special devices are resorted to. Many of the forms are quite visible to the naked eye.

If the lens is not symmetrical in form, i.e., for plano-convex meniscus and other lenses, the simple figures above discussed become more complicated fand the fringes multi-annular.

4. *Index of refraction, irrespective of form.*—If a plane-parallel trough containing a solution of mercury potassic iodide is placed between m and m', figure 1, normally to the rays, neither the achromatics nor the spectrum fringes (broad slit admissible) are affected. Inclination to the normal position will change the size of fringes only. Hence if a piece of glass is inserted into the trough with the rays separated as at 2, 3 and 8, 7 in figure 1, one of them (say 8, 7) only passing through the glass, the spectrum fringes will change form or vanish except at that part of the spectrum in which the index of refraction of the glass and of the solution are identical supposing the dispersion coefficients to be nearly enough the same. It is thus of interest to determine to what degree this method can be practically utilized.

In the case of bodies of regular form, like lenses, spectrum fringes and even achromatics will usually appear, when the sharply seen, fine slit images coincide in the principal focus (i.e., the position of the ocular for parallel rays). But the fringes will as a rule be in other focal planes.

5. *The same. Glass plate.*—In relation to the principle, §4, in question, if a plate of glass of higher refractive index is introduced into the solution and traversed by one ray only, the original intensely black, nearly horizontal bands in the spectrum are changed to much finer lines, at a considerable angle (45°, etc.) to the horizontal. This inclination is symmetrically down toward the red, or up toward the red, according as one beam or the other traverses the glass. Moreover the size of the fringes now decreases, in much more pronounced ratio from red to violet. The achromatics have necessarily vanished. It would need special compensation (horizontal spectrum fringes) to restore them and from this compensation the difference of index between solution and glass could be computed. Had there been no difference, the horizontal fringes would have been retained in that part of the spectrum. Such experiments may be made with astonishing ease and accuracy, and I hope soon to communicate quantitative results bearing on this and the preceding paragraph.

[1] *Amer. J. Sci., New Haven*, 31, 1886, (377).
[2] *Proc. Amsterdam Acad.*, 17, 1914, (445); and 18, 1915, (1).
[3] This modification is essential, as the symmetrical apparatus of Michelson aud Morley does not admit of ray separation.

A COMPARISON OF WHITE AND COLORED TROOPS IN RESPECT TO INCIDENCE OF DISEASE

By Lieutenant-Colonel A. G. Love, M.C. U. S. A. and Major C. B. Davenport, Sanitary Corps, U. S. A.

Read before the Academy, November 18, 1918

This paper is an analysis of over half a million admissions to sick report of troops in camps in the United States. Of these 531,445 were for whites and 15,186 for colored troops. The relative frequency of disease in the two 'races' was as 974 to 1155; that is, the colored troops were about 19% more liable to go on sick report than the white troops.

The grounds of sick report are very numerous; so we shall consider only the commoner ones. All comparisons will be of the so-called mean annual rate per 1000 men. The data are those collected for the Report of the Surgeon General of the Army, 1918. Some comparative data have occasionally been included derived from the nine preceding annual reports.

In many cases the morbidity rate is almost the same for the white and colored troops. In this paper particular attention is paid to those diseases that have a strikingly different rate in the two races. It is to be kept in mind that all troops had been subjected to the same examination at induction, in order to exclude chronic diseases; and that they lived under equally good sanitary conditions.

First, may be considered the diseases that are commoner in colored than in white troops. These fall into four groups (a), those in which the excess is due to the fact that fewer of the colored men had become artificially immunized by inoculation before, or at, mobilization. They probably brought their disease to camp; or failed to get a reaction there; (b) those in which there is a lower natural immunity in the negro; (c) venereal disease and its complications, (d) other diseases.

(a) Of the diseases that are due to lack of acquired immunity, smallpox is the most striking. The morbidity rate for this disease is for colored troops 9 times that for white troops (there being 146 admissions altogether). Chickenpox was relatively 8 times as common in colored as in white troops.

(b) Of the diseases for which negroes lack relative natural resistance tuberculosis of the lungs and pneumonia take first places. There were over 2½ times the admission rate for tuberculosis of the lungs in colored as in white troops in 1917. This is a little greater difference than the average of the past

ten years,—where the colored have about 2 times the admission rate of white troops. In tuberculosis of other organs the rate for colored troops is twice that for white.

In both lobar and broncho-pneumonia the morbidity rate of colored men exceeds that of white. For 1917 the rate for lobar pneumonia was 4½ times, for broncho-pneumonia 3 times, as great for colored as for white men. In the ten year record the colored rate is 2.2 times the white rate for broncho-pneumonia. Given, then, troops under similar environment in the presence of conditions favoring pneumonia, there are relatively two to three times as many colored troops as white who show symptoms of the disease.

(c) *Venereal disease.*—In the Army that was mobilized in 1917, the rate for syphilis was about 4 times as great in colored as white troops, for chancroid 4½ times, for gonococcus infection 2½ times. Combining the data of the last ten years the rate for all venereal diseases for colored troops is a little less than double that for whites. The difference between the races in incidence of venereal diseases is probably due partly to a difference in social pressure, partly to a difference in ability to control the sex instinct.

This greater infection of colored troops with venereal disease leads to the greater incidence in that race of various complications of those diseases. Thus, retinitis, iritis, cerebro-spinal meningitis, various diseases of the spinal cord—largely, probably, complication of syphilis—are commoner among the colored soldiers. Similarly, arthritis, osteomyelitis, endocarditis, nephritis and urethritis—complications of gonorrhea—are 2 to 3 times commoner in colored troops than in white.

Of other diseases there are few in which the colored rate is markedly greater than the white. The colored indeed show about 100% more epilepsy and hysteria than the whites, more drug addiction and double the amount of neuralgia and hemorrhoids; and that is about all.

Of diseases that are less common in colored than in white troops there are, first of all, skin diseases. Thus acute abscesses and inflammations of the connective tissue of the skin are about one-third as common in colored as white soldiers; boils are one-fourth as common; dermatitis from traumatisms one-third as common and venomous bites and stings have much less effect. As is well known, the reaction of the skin to cuts differs in the two races. Colored persons tend to form keloid tumors, or skin ridges, along the scar. In civil life deaths from skin cancer are one-fourth as common in colored as white citizens. This by no means exhausts the catalogue of skin differences in the two races.

Also, in the quality of the teeth there is a difference. It is not rare to find negroes who at twenty to thirty years have lost nearly all their teeth through the use of mercury. But, on the other hand, it is much commoner to find negroes than Anglo-Saxons who, at twenty to thirty years, have teeth that show no sign of decay, even where no especial care has been taken of them. The negro teeth are naturally resistant to the organism of tooth caries.

Diphtheria, scarlet fever and German measles are from one-half to one-fourth as common in colored as white men. This experience with morbidity in camps agrees with the census mortality rates of civil life. Thus, for the registration area of the United States (1915) the rates for white and colored respectively are: for scarlet fever, 3.7 and 1.3; for diseases of the larynx, 8 and 4. Also while admissions in camp for measles-complicated-by-lobar-pneumonia are slightly greater in colored than in white troops, cases of measles-complicated-by-broncho-pneumonia are only about half as common in colored troops.

Influenza in the troops during 1917 had a rate of 28 for colored men—about half that for white men (58); and during the past ten years only about one-fourth. It is commonly held that colored children are much less apt to be attacked by the organism of infantile paralysis, which, like influenza, enters the body through the naso-pharynx. Thus, in general, the diseases of the upper respiratory tract are much less common in colored than white people.

In general the skin, not only on the surface of the body but also that which infolded to form the lining of the mouth and naso-pharynx, is much more resistant to micro-organism in negroes than in whites. The white skin seems to be relatively a degenerate skin in this respect.

Moreover, the nervous system of the uninfected negroes shows fewer cases of instability than that of whites. Thus, there is only about one-third as much neurasthenia recorded in camps in 1917; about one-third as many cases of constitutional psychopathic state and one-half as much alcoholism in colored as in white troops. Functional cardiac disturbances of nervous origin are half as common. Eye defects are relatively few, and there is about half as much liability to refractive errors as in whites. Ear diseases are about half as common.

Metabolically, the uninfected colored troops seem less liable to disturbances than white troops. Diabetes is only half as common in the negroes as the whites and about the same is true of urinary calculi. Inflammations of the gall bladder are also much less frequent.

To sum up: The colored troops are relatively less resistant to diseases of the lungs and pleura as well as to certain general diseases, like tuberculosis and smallpox; they are also much more frequently infected with venereal diseases and suffer wide spread complications of these diseases.

But the uninfected negro is highly resistant to diseases of the skin, mouth and throat. He seems to have more stable nerves, has better eyes and metabolizes better. Thus, in many respects the uninfected colored troops show themselves to be constitutionally better physiological machines than the white men.

(Publication approved by the Office of the Surgeon General.)

Admissions to Sick Report, United States Troops, for Year 1917
(*From Table No. XV, Report Surgeon General, U. S. Army, 1918*)

Numbers of International Classifications	United States		
		White	Colored
	Mean strength	545,518	13,150
	Cause of admission to sick report	Ratio per 1000 of mean strength	
	I. Infectious Diseases (Excluding Tuberculosis and Venereal)		
1	Typhoid fever	0.41	0.23
2	Typhus fever	0.01	
3	Relapsing fever (all)	0.05	
4	Malarial fevers (all)	7.50	5.85
4a	Malarial fever, estivo-autumnal	0.20	0.38
4b	Malarial fever, mixed	0.01	0.08
4c	Malarial fever, quartan	0.10	0.08
4d	Malarial fever, tertian	4.10	3.19
4e	Malarial fever, undetermined	3.09	2.13
5	Smallpox	0.22	1.98
6	Measles (all)	83.11	87.07
6a	Measles, uncomplicated	78.96	82.28
6c	Measles with broncho pneumonia	2.48	1.37
6d	Measles with lobar pneumonia	1.25	1.75
6f	Measles with other complications	2.42	1.67
7	Scarlet fever	3.58	0.84
8	Whooping cough	0.04	0.08
9	Diphtheria and results	2.12	0.46
10	Influenza (all)	58.44	27.83
14	Dysentery (all)	0.87	0.84
14a	Dysentery, bacillary	0.09	0.08
14b	Dysentery, balantidic		
14c	Dysentery, entamæbic	0.35	0.30
14d	Dysentery (other protozoal)		
14e	Dysentery, unclassified	0.42	0.46
15	Plague	0.01	
17	Leprosy	0.01	
18	Erysipelas	1.17	1.44
18b	Meningitis, cerebrospinal (epidemic)	1.68	4.11
19a	Chicken pox	0.44	3.42
19b	Diphtheria bacillus carrier	0.45	1.14
19c	German measles	16.36	4.41
19d	Hemoglobinuric fever (nonmalarial)	0.01	0.08
19e	Paratyphoid fever A	0.02	
19f	Paratyphoid fever B		
19g	Trench fever		
19i	Typhoid bacillus carrier	0.02	
19j	Typhoid vaccination	17.65	11.25
19k	Vaccinia	15.69	6.16
19l	Mumps	38.93	37.26

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission in sick report	Ratio per 1000 of mean strength	
20	Septicæmia, general	0.09	
20b	Pyæmia, surgical	0.01	
21	Glanders	0.01	
22	Anthrax	0.01	
24	Tetanus	0.01	
25	Trench foot		
25a	Mycoses (others)	0.04	
26	Pellagra	0.04	0.23
27	Beriberi		
47	Rheumatic fever	8.68	6.39
11, 19, 18a, 18c, 20a	Infectious disease (others)	2.48	3.19
	II. Tuberculosis (All)		
28	Tuberculosis of lungs	11.69	32.02
29	Miliary tuberculosis, acute	0.07	
30–35	Tuberculosis (others)	0.76	1.90
	III. Venereal Diseases (All)		
37	Syphilis (all)	14.22	54.68
37a	Syphilis, hereditary	0.03	
37b	Syphilis, primary	3.85	20.08
37c	Syphilis, secondary	8.34	22.13
37d	Syphilis, tertiary	2.00	12.47
38a	Chancroid	12.39	52.78
38b	Gonococcus infection	82.47	203.04
	IV. General Diseases (Others)		
36	Rickets	0.12	
39–45	Cancer and other malignant tumors	0.15	
46	Benign tumors	1.17	1.75
48	Arthritis	5.74	10.58
48a	Gout	0.03	
49	Scurvy	0.01	
50	Diabetes mellitus	0.18	0.08
51a	Diseases of the thyroid gland (goiter simple excepted)	1.09	0.38
51b, 52, 53b, 116	Diseases of the ductless glands (thyroid excepted)	0.07	0.08
53a	Leukemia	0.01	
54	Anemia	0.20	0.23
55a	Purpura rheumatica	0.01	
56	Inebriety, drug addiction	0.75	1.30
56a	Alcoholism	2.45	1.06
57–59	Poisoning, chronic	0.04	
55, 88	General diseases (others)	5.12	8.14

ADMISSIONS TO SICK REPORT—*Continued*

Numbers of international classifications	United States		
		White	Colored
	Mean strength	445,528	13,150
	Causes of admission to sick report	Ratio per 1000 of mean strength	
	V. Nervous System, Diseases of (All)		
61	Meningitis, simple	0.10	0.23
62	Locomotor ataxia	0.07	
63	Diseases of the spinal cord (others)	0.21	0.53
64	Apoplexy	0.04	
66	Paralysis, cause not specified	0.38	0.46
69	Epilepsy	2.88	6.54
72	Chorea	0.14	
73	Hysteria and results (psychoses excepted)	1.17	2.05
73a	Neuralgia	1.83	3.57
73b	Neuritis	1.13	1.06
74	Neurasthenia	1.50	0.46
60, 65, 70, 73, 74a	Diseases of the nervous system (others)	2.21	2.59
	VI. Mental Alienation		
67	General paralysis of the insane	0.05	0.08
68a	Constitutional psychopathic states	0.86	0.30
68b	Dementia precox	1.46	1.75
68c	Mental deficiency	2.80	2.89
68d	Psychasthenia	0.17	
68e	Psychoneurosis	0.42	0.30
56b	Psychoses (alcoholic)	0.25	0.23
68f	Psychoses (nonalcoholic)	0.82	1.37
68	Mental alienation (others)	0.21	0.30
75	*VII. Eyes and Their Annexa, Diseases of*		
75a	Conjunctivitis	7.50	8.44
75b	Trachoma	0.84	0.38
75c	Refractive errors	1.01	0.68
75e	Iritis	0.44	1.06
75f	Keratitis	0.52	0.91
75g	Choroiditis	0.08	
75h	Retinitis	0.10	0.76
75i	The eyes and annexa (other diseases of)	2.71	5.40
76	*VIII. Ear, Diseases of*		
76a	Mastoiditis	0.45	0.15
76b	Otitis media	10.64	5.48
76c	Ears, other diseases of	0.83	0.91
86	*IX. Nasal Fossæ, Diseases of*		
86a	Adenoids	0.38	0.30
86b	Deviation of nasal septum or spurs	2.04	0.30
86c	Sinusitis	1.49	1.60
86d	Nasal fossæ, other diseases of	13.16	10.04

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission to sick report	Ratio per 1000 of mean strength	
	X. Throat, Diseases of		
87	Larynx, diseases of	8.38	4.56
100	Pharynx, diseases of	14.54	12.47
100a	Tonsils, diseases of	77.64	50.95
	XI. Circulatory System, Diseases of		
77	Pericarditis	0.08	0.08
78	Endocarditis, acute	0.15	0.53
79	Valvular heart diseases	4.21	6.08
79a	Heart, other organic diseases of	1.94	4.56
80	Angina pectoris	0.13	0.35
81a	Aneurysm	0.02	
81	Arteries, other diseases of	0.13	0.15
82	Embolism and thrombosis	0.05	
83a	Hemorrhoids	6.55	14.60
83b	Varicocele	3.72	2.51
83	Veins, other diseases of	1.89	1.67
84a	Status lymphaticus		0.15
84	Lymphatic system, other diseases of	4.07	5.86
85a	Cardiac arrhythmias	0.06	
85b	Cardiac murmur, not organic	0.03	0.08
85c	Cardiac disorders, functional	0.19	0.08
85d	Hemorrhage	0.48	0.68
85	Circulatory system, other diseases of	1.08	2.36
	XII. Respiratory System, Diseases of		
89, 90	Bronchitis	74.10	61.75
91	Broncho-pneumonia	2.51	7.30
92	Pneumonia, lobar	11.47	53.69
92b	Pneumonia, unclassified	0.08	0.23
93	Pleurisy	5.17	9.43
95	Gangrene of lung	0.01	
98	Abscess of lung	0.01	
96, 97	Respiratory system, other diseases of	2.17	4.03
	XIII. Digestive System, Diseases of		
99	Mouth and annexa, diseases of	3.91	4.18
101	Esophagus, diseases of	0.01	
102	Ulcer of the stomach	0.49	0.30
103	Stomach, diseases of (ulcer and cancer excepted)	10.68	9.73
106	Ankylostomiasis	0.58	
107a	Intestinal parasites, others	0.41	0.38
108	Appendicitis	9.83	6.31
109	Hernia	8.98	10.57

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES	White	Colored
	Mean strength	545,518	13,150
	Causes of admissions to sick report	Ratio per 1000 of mean strength	
109a	Intestinal obstruction	0.15	0.23
105, 110	Intestines, other diseases of	47.35	33.76
112	Hydatid tumor of liver		
113	Cirrhosis of the liver	0.05	
114	Cholelithiasis	0.08	
115	Cholecystitis	0.87	0.38
115a	Liver, other diseases of	1.97	0.61
117	Peritonitis, acute diffuse	0.02	
117a	Peritonitis, acute local	0.02	
117b	Peritonitis, chronic	0.01	0.08
117c	Peritoneal adhesions	0.37	
118	Other diseases of the digestive system	0.23	0.30
118a	Pancreatitis		
	XIV. Genito-Urinary System and Annexa, Nonvenereal Diseases of		
119	Nephritis, acute	0.47	0.76
120	Nephritis, chronic (Bright's disease)	0.71	0.91
122	Kidney and annexa, other diseases of	0.72	0.91
123	Calculi of urinary passages	0.43	0.23
124	Bladder, diseases of	3.24	4.41
125	Urethra, diseases of	0.83	1.37
126	Prostate, diseases of	0.19	0.15
127	Male genital organs, diseases of (nonvenereal)	4.47	6.54
	XV. Skin and Cellular Tissue, Diseases of		
142	Gangrene	0.03	
143	Furuncle	8.78	2.13
144	Acute abscess	5.83	2.89
144a	Cellulitis	4.52	1.14
145	Skin and annexa, other diseases of	17.29	11.86
	XVI. Bones and the Organs of Locomotion, Diseases of		
146	Osteomyelitis	0.36	0.61
146a	Bones, other diseases of (tuberculosis excepted)	0.93	1.44
147	Joint, diseases of (rheumatism and tuberculosis excepted)	1.59	1.98
149	Pes planus	8.40	9.28
149a	Organs of locomotion, other diseases of	10.71	9.89
	XVII. Congenital Malformations and Ill-Defined Diseases		
150	Congenital malformations	8.23	12.09
189	Not specified or ill-defined diseases	36.17	51.41
	Total for diseases	882.54	1064.41

ADMISSIONS TO SICK REPORT—*Continued*

NUMBERS OF INTERNATIONAL CLASSIFICATIONS	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission to sick report	Ratio per 1000 of mean strength	
Office classification	*Traumatism Produced by External Causes* Total	91.68	90.42
03	Bites	0.13	
04	Burns	1.54	1.06
05, 06	Burns, chemical and X-ray	0.34	0.08
09	Compression (brain)	0.01	
10	Concussion (brain)	0.63	0.23
12	Contusion	3.41	0.68
14	Crushing	0.43	0.23
17–20	Dermatitis from traumatism	2.23	0.68
21	Dislocation of joint	1.84	1.52
62	Dislocation, articular, cartilage of knee	0.16	
22, 23	Drowning (except suicides)	0.04	
28	Epiphyseal separation	0.01	
08, 11, 13, 32,33, 68,69	Eye and annexa, traumatisms	0.34	0.46
34	Foreign bodies	0.25	0.30
36	Fracture, simple	7.63	4.41
35	Fracture, compound	0.52	0.46
38–48	Fracture, faulty union	0.13	0.23
0x	Fracture near joint, with dislocation	0.02	
37	Fracture, comminuted	0.22	0.30
50	Gases deleterious, absorption of	0.03	
51	Gunshot wound	1.02	1.44
552–56	Hemorrhage and results	0.09	
59	Homicides	0.01	0.23
66	Poisoning by food	1.04	0.61
67	Poisoning, other acute	0.22	0.08
96	Secondary result of injury	5.55	11.79
73	Sprain of joint	18.69	16.73
75	Strain	2.88	3.19
79	Suicide	0.05	0.15
81	Synovitis, traumatic, cause undetermined	0.97	0.91
85	Venomous bites and stings	0.31	0.08
93	Wounds, multiple	0.07	0.08
94	Wounds, penetrating	0.16	0.53
92	Wounds, extensive	0.01	
95	Wounds, perforating	0.90	1.82
86–91	Wounds, others	30.62	32.02

ADMISSIONS TO SICK REPORT—*Concluded*

NUMBERS OF INTERNATIONAL CLASSIFICATIONT	UNITED STATES		
		White	Colored
	Mean strength	545,518	13,150
	Causes of admission to sick report	Ratio per 1000 of mean strength	
00–02, 07, 15, 16, 24–27, 29–31, 49, 57, 58, 60, 61, 63–65, 70–72, 74, 76–78, 82–84, 97–99	Traumatisms, others	9.14	10.04
80	Sunstroke	0.04	0.08
	Total for diseases	882.51	1064.41
	Total for external causes	91.69	90.42
	Grand total	974.19	1154.83

PERTURBATIONS AND TABLES OF THE MINOR PLANETS DISCOVERED BY JAMES C. WATSON

By ARMIN O. LEUSCHNER

BERKELEY ASTRONOMICAL DEPARTMENT, UNIVERSITY OF CALIFORNIA

Read before the Academy, April 16, 1916

Among the many important contributions to Astronomical Science by James C. Watson, one of the original members of the National Academy of Sciences, is the discovery of twenty-two minor planets commonly known as the Watson asteriods. The first of these, (79) Eurynome, was discovered at Ann Arbor on September 14, 1863, and the last, (179) Klytaemnestra, on November 11, 1877, three years before his death. By his will a fund was bequeathed in trust to the National Academy of Sciences for the purpose of promoting astronomical research. One of the objects specifically designated was the construction of tables of the perturbations of the minor planets discovered by the testator. From the beginning Prof. Simon Newcomb was a

member of the board of Watson Trustees; later he became chairman of the board, and remained in that capacity on the board until his death in 1908, being succeeded by Prof. E. C. Pickering, whose associate trustees now are Professors E. B. Frost and W. L. Elkin. The National Academy of Sciences through its various boards of trustees has at all times made the most persistent efforts to discharge the chief obligation imposed by the trust, by providing tables of the perturbations of the Watson asteroids.

My own connection with this great undertaking is best explained by the following substance from a letter addressed to me by Professor Newcomb under date of June 4, 1901.

> I have consulted with the Watson Trustees on the subject of having the perturbations of certain Watson asteroids computed under your direction. We are all favorable to the project provided we can have some assurance of success. We have had the coöperation of many capable men but so far the result has not been satisfactory. The general outcome of the matter is that we are supplied with a mass of papers, computations, perturbations, and perhaps tables, but nothing has yet reached us in a form complete and perfect for publication as tables. We had just about reached the conclusion that the work must be done here at Washington under my personal direction and according to formulae which I should supply.

For further details in regard to the earlier history of the work I must refer to the introduction written by Professor Newcomb in March, 1908, to the tables of twelve Watson planets completed under my direction in 1907 and published in 1910 as volume X, Seventh Memoir of the National Academy of Sciences.

Actual work was commenced by myself with the assistance of Drs. Russell Tracy Crawford and Frank Elmore Ross according to a somewhat definite program carefully arranged by Professor Newcomb. Our investigations were to be confined to those thirteen planets, for which so far no attempt for derivation of the perturbations had been made, Professor Newcomb's purpose being to have the difficulties encountered with the remaining planets investigated under his personal direction at Washington. One planet, (132) Aethra, was to be entirely excluded from the program, having remained lost since the year of its discovery in 1873. But in April, 1903, it was agreed that all the previous investigations should be turned over to me. At the same time it had become apparent that the success of the undertaking demanded that I should be given the utmost freedom in planning and conducting the investigations, to which the trustees readily consented.

Today it is my great privilege to be able to report to you, that except for the completion of the manuscript for publication, and some minor numerical work, the task is accomplished and that the obligations of the National Academy under the Watson Trust pertaining to the perturbations and tables of the minor planets discovered by Watson, will soon be fully discharged.

It is impossible in this brief space of time to give a complete history of the work, including the part taken by my numerous assistants. I must also refrain from reporting on the many scientific results attained as to methods of attack

and solution, as these involve mathematical detail. I can touch here only on a few points which must serve to illustrate the general character of the work. But first of all it would seem necessary briefly to review what objects are to be served in determining perturbations.

When a planet moves solely under the central attraction of the Sun it describes an ellipse defined in space by six elements or constants, of which for our present purposes three are of prime importance, namely the mean daily motion, or briefly, mean motion, which is the average angular velocity about the Sun, or 360° divided by the period of revolution, in days; the eccentricity of the ellipse; and the inclination of the plane of its motion to the ecliptic. The six elements at any instant depend on the three coördinates defining the position of the planet at the instant and on the three components of velocity. When the constant elements are ascertained and the body continues to move solely under the Sun's attraction, the position and velocity of the planet may be obtained from them at any other time and vice versa. Such positions and velocities are called undisturbed positions and velocities. But if to the Sun's attraction is added that of one or more major planets, like Jupiter, then the velocity is changed and the planet will depart from its undisturbed ellipse. These departures are called perturbations. From a disturbed position and velocity at any instant elements may again be determined for that instant. They represent disturbed elements, as compared with the former. Thus different elliptic elements may be computed for different instants from the corresponding positions and velocities attained under the attraction of the Sun and major planets. These are called osculating elements. Any one set may be adopted as undisturbed and the differences of the others from that set represent the perturbations in the elements.

Whether the perturbations are expressed in the elements or in the coördinates, they are determined by the integration of differential equations, the solution of which in the present state of astronomical science involves expansion in trigonometrical series. The integration introduces divisors which become very small, when the mean motion of the minor planet is in a commensurable ratio to that of a disturbing planet. Therefore since Jupiter's mean motion is 299″ or nearly 300,″ such series fail for instance for a planet with mean motion nearly 300″ or 600″ or 900″. Since Hansen's method, to the application of which our work was originally to be restricted, is based on series of the character referred to, it was bound to fail for planets with a commensurable ratio. This was our actual experience with the first planet undertaken.

This difficulty, however, is purely a mathematical one and has been overcome by Bohlin in his "Gruppenweise Berechnung der Stoerungen" for the group 1/3 by the introduction of the simple expedient of using the exponential for the trigonometrical form in the series. In fact he has published tables for the exact commensurability 1/3 in which all of the elements appear explicitly in the coefficients. The series referred to progress according to

powers of the eccentricities of the disturbed and disturbing planets and of the mutual inclination of their orbit planes. In order to avoid unmanageable expressions Bohlin had to confine himself to terms of the second and sometimes third degree in these quantities, while Hansen's method with the aid of Bessel's functions for the eccentricities imposes no restrictions in this respect.

Bohlin's method otherwise closely resembles Hansen's in the treatment of the perturbations, the only other distinctive feature being that for planets with mean motion nearly commensurable with Jupiter's, the perturbations are developed by Taylor's theorem in ascending powers of a quantity w which depends on the difference between a multiple of a planet's actual mean motion and a multiple of Jupiter's so that ultimately series within series progress according to powers of the mass of the disturbing body, of the eccentricities of the disturbed and disturbing bodies, of the mutual inclination of their orbit planes, and of the quantity w.

Now several of the Watson planets belong to the Hecuba Group or Group 1/2, having a mean motion of nearly $600''$ or about twice that of Jupiter, and as no theory then existed for this group it was decided to develop the theory and tables for this group 1/2 on Bohlin's plan for the group 1/3. But owing to the great complexity of the problem and the intricate transformations involved it was thought wise first of all to assure ourselves of an exact understanding of Bohlin's method by reproducing selected values of his tables for the group 1/3. In this we failed in many instances. After much fruitless search for the cause of the discrepancies these were called to the attention of Bohlin, who promptly replied that he had become aware of inaccuracies in his theory and tables and that he had already completed a revision of his work, sending at the same time advance proof sheets, verifying our conclusions. We now felt safe in attacking the mathematical theory of group 1/2 and after another year's work on the theory and tables of group 1/2 preparatory to the application of Bohlin's method to the Watson planets of that group, we learned from Bohlin that von Zeipel was engaged in the same task. A little later we received von Zeipel's printed tables. These we at once compared with our own, many transformations from one to the other being necessary on account of the difference of developments used, but to our dismay we discovered many disagreements. By correspondence these have practically all been cleared up and thanks to the careful system of checks adopted in our work we found it unnecessary to change any of our results. In some minor respects we still differ, but the expressions on which the numbers in question are based are so complicated that von Zeipel doubts whether he can remember how he has obtained his values. We are thus abiding by our own results, which have been fully verified as I shall show a little later.

The mathematical and numerical work involved in the revision of von Zeipel's theory has been performed under my direction by Miss Anna Estelle Glancy and Miss Sophia H. Levy. The former has also prepared a complete

set of directions for the application of von Zeipel's theory for the Hecuba Group and has illustrated the same with full numerical details for his example, 10 Hygiea, for the guidance of computers.

The work to which I have just referred serves to indicate how the original program of routine computation to the first order of mass with respect to Jupiter by Hansen's established formulae was completely changed to include extensive theoretical investigations.

Although Hansen's and Bohlin's methods—the latter extended by von Zeipel and ourselves, and also by D. F. Wilson, to provide tables for groups 1/2, and 5/2 in addition to group 1/3—were entirely sufficient to master the Watson planets, for which, however, the group 5/2 was not applied, an extensive theoretical analysis has also been made of Brendel's recent methods based on the researches of Gyldèn. These methods appeared while our work was well under way. In passing I might state that all this theoretical work was done independently of the Watson funds.

Aside from these purely theoretical problems the Watson work naturally falls under two heads:

(1) The investigation of the causes of the failure of the earlier work as referred to by Newcomb.

(2) The application of the most suitable methods to the planets on which work had not been undertaken.

In addition the ultimate investigation of the cause of the loss of (132) Aethra was constantly kept in mind.

Under the first head I wish to make especial reference to the work done by Eichelberger on (93) Minerva. Professor Newcomb naturally suspected an error of computation on account of an apparent motion of the node indicated by the outstanding discrepancies in the representation of the observations. Newcomb himself attempted to correct Eichelberger's investigations by further computations at Washington. But later I found the whole trouble to be due to the fact that too much had been expected from first order perturbations by Jupiter, that is, perturbations involving only the first power of the mass. I suggested that a repetition of the work on Minerva to only four or five places would accomplish as much as the previous investigation to seven figures and that the residuals of the observations would reveal the same implied motion of the node; that the latter was of a higher order; and that it could be determined empirically either from our own or Eichelberger's outstanding residuals. Professor Newcomb welcomed this test and all of Eichelberger's work was repeated under the plan proposed, but on the basis of mean instead of Eichelberger's osculating elements and with the mean instead of the eccentric anomaly as independent variable, with the result that the suspected cause of the trouble was fully substantiated. The outstanding discrepancies which were really negligible under the original program but appeared large because of the great accuracy with which Eichelberger had computed, were then removed by an empirical determination of the still remaining motion of the node.

At first Professor Newcomb was greatly surprised by the considerable difference between our coefficients of the perturbations and those of Eichelberger, but formulae were soon derived which enabled me to deduce one from the other on the basis of the difference in elements used and thus to verify both sets of developments. After this investigation Eichelberger's original tables were published in an abridged form.

More serious complications were encountered in the revision of other computations, which were due to misconceptions of theory and numerical errors.

The integration of the differential equations for the perturbations gives rise to certain constants in the developments, to terms multiplied by the time (secular terms), and to purely trigonometrical or periodic terms. All of these added to the undisturbed coördinates give the disturbed place. Some of these terms are of the same form as terms employed in computing the undisturbed positions and may be combined with them or introduced in the elements. It then becomes merely a matter of arbitrary classification or combination, as to what part of the disturbed coördinate shall be directly computed from properly changed elements and what part shall be left over as perturbations. When all but the purely periodic terms are put into the elements, we may deduce a sort of mean elements, and we can thus speak also of a mean mean motion. These elements involve properly chosen constants and are not of the sort that they represent the actual position and velocity at any particular time. It was found that some of my predecessors in the work had misinterpreted the terms they were dealing with although their developments were numerically correct.

The tracing of these and other inaccuracies for the planets to be revised have absorbed more time and effort than all the work on the planets that had not been previously undertaken. To simplify the determination of the constants and to secure greater accuracy Professor Newcomb's program originally called for adopting the mean of the several sets of elements printed for each planet in the B. J. as approximate mean elements, but later we shall see that in the case of Andromache the actual mean elements lie outside of the osculating elements obtained from observations and special perturbations over a period of thirty-three years. Thus we actually at first started with elements which were neither osculating nor mean elements, which involved corresponding difficulties later. But it should not be forgotten that at that time the whole problem of proper attack was still in an experimental stage.

With reference to Brendel's very exhaustive and brilliant methods I may remark that I have come to the conclusion that even if they stand the test of the very critical cases of near commensurability, for practical purposes they do not have the advantage of the same simplicity as a proper choice between Hansen's and Bohlin's methods.

For six of the twelve sets of tables published in 1910 comparison of observations taken in 1912–13 have been made with theory by Miss A. R. Kidder, with an agreement of less than the proposed 1′ of arc, excepting in one

case, (133) Cyrene, which although approaching the Hecuba type had been developed by Hansen's method in the early stages of our investigations. As the observation used in this case, however, is only approximate, a further test may show the slight discrepancy to be erroneous. These comparisons represent a remarkable result for the published tables, as in these cases some twelve years had elapsed since the last opposition used for the correction of the elements and for the tables.

In conclusion I may refer briefly to two of the most interesting Watson planets.

The Watson planet which promises to become one of the most interesting of the twenty-two under consideration, if not of all the 1400 or more so far discovered, is (175) Andromache, which belongs to the Hecuba group, or group 1/2, the mean daily motion being approximately twice that of Jupiter. Andromache was found by Watson on October 1, 1877 and was the last but one discovered by him, his latest discovery, as stated before, being (179) Klytaemnestra on November 11, 1877. Andromache was assiduously observed for one month, 43 observations at widely scattered observatories being secured by the most competent observers. The calculation of the orbit yielded a mean motion of approximately 550″. In spite of diligent search the planet was not found again from any predictions based on the original elements. This failure to find Andromache is a striking illustration of the fallacy of some of the orbits included as thoroughly reliable in our published lists. On May 19, 1893 a presumably new planet was discovered at Nice and given the provisional designation 1893 Z. From observations extending over more than two months an orbit resulted resembling the orbit of Andromache, except for the mean motion which was found to be at first 617″ and later 610.″

The following possibilities existed then with reference to Andromache and 1893 Z. (1) the planets might be identical, the adopted mean motion of 550″ of Andromache being erroneous. (2) The planets might be identical and the mean motion might have changed from the smaller value of 550″ to the larger value of 610,″ passing through 598″ which is twice the mean motion of Jupiter. If this were true a phenomenon of the utmost importance to science, known as libration, would come under consideration. (3) The planets might not be identical.

A study of the observations of Andromache revealed an uncertainty of the comparison star of one of the fundamental positions. From the orbit of 1893 Z positions were then calculated for the dates of observation of Andromache in 1877 and a satisfactory agreement found. The identity of the two planets was thus established in 1893–94. At the same time the hope for the occurrence of the first case of libration for a planet of the group 1/2 was shattered, for the mean motion in 1877 turned out larger by several seconds than the 610″ calculated for 1893. On the basis of these computations Andromache was also found on some Heidelberg plates taken in 1892.

The close commensurability of the mean motion with twice Jupiter's mean motion made it apparent that a satisfactory determination of the perturbations of this planet by Hansen's method was entirely out of the question, in fact impossible. After the revision of von Zeipel's tables for the Hecuba group, to which I have already made reference, this, our most difficult and striking case of the Hecuba group, was selected as a test case for our tables. After applying our perturbations, the least squares correction was based on ten oppositions from 1894 to 1907 with eminently satisfactory results, not only for these thirteen years but also for the 1877 oppositions, seventeen years before, and for a subsequent opposition in 1911. According to Miss Sophia H. Levy's computations, the perturbations of this planet are the largest we have experienced, the coefficient of the largest term in the perturbation of the mean anomaly reaching nearly 30°. For the 1877 opposition the perburbation in the mean anomaly is 24°. These amounts generally produce double the displacement in geocentric position at opposition and yet an approximate right ascension and declination published by Wolf in 1911 is represented by our theory to within 4′ of arc in right ascension and less than 1′ in declination. A further very slight revision of the theory is contemplated before publication to include some terms of higher degree depending on the very large eccentricity of 20°. This revision will make this representation still more satisfactory, although the larger part of all outstanding differences is due to perturbations by Saturn.

Even then our representation is much closer than was to be expected. Another approximate position has recently been published by Wolf for January 16, 1914. This is equally well represented, showing that the theory has definitely been verified by observation. The importance of this result lies in the fact, that with the most difficult case of the Hecuba type conquered, the revised tables of von Zeipel now provide a ready and accurate means of representing the motions of all planets of this type at present known. Confirmation of this statement is afforded by our subsequent work on (104) Clymene, (106) Dione, and (168) Sybilla of the same group with mean motions of 634″, 629″, and 572″.

But this planet has another striking significance. To prevent its loss, pending the computation of its theory under the Watson Fund, Berberich of the Kgl. Rechensinstitut, Berlin, has applied the laborious process of special perturbations by Jupiter and Saturn for the determination of osculating elements. The extent of his unpublished work was not known to me, until some years ago I addressed an inquiry to Berlin in regard to any unpublished data, particularly the Saturn perturbations.

Among the unpublished data are osculating elements for epochs in 1877, 1892 and from then on for practically every year until 1910. According to these elements the mean motion in 1877 was 617″.7. In 1892, fourteen and a half years later, it was 614″. For three and a half years it oscillated about this figure and then in another fourteen and a half years to 1910 gradually

diminished to 608″.2, the last figure available. Thus in 1910 Hecuba had already encroached well within the previously known gap about 600″.

In von Zeipel's theory a transformation of the given osculating elements is made at the outset with the aid of the general tables for the group 1/2. The resultant elements which are retained as constants until the theory is completed, are not exactly mean elements in the sense defined above, but they approximate the same. Since from 1877 to 1910, or in thirty-three years, the osculating mean motion has gradually diminished by approximately 10″, it might be supposed that our own approximate starting mean mean motion would lie somewhere within this range, or at least be less than the original maximum value of 617″.7 in 1877, but as a matter of fact our approximate mean mean motion is larger by 1″.8. The numerical developments indicate that the mean motion may continue to diminish. Whether the mean motion of 598″ will actually be passed, will form a significant problem of research in the theory of motion of minor planets. But the following facts stand out prominently even now: (1) The extent of the hitherto known gap about 600″ has been greatly diminished; (2) stability has not been impaired thereby as our representation of the 1914 observation shows, for which the perturbation in the mean anomaly has now diminished from the original 24° in 1877 to a comparatively small quantity. (3) E. W. Brown's conclusion stated in his vice-presidential address before the Section A of the A. A. A. S. that if instability exists at or near 598″, it must be for mean motion exceedingly close to that figure has been verified, and (4) if Andromache's mean motion should pass through 598″ stability will probably not be impaired, because our developments indicate no discontinuity for that condition.

Another planet to which I desire to refer is the lost planet Aethra. From investigations by Dr. Dinsmore Alter the cause of the loss of Aethra is due to the fact that the elements on which extensive search has been based for several decades are wholly unreliable, although published to seven figures. In fact the observations can be represented by orbits ranging in mean motion from 800″ to 900″ and more. All that can be said about this planet at the present time is that it may be anywhere at any time within a belt of the celestial sphere 2°20′ wide and that its rediscovery and identification will be a matter of chance, unless it is located on the supposition that an unknown planet discovered in 1913 at the Lowell Observatory, but observed only once, was Aethra. An orbit representing all the original observations of Aethra from June 13 to July 5, 1873, as well as the position of the unknown object in 1913 would make the average mean motion in the interval of forty years equal to about 883″. Details of the investigation and a search ephemeris extending to March 1 of this year, were published in a Lick Observatory Bulletin in November, 1915, but war conditions in Europe and unfavorable weather in California seem to have prevented adequate search. (An exhaustive, but unsuccessful, search has since been reported by Dr. Anna Estelle Glancy in *Astronomical Journal*, **31**, No. 723, April, 1918.)

With reference to the other Watson planets it may safely be asserted that excepting the need of further observations of Andromache for reasons stated above, all of these planets may be stricken from any observing program for decades to come, since they may readily be identified if science so demands, on the basis of the orbits and perturbations determined under the auspices of this academy.

CONGENITAL ARTERIO-VENOUS AND LYMPHATICO-VENOUS FISTULAE. UNIQUE CLINICAL AND EXPERIMENTAL OBSERVATIONS.

By William S. Halsted

Medical School, Johns Hopkins University

Read before the Academy, November 18, 1918

A. Advance of proximal arterial dilation conforming to shifted position of fistula.—Thanks to the assistance of highly competent secretaries I have abstracts of about 400 cases of arterio-venous fistula. These have been studied with special reference to occasional observations on the dilation of the arteries. In 52 instances proximal dilation of the arterial trunk has been noted. I am quite sure that in almost every instance in which the fistula had existed for two or more months proximal dilation of the artery would have been demonstrable if looked for.

Congenital arterio-venous fistula is rare, particularly so when unassociated with naevus. We have been able to find reports of only 3 cases without and 6 with naevus. Of the former none has been cured, unless we except a case (von Eiselsberg's) in which an attempt to cure a fistula between the popliteal artery and vein was followed by gangrene, necessitating amputation of the thigh.

The following case, unique in several particulars, is reported to record the arterial changes observed at 2 operations, the second performed six and one-half years after the first. The patient, a girl aged eleven years, was operated upon November 15, 1911, for a congenital arterio-venous fistula below the angle of the jaw on the right side. After the removal of enormously dilated veins (lantern slide) it was found that the fistula was between one of these and the external carotid artery near the bifurcation or ventricle of the common carotid. Fortunately a careful note was made of a very small anomalous, ascending branch given off from the external carotid just proximal to the fistula (lantern slide). There was great dilation of the common carotid artery and of the external carotid proximal to the fistula, whereas the internal carotid was surprisingly small (lantern slide). The vessels concerned in the fistula formation were excised, the aberrant artery happily being spared. The child was relieved by the operation of very distressing symptoms, but a few

weeks later signs of a second, smaller fistula developed at a distal point just below and in front of the ear. A second operation, proposed frequently, was not acceded to until last spring, six and one-half years after the first. At this operation, performed by Dr. Mont Reid in my presence, remarkably interesting observations were made. The tiny aberrant artery had become dilated almost to the size of a goose quill, and the internal carotid which at the first operation was strikingly small, was as large as normal, or larger (lantern slide). The explanation of the sequence of events is, I think, clear. There were originally two fistulae. The chief of these being eliminated at the first operation, the second, distal to the first, functioned more and more freely in the course of the six and one-half years. The internal carotid, small at the first operation because it was central to the main fistula, became dilated after the subordinate or distal fistula became active; and the anomalous artery, also central to the main fistula, became greatly dilated consequent upon the shifting of the fistula to a distal position.

B. Enlargement of the heart in cases of arterio-venous fistula and persistent ductus arteriosus.—A particularly interesting result of our clinical and experimental studies of arterio-venous fistula is the discovery that enlargement of the heart probably occurs, after a time, as a rule in the major cases. For ten years or more we have noted the condition of the heart in our patients with arterio-venous fistula and have, I believe, quite invariably found the heart enlarged—strikingly so in several instances. Dr. Mont Reid, of our Surgical Staff, has in preparation a report upon his experimental and clinical work on arterio-venous fistula in which he will offer convincing proof of our view that the fistula in its consequences may profoundly affect the heart as well as the veins and arteries. The skiagraphs (exhibit) show the effects of a fistula made three and one-half years ago by Dr. Reid between the carotid artery and external jugular vein of a dog. The veins of the neck on both sides are dilated and the carotid artery is dilated central to the fistula. The heart after two years shows slight enlargement, and after three years, as you observe, it has become pronouncedly increased in size. If the assumption is correct that the heart dilates in consequence of arterio-venous fistula it is important that the fact should be brought to the attention not only of surgeons but also of pathologists and internists who evidently have overlooked it.

Our experimental and clinical observations on arterio-venous fistula and partial occlusion of large arteries may ultimately aid in the explanation of the sequelae of certain congenital anomalies of the heart and aorta. May we not regard the persistent ductus arteriosis as an arterio-venous fistula, the pulmonary artery and the right heart representing the venous side of the fistula? The enlargement of the left heart we might assume to be analagous to the dilation of the artery proximal to a fistula; and in the dilation of the right heart and pulmonary artery we recall the dilation of the veins. My studies on the subject of the dilation of an artery which we find occurs distal to a constricting metal band and distal to the compression exercised by a cervical rib have

led me to investigate the results of the congenital coarctations of the aorta at or beyond its isthmus. I have been interested to find that in a large percentage of these cases of coarcted aorta there is dilation beyond the site of the coarctation. The generally accepted view that this dilation is to enable the aorta to better carry on the anastomotic circulation must, it seems to me, be erroneous. When we shall have ascertained the cause of the arterial dilation obtained experimentally below constricting bands and of the dilation of the artery proximal to an arterio-venous fistula we may be able to explain the dilation of the aorta beyond the congenital coarctation.

C. Plausible explanation of the presence of blood in lymph-cysts at the second and subsequent tappings.—A few years ago, assisted by Dr. Heuer, I removed from the abdomen of a woman about forty years of age a huge congenital hygroma or lymph-cyst. The diaphragm was pushed high up into the right thorax and the liver was displaced far to the right and so rotated on its vertical axis that its inferior border instead of being transverse was parallel and almost in line with the linea alba. The enucleation of the greater part of the cyst was easily accomplished, the few adhesions being disposed of by gentle, blunt dissection. Finally, when there remained only a few filamentous fibers binding the sac to the right adrenal gland[1] and the inferior vena cava we proceeded with even more deliberation and caution. The adhesions to the vein were so delicate that the gentlest manipulation with the handle of the scalpel sufficed to break them. We had an unusually free and clear exposure of the vein and were operating without embarrassment. Suddenly blood gushed from a linear defect about 3 mm. long in the vena cava. The hemorrhage was promptly controlled and the slit in the vessel sutured. Proceeding thereafter with perhaps even greater delicacy, we were again confronted with a gush of blood from the vena cava at a higher point. Here we found a slit from 1.5 to 2 cm. long in this vein. The edges of the slit were smooth, the linear defect being clearly not due to a tear or cut. The gap in the vein was closed by suture. Dr. Heuer and I satisfactorily assured ourselves that there was no defect or special thinning of the wall of the cyst at the point contraposed to the larger of the two defects in the wall of the vena cava.[2]

The slits in the wall of the vena cava were surely not artefacts. They represented, I believe, imperfectly closed embryonic communications between the vein and lymph buds or lymphatic vessels. Dr. Florence Sabin to whom we owe so much for our knowledge of the origin and development of the lymphatic system very kindly writes me in regard to this case as follows:

> Recent work on the lymphatic system serves to demonstrate that lymphatic vessels are modified veins. It has been shown that lymphatic vessels occur first in the neck as sacs, lined with endothelium and packed with blood, which lie close to the jugular veins. The abdominal lymphatics begin as a sac which lies close to that part of the inferior cava which connects the two Wolffian bodies. Baetjer showed in 1908 that in the pig this sac which is the forerunner of the retroperitoneal lymphatics communicates for a time with the inferior vena cava. These communications between the lymphatics and the abdominal veins which are transitory in the pig were then shown to be permanent in the South American monkeys

by Silvester in 1912 (lantern slides), while in 1915 Job demonstrated similar permanent connections in rodents. Thus the study of the development of the lymphatic system affords an explanation of anomalies involving connections between the lymphatic vessels and both the renal veins and the inferior vena cava.

The statement has repeatedly been made that hygromata which at the first tapping have yielded a clear fluid may be found at all subsequent tappings to contain more or less blood. Only one explanation has been offered for the presence of the blood, viz., trauma of the wall of the cyst. This explanation has always seemed to me an unsatisfactory one because the walls of these cysts are as a rule very thin and non-vascular. May it not, in view of the findings in our case, be possible that vestigia of lymphatico-venous communications are responsible for the admixture of blood which has occasionally been noted at only the second and subsequent tappings of lymph-cysts and is more frequently found at the first tapping? The negative pressure consequent upon the aspiration of fluid from the cyst might divert for the moment a little blood from the vein which had given origin to the hygroma's lymphatic bud or vessel. Thus the contents of the sac, clear at the first withdrawal, would be blood stained at the second. Thereafter, with each tapping blood would be aspirated into the sac and hence clear fluid might never again be obtained.

[1] The relation of the cyst to the right adrenal gland was remarkable. In the course of stripping the sac's final delicate attachments we exposed a flat, black surface, evidently the spread-out medulla of the adrenal, about the size of a half dollar. Parenchymatous oozing from this surface required for its arrest a few mattress sutures of fine silk.

[2] The patient recovered promptly and has enjoyed excellent health since the operation.

TABLES OF THE ZONAL SPHERICAL HARMONIC OF THE SECOND KIND $Q_1(z)$ AND $Q_1'(z)$

By A. G. Webster and Willard Fisher

Ballistic Institute, Clark University*

Communicated, January 9, 1919

In connection with certain investigations that I was asked to make with regard to submarines it seemed convenient to have tables of the zonal harmonic of the second kind. These are desirable in various problems in mathematical physics connected with ellipsoids of revolution, such as problems in magnetism. Failing to find such tables in the literature of the subject, I requested Dr. Fisher to calculate tables of the function of the first order and its derivative for an interval that will be sufficient for most practical problems. Dr. Fisher has now left the country, and the stress of other work that prevented their publication being over, I trust that they may be available for peaceful purposes.—A. G. W.

* Contribution from the Ballistic Institute, Clark University, No. 1.

$$Q_1(z) = \frac{z}{2}\log_e\frac{z+1}{z-1} - 1.$$

z	0	1	2	3	4	5	4	7	8	9
1.10	0.6745	0.6708	0.6671	0.6635	0.6600	0.6565	0.6530	0.6495	0.6461	0.6428
1.11	.6394	.6362	.6329	.6297	.6265	.6234	.6203	.6172	.6141	.6111
1.12	.6081	.6052	.6023	.5994	.5965	.5937	.5909	.5881	.5854	.5826
1.13	.5799	.5773	.5746	.5720	.5694	.5668	.5643	.5618	.5593	.5568
1.14	.5543	.5519	.5495	.5471	.5448	.5424	.5401	.5378	.5355	.5332
1.15	.5310	.5288	.5266	.5244	.5222	.5200	.5179	.5158	.5137	.5116
1.16	.5096	.5075	.5055	.5035	.5015	.4995	.4975	.4956	.4936	.4917
1.17	.4898	.4879	.4860	.4842	.4823	.4805	.4787	.4769	.4751	.4733
1.18	.4715	.4698	.4680	.4663	.4646	.4629	.4612	.4595	.4579	.4562
1.19	.4546	.4529	.4513	.4497	.4481	.4465	.4449	.4434	.4418	.4403
1.20	0.4387	0.4372	0.4357	0.4342	0.4327	0.4312	0.4297	0.4283	0.4268	0.4254
1.21	.4240	.4225	.4211	.4197	.4183	.4169	.4155	.4142	.4128	.4114
1.22	.4101	.4088	.4074	.4061	.4048	.4035	.4022	.4009	.3996	.3983
1.23	.3971	.3958	.3946	.3933	.3921	.3909	.3896	.3884	.3872	.3860
1.24	.3848	.3836	.3825	.3813	.3801	.3790	.3778	.3767	.3755	.3744
1.25	.3733	.3721	.3710	.3699	.3688	.3677	.3666	.3656	.3645	.3634
1.26	.3623	.3613	.3602	.3592	.3581	.3571	.3561	.3550	.3540	.3530
1.27	.3520	.3510	.3500	.3490	.3480	.3470	.3460	.3451	.3441	.3431
1.28	.3422	.3412	.3403	.3393	.3384	.3374	.3365	.3356	.3347	.3338
1.29	.3328	.3319	.3310	.3301	.3392	.3284	.3275	.3266	.3257	.3248
1.3	0.3240	0.3155	0.3075	0.2998	0.2924	0.2854	0.2876	0.2721	0.2659	0.2600
1.4	.2542	.2487	.2434	.2383	.2333	.2286	.2240	.2195	.2152	.2111
1.5	.2071	.2032	.1994	.1958	.1922	.1888	.1855	.1822	.1791	.1760
1.6	.1731	.1702	.1674	.1647	.1620	.1594	.1569	.1544	.1520	.1497
1.7	.1474	.1452	.1431	.1409	.1389	.1369	.1349	.1330	.1311	.1293
1.8	.1275	.1257	.1240	.1223	.1207	.1191	.1175	.1160	.1145	.1130
1.9	.1116	.1102	.1088	.1074	.1061	.1048	.1035	.1022	.1010	.0998
2.0	0.0986	0.0974	0.0963	0.0952	0.0941	0.0930	0.0920	0.0909	0.0899	0.0889
2.1	.0879	.0869	.0860	.0850	.0841	.0832	.0823	.0814	.0806	.0797
2.2	.0789	.0781	.0773	.0765	.0757	.0750	.0742	.0735	.0727	.0720
2.3	.0713	.0706	.0699	.0692	.0686	.0679	.0673	.0666	.0660	.0654
2.4	.0648	.0642	.0636	.0630	.0624	.0618	.0613	.0607	.0602	.0596
2.5	.0591	.0586	.0581	.0576	.0571	.0566	.0561	.0556	.0551	.0547
2.6	.0542	.0538	.0533	.0529	.0524	.0520	.0516	.0511	.0507	.0503
2.7	.0499	.0495	.0491	.0487	.0483	.0479	.0476	.0472	.0468	.0465
2.8	.0461	.0457	.0454	.0450	.0447	.0444	.0440	.0437	.0434	.0430
2.9	.0427	.0424	.0421	.0418	.0415	.0412	.0409	.0406	.0403	.0400
3.0	0.0397	0.0394	0.0392	0.0389	0.0386	0.0383	0.0381	0.0378	0.0375	0.0373
3.1	.0370	.0368	.0365	.0363	.0360	.0358	.0355	.0353	.0351	.0348
3.0	0.0397	0.0370	0.0346	0.0324	0.0304	0.0286	0.0270	0.0255	0.0241	0.0228
4.0	.0217	.0206	.0196	.0186	.0178	.0170	.0162	.0155	.0148	.0142
5.0	.0137	.0131	.0126	.0121	.0117	.0113	.0108	.0104	.0101	.0098
	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	
	0.0094	0.0080	0.0069	0.0060	0.0052	0.0047	0.0042	0.0037	0.0034	

$$Q_1'(z) = \frac{1}{2}\log_e\frac{z+1}{z-1} - \frac{z}{z^2-1}$$

Below the line * * * *, second differences are negligible. Prefix the sign − to every value

z	0	1	2	3	4	5	6	7	8	9
1.10	3.7158	3.6710	3.6270	3.5839	3.5417	3.5004	3.4598	3.4201	3.3811	3.3429
1.11	3.3054	3.2687	3.2326	3.1972	3.1624	3.1283	3.0948	3.0619	3.0296	2.9979
1.12	2.9667	2.9360	2.9059	2.8764	2.8473	2.8187	2.7906	2.7629	2.7358	2.7090
	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *	* * *
1.13	2.6827	2.6569	2.6314	2.6063	2.5817	2.5574	2.5335	2.5100	2.4869	2.4641
1.14	.4416	.4195	.3977	.3763	.3551	.3343	.3138	.2935	.2736	.2540
1.15	.2346	.2155	.1967	.1781	.1598	.1418	.1240	.1064	.0891	.0720
1.16	.0551	.0385	.0221	.0059	1.9899	1.9741	1.9586	1.9432	1.9280	1.9130
1.17	1.8982	1.8836	1.8692	1.8550	.8409	.8270	.8133	.7998	.7864	.7731
1.18	.7601	.7472	.7344	.7218	.7093	.6970	.6849	.6728	.6609	.6492
1.19	.6376	.6261	.6147	.6035	.5924	.5814	.5706	.5598	.5492	.5387
1.20	1.5283	1.5180	1.5079	1.4978	1.4879	1.4870	1.4683	1.4587	1.4491	1.4397
1.21	.4304	.4211	.4120	.4030	.3940	.3851	.3764	.3677	.3589	.3506
1.22	.3421	.3338	.3255	.3174	.3093	.3012	.2933	.2854	.2776	.2699
1.23	.2623	.2547	.2472	.2398	.2325	.2252	.2180	.2108	.2037	.1967
1.24	.1898	.1829	.1760	.1693	.1626	.1559	.1494	.1428	.1364	.1300
1.25	.1236	.1173	.1111	.1049	.0988	.0927	.0867	.0807	.0748	.0689
1.26	.0631	.0573	.0516	.0459	.0403	.0347	.0292	.0237	.0183	.0129
1.27	.0076	.0023	0.9970	0.9918	0.9866	0.9815	0.9764	0.9713	0.9663	0.9614
1.28	0.9564	0.9516	.9467	.9419	.9371	.9324	.9377	.9230	.9184	.9138
1.29	.9093	.9047	.9003	.8958	.8914	.8870	.8827	.8784	.8741	.8698
1.30	0.8656	0.8614	0.8573	0.8532	0.8491	0.8450	0.8410	0.8370	0.8330	0.8291
1.31	.8251	.8212	.8174	.8136	.8098	.8060	.8022	.7985	.7948	.7912
1.32	.7875	.7839	.7803	.7767	.7732	.7697	.7662	.7627	.7593	.7559
1.33	.7525	.7491	.7458	.7424	.7391	.7359	.7326	.7294	.7261	.7230
1.34	.7198	.7166	.7135	.7104	.7073	.7042	.7012	.6982	.6952	.6922
1.35	.6892	.6863	.6833	.6804	.6775	.6747	.6718	.6690	.6662	.6634
1.36	.6606	.6578	.6551	.6524	.6497	.6470	.6443	.6416	.6390	.6364
1.37	.6338	.6312	.6286	.6260	.6235	.6209	.6184	.6159	.6134	.6110
1.38	.6085	.6061	.6037	.6013	.5989	.5965	.5941	.5918	.5894	.5871
1.39	.5848	.5825	.5802	.5780	.5757	.5735	.5712	.5690	.5668	.5646
1.40	0.5625	0.5603	0.5581	0.5560	0.5539	0.5518	0.5497	0.5476	0.5455	0.5434
1.41	.5414	.5393	.5373	.5353	.5333	.5313	.5293	.5273	.5253	.5234
1.42	.5215	.5195	.5176	.5157	.5138	.5119	.5100	.5082	.5063	.5045
1.43	.5026	.5008	.4990	.4972	.4954	.4936	.4918	.4900	.4883	.4865
1.44	.4848	.4831	.4813	.4796	.4779	.4762	.4745	.4729	.4712	.4695
1.45	.4679	.4663	.4646	.4630	.4614	.4598	.4582	.4566	.4550	.4534
1.46	.4519	.4503	.4488	.4472	.4457	.4442	.4426	.4411	.4396	.4381
1.47	.4366	.4352	.4337	.4322	.4308	.4293	.4279	.4264	.4250	.4236
1.48	.4222	.4208	.4194	.4180	.4166	.4152	.4138	.4125	.4111	.4097
1.49	.4084	.4071	.4057	.4044	.4031	.4018	.4005	.3991	.3979	.3966

x	0	1	2	3	4	5	6	7	8	9
1.50	0.3953	0.3940	0.3927	0.3915	0.3902	0.3890	0.3877	0.3865	0.3852	0.3840
1.51	.3828	.3816	.3804	.3791	.3779	.3768	.3756	.3744	.3732	.3720
1.52	.3709	.3697	.3685	.3674	.3662	.3651	.3640	.3628	.3617	.3606
1.53	.3595	.3584	.3573	.3562	.3551	.3540	.3529	.3518	.3507	.3497
1.54	.3486	.3475	.3465	.3454	.3444	.3433	.3423	.3413	.3402	.3392
1.55	.3382	.3372	.3362	.3352	.3342	.3332	.3322	.3312	.3302	.3292
1.56	.3283	.3273	.3263	.3254	.3244	.3234	.3225	.3215	.3206	.3197
1.57	.3187	.3178	.3169	.3160	.3150	.3141	.3132	.3123	.3114	.3105
1.58	.3096	.3087	.3078	.3069	.3061	.3052	.3043	.3034	.3026	.3017
1.59	.3009	.3000	.2992	.2983	.2975	.2966	.2958	.2950	.2941	.2933
1.60	0.2925	0.2917	0.2908	0.2900	0.2892	0.2884	0.2876	0.2868	0.2860	0.2852
1.61	.2844	.2836	.2828	.2821	.2813	.2805	.2797	.2790	.2782	.2774
1.6	0.2925	0.2844	0.2767	0.2693	0.2621	0.2552	0.2486	0.2423	0.2361	0.2302
1.7	.2245	.2190	.2137	.2086	.2036	.1988	.1942	.1898	.1854	.1812
1.8	.1772	.1733	.1695	.1658	.1622	.1588	.1554	.1521	.1490	.1459
1.9	.1429	.1400	.1372	.1345	.1318	.1293	.1267	.1243	.1219	.1196
2.0	.1174	.1152	.1130	.1110	.1089	.1069	.1050	.1031	.1013	.0995
2.1	.0978	.0961	.0944	.0928	.0913	.0897	.0882	.0867	.0853	.0839
2.2	.0825	.0812	.0799	.0786	.0773	.0761	.0749	.0737	.0726	.0714
2.3	.0704	.0693	.0682	.0672	.0662	.0652	.0642	.0633	.0624	.0614
2.4	.0605	.0597	.0588	.0580	.0572	.0563	.0556	.0548	.0540	.0533
2.5	.0525	.0518	.0511	.0504	.0497	.0491	.0484	.0478	.0472	.0465
2.6	.0459	.0453	.0447	.0442	.0436	.0430	.0425	.0420	.0414	.0409
2.7	.0404	.0399	.0394	.0389	.0384	.0380	.0375	.0371	.0366	.0362
2.8	.0357	.0353	.0349	.0345	.0341	.0337	.0333	.0329	.0325	.0322
2.9	.0318	.0314	.0311	.0307	.0304	.0300	.0297	.0294	.0291	.0287
3.0	.0284	.0281	.0278	.0275	.0272	.0269	.0266	.0263	.0261	.0258
3.1	0.0255	0.0253	0.0250	0.0247	0.0245	0.0242	0.0240	0.0237	0.0235	0.0232
3.0	0.0284	0.0255	0.0230	0.0208	0.0189	0.0172	0.0157	0.0144	0.0132	0.0122
4.0	.0113	.0104	.0096	.0090	.0083	.0078	.0073	.0068	.0064	.0060
5.0	.0056	.0053	.0050	.0047	.0044	.0042	.0039	.0037	.0035	.0034
	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	
	0.0032	0.0025	0.0020	0.0016	0.0013	0.0011	0.0009	0.0008	0.0007	

The above tables were computed to five places with Barlow's Tables of squares and Naperian logarithms, checked by differences and the last figure cut off. When the figure cut off was exactly 5, the number was recomputed to six places or seven places.

ON THE MEAN AGE AT DEATH OF CENTENARIANS*

By Raymond Pearl

School of Hygiene and Public Health, Johns Hopkins University

Communicated, January 28, 1919

Some years ago I investigated the mean age at death of infants dying at very early ages.[1] The purpose of that study was to determine with accuracy where the deaths under one year should be centered in statistical computations involving the computation of moments, where one had to know the center

TABLE 1

Age Distribution of Centenarian Deaths in the Registration Area, 1916

Age.	Males		Females	
	White	Colored	White	Colored
years				
100	48	42	72	85
101	15	4	30	8
102	14	10	25	19
103	12	11	15	14
104	17	5	9	14
105	14	8	18	27
106	2	7	5	8
107	5	1	1	2
108	2	6	1	5
109	1	1	0	2
110	4	7	2	15
111	1	4	0	3
112	0	1	0	3
113	0	2	0	0
114	2	0	1	1
115	0	2	1	4
116	0	1	0	3
118	0	2	0	0
120	0	2	0	2
134	0	0	0	1
Totals............	137	116	180	216

of gravity of each frequency area over given base units. In an investigation which I am now undertaking I have been confronted with the same problem at the other end of the life span. In the "Mortality Statistics" published by the Bureau of the Census, the deaths occurring at ages of 100 and over are lumped in a single class. In calculating the frequency constants of mor-

* Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 3.

tality curves, it is obvious that these centenarian deaths must be centered at some point. The finding of as close an approximation as possible to the correct centering point has been attacked along two lines, with the results set forth in this paper.

In the first place, actual deaths occurring in the year 1916 in the Registration Area of individuals reported to the Census Bureau as having died at age 100 or more were studied. Detailed statistics giving the distribution of these deaths by years was very kindly furnished me by Dr. William H. Davis, Chief Statistician of the Vital Statistics Division, Bureau of the Census. I wish to record here my obligation to Doctor Davis for furnishing me with this tabulation.

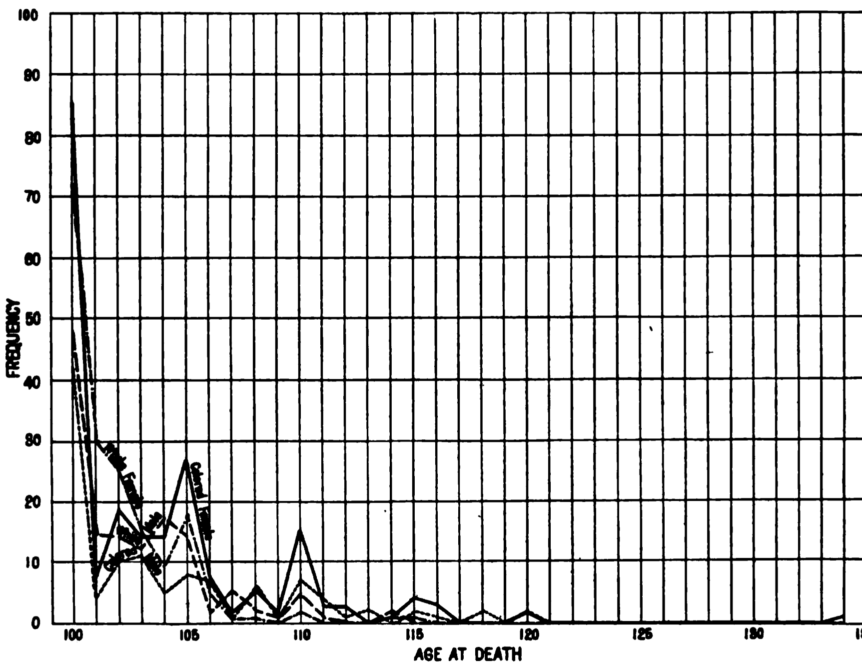


FIG. 1. DISTRIBUTION OF THE DEPTHS IN THE REGISTRATION AREA IN 1916, AT AGE 100 OR MORE

These figures are set forth in table 1, and graphically in figure 1.

It is apparent at once from the table and diagram that there are gross errors in the records of ages in these centenarian deaths. Unquestionably, some people do live to very advanced ages[2] but it is equally clear that they do not occur in such numbers, nor at such advanced ages as would be indicated by these figures, taken at their face values.

Particularly are the figures for the colored population obviously incorrect. One notes the outstanding humps in the colored female line at the quinquennial years, 100, 105, 110, etc. These, of course, have no foundation in fact. In

the case of the whites, the statistics are undoubtedly more accurate, and one would certainly not be justfied in going so far as does Clark[3] in denying the existence of centenarians except in the very rarest cases.

From the unmodified data in table 1, the constants shown in table 2 have been deduced.

Before commenting on the data of table 2, it is desirable to present the results of another method of approach to the matter. That method is to take the number of deaths for each year over 100 from the d_x column of a properly graduated mortality table, and calculate from such figures the means and standard deviations. The mortality tables which have been used for this

TABLE 2

CONSTANTS FOR DEATHS OF CENTENARIANS IN THE REGISTRATION AREA IN 1916

GROUP	MEAN	STANDARD DEVIATION
	years	*years*
White males	103.19 ± 0.17	3.02 ± 0.12
White females	102.42 ± 0.12	2.49 ± 0.09
White, both sexes together	102.75 ± 0.10	2.76 ± 0.07
Colored, both sexes together	104.45 ± 0.18	4.89 ± 0.13

TABLE 3

DEATHS OF CENTENARIANS FROM UNITED STATES LIFE TABLES

x TO $x+1$	d_x OF 100,000 BORN ALIVE			
	White males	White females	Negro males	Negro females
100–101	13	19	13	34
101–102	8	11	10	26
102–103	5	6	7	18
103–104	3	4	4	13
104–105	1	2	3	9
105–106	1	1	1	5
106–107	0	1	1	3
107–108	0	0	1	2
108–109	0	0	0	1
109–110	0	0	0	1

purpose are Glover's[4] 1910 American mortality tables. The distributions of death which these tables give for white and colored are exhibited in table 3.

As would be expected the distributions in table 3 are much smoother than those of table 1. Furthermore, their range is less extended at the upper end. The constants for the distributions in table 3 are exhibited in table 4.

From these tables the following points are to be noted:

1. As was to be expected the mean ages at death are sensibly lower, and the standard deviations smaller, when the graduated data of the life tables are used instead of the raw statistics.

2. Inasmuch as the life table values given in table 4 are certainly nearer the true values than those of table 2, they may well be adopted as the basis of centering deaths over 100 in biostatistical computations.

3. Accordingly in this laboratory the deaths of whites, both male and female, occurring at 100 years and over will be assumed to center at 101.7 years, and those of negroes at 102.0. The latter value is probably still too high, but unless one repudiates the statistical data entirely, which would amount to throwing away the child with the bath, there is no warrant for arbitrarily taking a lower value.

TABLE 4

MEAN AGE AT DEATH AND STANDARD DEVIATION IN AGE AT DEATH OF CENTENARIANS, FROM UNITED STATES LIFE TABLE FIGURES

GROUP	MEAN	STANDARD DEVIATION
White males	101.66	1.32
White females	101.73	1.49
Whites, both sexes	101.70	1.42
Negroes, both sexes	102.32	1.92

[1] Pearl, R., *Biometrika*, 4, 1906, (510–516).

[2] Cf. the authentic case reported by Rodriquez (in *Siglo med., Madrid*, 53, 1906, (341–343) > of death at 124 years of age.

[3] Clark, F. C., *Providence, Med. J.*, 10, 1909, (143–158).

[4] Glover, J. W., United States Life Tables, 1910. Bureau of the Census 1916.

REPORT OF THE AUTUMN MEETING

Prepared by the Acting Home Secretary

The Autumn Meeting of the Academy was held at the Johns Hopkins University in Baltimore, November 18 and 19, 1918.

Forty-one members were present as follows: C. G. Abbot, Abel, Ames, Bliss, D. H. Campbell, Castle, F. W. Clarke, J. M. Clarke, Conklin, Cross, Davenport, Davis, Franklin, Freeman, Gomberg, Halsted, Holmes, Howard, Howe, Howell, Iddings, Jennings, Jones, Leuschner, Lewis, Lindgren, C. E. Mendenhall, J. C. Merriam, Michelson, Moulton, E. F. Nichols, Pupin, Ransome, Russell, Erwin F. Smith, Thorndyke, Vaughan, Walcott, Webster, Welch, R. W. Wood.

BUSINESS SESSIONS

The President announced the death of S. W. Williston and of G. K. Gilbert.

The following assignments of Biographical Memoirs was announced: Memoirs of G. K. Gilbert to William M. Davis, of Richmond Mayo Smith to Edwin R. A. Sigman, and of Samuel W. Williston to Richard S. Lull.

Announcement was made of the following funds available for the National Research Council:

Carnegie Institution	$100,000.00
W. W. Keen	483.57
Rockefeller Foundation	50,000.00
President's Fund	120,000.00
Total available for the fiscal year 1919	$270,483.57

The President announced the following appointments to the Board of Editors of the Proceedings with terms expiring December, 1921: J. M. Clarke, Ludvig Hektoen, H. S. Jennings, R. A. Millikan, W. A. Noyes.

The Managing Editor of the Proceedings, Mr. E. B. Wilson, was re-elected for the year ending November 30, 1919.

The following communication from the Home Secretary, Mr. Arthur L. Day, was read and acted upon:

Dr. Charles D. Walcott
President, National Academy of Sciences,
Washington, D. C.

9 Sept. 1918

Dear Dr. Walcott:

Twice within the year, under the pressure of war activities which have kept me away from Washington considerably more than half the time, I have asked to be relieved of the duties of Home Secretary of the Academy. Both times the Council has done me the honor to request that the resignation be withdrawn, even though the duties which I have hitherto performed are required to be otherwise provided for. I have appreciated this very unusual

mark of confidence most keenly, and notwithstanding some twinges of a New England conscience about receiving credit for work done by some one else, I have been very grateful to the Council for its complimentary action.

More recently, it may have come to your ears that I have undertaken a very much greater war responsibility than any with which I have heretofore had to do. It is in connection with the Corning Glass Works at Corning, N. Y., and I have accordingly decided to move my family there.

It is a very necessary though unwritten law that the Home Secretary should be a resident of Washington, which after October 1 I shall no longer be. I therefore respectfully offer a final request to be relieved of the duties of Home Secretary from October 1, 1918.

With kindest regards, believe me,

Very sincerely yours,
ARTHUR L. DAY, *Home Secretary.*

The recommendation of the Council that the resignation of Mr. A. L. Day as Home Secretary be accepted with an expression of appreciation of his valuable services in that office and that the election of a Home Secretary as provided for under the Constitution be held at the next stated meeting of the Academy was approved.

The President presented the following communication from the President of the United States relative to the *National Research Council:*

Executive Order Issued by the President of the United States, May 11, 1918

The National Research Council was organized in 1916 at the request of the President of the National Academy of Sciences, under its Congressional charter, as a measure of national preparedness. The work accomplished by the Council in organizing research and in securing coöperation of military and civilian agencies in the solution of military problems demonstrates its capacity for larger service. The National Academy of Sciences is therefore requested to perpetuate the National Research Council, the duties of which shall be as follows:

1. In general, to stimulate research in the mathematical, physical and biological sciences, and in the application of these sciences to engineering, agriculture, medicine and other useful arts, with the object of increasing knowledge, of strengthening the national defense, and of contributing in other ways to the public welfare.

2. To survey the larger possibilities of science, to formulate comprehensive projects of research, and to develop effective means of utilizing the scientific and technical resources of the country for dealing with these objects.

3. To promote coöperation in research, at home and abroad, in order to secure concentration of effort, minimize duplication, and stimulate progress; but in all coöperative undertakings to give encouragement to individual initiative, as fundamentally important to the advancement of science.

4. To serve as a means of bringing American and foreign investigators into active coöperation with the scientific and technical services of the War and Navy Departments and with those of the civil branches of the government.

5. To direct the attention of scientific and technical investigators to the present importance of military and industrial problems in connection with the war, and to aid in the solution of these problems by organizing specific researches.

6. To gather and collate scientific and technical information at home and abroad, in coöperation with Governmental and other agencies and to render such information available to duly accredited persons.

Effective prosecution of the Council's work requires the cordial collaboration of the scientific and technical branches of the Government, both military and civil. To this end

representatives of the Government, upon the nomination of the President of the National Academy of Sciences, will be designated by the President as members of the Council, as heretofore, and the heads of the departments immediately concerned will continue to coöperate in every way that may be required.

WOODROW WILSON.

THE WHITE HOUSE, *May 11, 1918.*

The President read the following resolutions adopted at the Inter-Allied Conference, held at London, October 9, 1918, received from the Foreign Secretary, Mr. Hale, who was also Chairman of the Delegation, the other members being Messrs. A. A. Noyes, Durand, Flexner, Bumstead, and Carty.

RESOLVED THAT: A Committee of Enquiry be constituted by the Conference, the Academies of the countries at war with the Central Powers having power to add further members. This Committee shall prepare a general scheme of international organizations to meet the requirements of the various branches of scientific and industrial research including those relating to national defence. (The Committee will meet in Paris during the second fortnight in November.)

Each of the Academies represented at the Conference shall be invited to initiate the formation of a National Council for the promotion of the researches specified in Resolution 4.

An International Council, having as nucleus the Committee specified in Resolution 4, shall be formed by the federation of the National Councils.

The Conference being of opinion that all industrial, agricultural and medical progress depends on pure science, draws the attention of the various governments to the importance of theoretical and disinterested researches, which after the restoration of peace should be supported by large endowments. The Conference urges similarly the creation of large laboratories for experimental science, both private and national.

The action of the Academy delegates in having the above resolutions adopted by the Inter-Allied Conference in London was approved by the Academy.

On the recommendation of the Council the Academy placed itself on record as welcoming the publications of a scientific and technical character to go into the building that is contemplated.

Mr. Davis presented the following forms, asking that the members of the Academy sign the petition. Many members present did so, and after the meeting, with the approval of the President of the Academy, additional signatures of members were obtained by correspondence to a total number of about 100.

The undersigned Members of the National Academy of Sciences, meeting in Baltimore November 18, 1918, petition the Congress of the United States to take action in consultation with the governments of many other countries, toward the formation at as early a date as possible of a League of Nations for the maintenance of Peace.

The undersigned Members of the National Academy of Sciences, meeting in Baltimore November 18, 1918, having petitioned the Congress of the United States to take action, in consultation with the governments of many other countries, toward the formation at as early a date as possible of a League for the *maintenance* of Peace, hereby urge the members of other learned societies in the United States to do likewise.

The following motion was offered:

MOVED: That the Home Secretary be requested to transmit the thanks of the Academy to the President and Trustees of Johns Hopkins University, the members of the Johns Hopkins Club and the members of the Academy resident in Baltimore serving as the local committee, for the courtesies extended to the members of the National Academy of Sciences at the Autumn Meeting, 1918. (*Adopted.*)

SCIENTIFIC SESSIONS

Three public scientific sessions were held on November 18 and 19 at which the following papers were presented.

C. G. ABBOT: Cloud reflection and the albedo of the earth and Venus.

A. H. PFUND (introduced by J. S. AMES): Colorimetry of white surfaces.

F. W. CLARKE and G. STEIGER: The inorganic constituents of lobster shells.

R. W. HEGNER (introduced by H. S. JENNINGS): Quantitative relations between chromatin and cytoplasm in the genus Arcella, with their relations to external characters.

D. R. HOOKER (introduced by W. H. HOWELL): The physiological effects of air-concussion.

W. H. HOWELL: Two new factors in blood coagulation.

C. B. DAVENPORT and ALBERT G. LOVE: Comparative morbidity of white and colored troops.

A. G. WEBSTER: Theory of wind instruments.

A. G. WEBSTER: On the ballistic resistance function.

A. G. WEBSTER: On the dynamics of the rifle fired from the shoulder.

G. P. MERRILL: Second report on the researches on the chemical and mineralogical composition of meteorites.

W. E. DANDY (introduced by W. S. HALSTED): Hydrocephalus: experimental and clinical study. Illustrated.

W. S HALSTED: Clinical and experimental observations in cases of arteriovenous and lymphatico-venous fistulae. Illustrated.

W. E. CASTLE: Is the arrangement of the genes in the chromosome linear? Illustrated.

E. F. SMITH: Cause of Phyllomania in Begonia. Illustrated.

W. G. MACCALLUM (introduced by H. S. JENNINGS): Recent epidemics of pneumonia in army camps. Illustrated.

G. A. BLISS: Differential corrections of ballistics.

S. B. WOLBACH: Some results of studies of Influenza at an army camp.

V. C. VAUGHAN: Communicable diseases in an army camp.

R. W. WOOD: Physiological effects of light of short wave-length.

*G. N. LEWIS: Kinetics and thermodynamics.

*A. O. LEUSCHNER: Perturbations and tables of the minor planets discovered by James C. Watson, Part II.

*C. E. VAN ORSTRAND: Mathematical tables.

*F. SLATE, Jr.: Biography of Eugene Waldemar Hilgard.

*W. W. CAMPBELL: Biographical memoir of George Davidson.

*C. R. CROSS: Biographical memoir of James Mason Crafts.

*T. D. A. COCKERELL: Biographical memoir of Alpheus Spring Packard.

*F. SLATE, Jr.: Biographical memoir of Eugene Waldemar Hilgard.

*The asterisk denotes presentation by title.

NATIONAL RESEARCH COUNCIL

MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

AT THE NATIONAL RESEARCH COUNCIL BUILDING, DECEMBER 21, 1918, AT 10 A.M.

Present: Messrs. Bogert, Cross, Flinn, Howe, Hussey, Johnston, Manning, Mendenhall, Merriam, Millikan, Pupin, Walcott, Washburn, Welch, Woods, Woodward. and Yerkes.

The Chairman presented a report of the Committee on Reconstruction Problems. This Committee at its first meeting on August 18, 1918, adopted a program calling for research into the agencies and activities dealing with after-war problems. It appears that many organizations have taken up, or are about to take up, some phase of this far-reaching matter; so that, in theory at least, there are relatively few points not being covered. The Committee decided therefore to limit its efforts to a specific undertaking; and recommends the preparation of a comprehensive report on the whole question of the supply and control of water in relation to food and power production and other industries, a question which enters largely into all reconstruction plans throughout the world. In the compilation of data and preparation of this report it is proposed that each member of the Committee should cover one of the major topics, and that experts in Government service be invited to cooperate; and that the whole should be edited by some one person, with due credit to the several collaborators and sources of information.

Moved: That the plan presented by the Committee on Reconstruction Problems be approved, provided that it does not conflict with similar activities of present Government organizations and that the manuscript of the report having to do with reconstruction be submitted to the Committee on Publicity and Publications for approval before it is sent to press. (*Adopted.*)

Mr. Merriam, Chairman of the Committee on Organization, presented a report, in substance as follows:

The period of the war emergency having come to an end, so far as activities in most phases of research are concerned, the National Research Council must take up for immediate consideration the extent of readjustment necessary in order to adapt its machinery most perfectly for functioning through the period of reconstruction and in normal times of peace.

In the evolution of the Research Council the first stage was that of a temporary organization intended to continue for a period not longer than one year, within which time it was assumed that the best mode of operation would become evident. The entrance of America into the war made necessary considerable modification of the temporary machinery in order to permit concentration of effort within the briefest limits of time and space. These shifts were not presumed to lead in all cases in the direction of permanent organization, and suggestions relating to the ultimate plan upon which the work of the Council was to be based continued under discussion through the period of the war. While it is not necessary to assume that either the aims or the plan of operation of the Council must be materi-

ally altered for the immediate future, it is certain that the tremendous change of conditions in the country incident to ending of the war makes necessary the immediate consideration of adaptability of the Council's present machine for handling the next problems to come before it. Not only must the question as to our present situation be answered for ourselves, but we must make clear to the research men of the country that the Council has fully considered this matter and that, recognizing changing conditions, it is fitting itself as rapidly as possible into the best position for future work.

The aims and purposes of the Research Council have from the beginning covered the broadest relations and applications of science. In recent months the work has been expressed in effort to assist in bringing science to bear upon problems of war. From this experience it has been clear that efficiency of the nation in war is in considerable measure dependent upon the most intimate correlation of scientific interests, such as is also needed in time of peace. With the cessation of hostilities, we find our problem differing largely in mode of application of results of research and in the larger freedom to consider stimulation and correlation of scientific investigation and initiation of new enterprises.

The skeleton of the present war organization of the Research Council appears in general to furnish a satisfactory basis for operation in time of peace. The principal modifications needed involve:

(1) The placing of larger emphasis upon the initiation, stimulation, and correlation of fundamental researches.

(2) Reorganization of the Divisions so as to secure a wide representation of the major research interests in the country regardless of geographic location of the members.

Owing to the immediate need for securing assistance of science in handling emergency problems, the work of the Council has recently tended toward the extreme of application of results of research, rather than toward the stimulation or initiation of work on new problems. With relief from urgent requirements for application of available research data in war, it becomes possible for the investigator to concern himself more particularly with researches in science and technology, depending upon his cooperation with the engineer for application of results secured. At the same time that we are relieved from extraordinary requirements in application of research we are given the possibility of bringing into conference with the Research Council a considerable group of men whose participation in the work of this organization has been difficult because of geographic situation.

Accordingly it is considered that the organization of the National Research Council should be such as to render it an effective federation of the research agencies of the country. To this end a large proportion of the membership of the Council will be nominated by the national scientific and technical societies. All members will be appointed for periods of three years, and will be grouped into a series of divisions each of which will deal with related branches of science and technology or with some specific phase of activity of the Council. Each Division will elect a Chairman, Vice-Chairman and Executive Committee to administer the affairs of the Division. The Chairman and Vice-Chairman of each of the several Divisions, together with certain other members, shall be the Executive Board of the Council; the officers of the Executive Board will be the officers of the Council as a whole. The Executive Board will determine the general policy of the Council; but each Division will be as autonomous as possible, and the policies of the several Divisions may differ in various respects provided that these do not conflict with the general policies of the Council.

Tentative plans, in accordance with the above general principles, were discussed at length; but no final conclusions were reached.

Moved: That the report of the Committee on Organization be approved with the recommendations contained therein. *(Adopted.)*

Moved: That the question of reorganization of the Divisions concerned with administrative, governmental and general relations, be referred to the Committee on Organization

for further consideration and that the Organization Committee in conference with the Divisions take up the problem of modification of the division organization. (*Adopted.*)

Moved: That the Executive Board ask for a conference at an early date with the Council of the National Academy of Sciences for the consideration of the recommendations set forth in the plans presented by the Committee on Organization. (*Adopted.*)

By request of Mr. Phinehas Stephens attention of the Research Council was called to the Smith-Howard Bill, the general purpose of which is to give grants of federal money to the several states for engineering experiment stations.

Moved: That a committee of six be appointed by the Chairman to recommend what action, if any, shall be taken by the National Research Council. (*Adopted.*)

The Chairman appointed as members of the Committee Messrs. Carty, Dunn (Chairman), Howe, Leuschner, Millikan and Pupin.

In response to a notice of the Secretary of the National Research Council dated November 29, a meeting of the Special Committee appointed to discuss the proposals in the Smith-Howard Bill for the establishment of engineering experiment stations was held at the offices of the National Research Council in Washington on Wednesday, December 4, 1918, at 9.30 a.m. There were present Messrs. Henry M. Howe, F. B. Jewett, John Johnston, A. O. Leuschner, J. C. Merriam, R. A. Millikan, M. I. Pupin, S. W. Stratton, C. D. Walcott, W. R. Whitney, and Gano Dunn, Chairman. Messrs. R. C. Maclaurin and Van H. Manning were unavoidably absent and Mr. Manning was represented by Mr. H. Foster Bain. By invitation Mr. Phinehas V. Stephens, author of the Smith-Howard Bill, attended the latter part of the meeting and participated in the discussion.

The text of the Smith-Howard Bill, as of November 8, 1918, was thoroughly analyzed and discussed and the text of a proposed substitute drafted by Major Leuschner was offered and considered. After much discussion certain principles were adopted as expressing the judgment of the Special Committee.

The report of the Special Committee was presented to the Executive Board and approved by it; and a small committee consisting of Messrs. Leuschner (Chairman), Howe, Johnston, and Pupin, was appointed to incorporate in a revised text of the Smith-Howard Bill the principles laid down by the Special Committee.

A revised text of the Smith-Howard Bill, presented to the Executive Board, was agreed to by the Board.

Mr. Howe reported that the Committee on Helmets and Body Armor had subscribed among themselves nearly $2500 for carrying on the work of that Committee.

Moved: That a vote of thanks of the National Research Council be given to the members of the Committee on Helmets and Body Armor for their interest and support in carrying forward the research work of that Committee. (*Adopted.*)

Moved: That Bradley Stoughton be elected as a Second Vice-Chairman of the Division of Engineering. (*Adopted.*)

Moved: That Armin O. Leuschner be elected to represent astronomy on the Executive Committee of the Division of Physics, Mathematics, Astronomy, and Geophysics. (*Adopted.*)

Moved: That the resignation of J. C. Merriam as Chairman of the Division of Geology and Geography be accepted with great regret; and that the National Research Council express its appreciation of the great services which he has rendered to the Council and convey to him a vote of thanks. (*Adopted.*)

Moved: That Mr. Whitman Cross be elected Chairman of the Division of Geology and Geography, and that Mr. P. S. Smith be elected Vice-Chairman. (*Adopted.*)

Mr. Rous stated that on December 4 the Rockefeller Foundation adopted the following resolution:

RESOLVED that the sum of Fifteen Thousand Dollars ($15,000) be, and it is hereby appropriated, of which so much as may be necessary shall be paid to the NATIONAL RESEARCH COUNCIL for the special work in connection with the war emergency and the demobilization period, of the *Division of Medicine and Related Sciences* for the year 1919.

Moved: That the National Research Council transmit to the Rockefeller Foundation a vote of appreciation and thanks for its splendid support of the Division of Medicine and Related Sciences in its work carried on during the war emergency and the period of demobilization. (*Adopted.*)

Moved: That the resignation, as of December 31, 1918, of R. M. Pearce and of Peyton Rous be accepted with regret; and that the National Research Council express its appreciation of the splendid work which they have accomplished as heads of the Division of Medicine and Related Sciences, and convey to them a vote of thanks for their assistance. (*Adopted.*)

Moved: That the election of Colonel Russell as Acting Chairman and that of Major Hussey as Vice-Chairman of the Division of Medicine and Related Sciences be approved. (*Adopted.*)

The meeting adjourned at 12.50.

PAUL BROCKETT, *Assistant Secretary.*

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 APRIL 15, 1919 Number 4

RESTORATION OF VITALITY THROUGH CONJUGATION

GARY N. CALKINS

DEPARTMENT OF ZOOLOGY, COLUMBIA UNIVERSITY

Communicated by Edmund B. Wilson, January 15, 1919

The physiological effect of conjugation, or fertilization, has been interpreted, in the main, along two lines of theory. One of these may be indicated by Bütschli's term *Verjüngung*, and by Maupas's corresponding one *rajeunissement*, terms indicating that the primary effect of conjugation is to restore vital activities to an optimum. The other theory, first fully elaborated by Weismann, assumes that the union of germ plasms (amphimixis), brought about by conjugation, is a source of variations.

These theories are not reciprocally exclusive, and it is possible that both are correct, although neither has been conclusively established.

Bütschli interpreted conjugation in the protozoa, as a means whereby waning vitality is restored to full metabolic activity. The problem thus suggested, involves three fundamental questions: (1) Does the protoplasm of a single individual protozoon and its progeny by division, undergo a progressive waning of vital activities leading to 'old age,' degeneration, and, finally, to natural death? (2) Does conjugation actually restore such weakening protoplasm to a condition of full metabolic activity? (3) If conjugation accomplishes this, what is the explanation of the result?

The first of these three questions was answered in the affirmative by the experiments of Maupas and of numerous subsequent investigators. The second has never been answered conclusively, although strong experimental evidence has accumulated in support of the affirmative. The third question, obviously, is dependent on the second and will be disregarded here.

In the present preliminary paper, I submit the results of experiments made during the last year and a half, which offer a positive answer to the second of these three questions, affording proof that conjugation, in the ciliated protozoon Uroleptus mobilis, actually restores waning vitality to full metabolic vigor. These results are based on the records of the progeny of a

single individual cell, which have been maintained under identical conditions and fed daily with the same standardized culture medium.

The usual method of isolating single individuals each day in fresh culture medium, has been used throughout; five "lines" of individuals of the same ancestry forming a 'series.' Being isolated each day, conjugation never occurs in the culture dishes.

After the daily isolations, the residual individuals are either thrown away, or are put together in larger culture dishes containing fresh culture medium. Here they are kept for a period of two weeks without renewal of the culture medium. At first there is an abundance of food and the organisms multiply by fission until three or four thousand are present. Later, the food becomes exhausted, and, as is well known in some other cases, the transition from rich feeding to starvation induces conjugation, provided the organisms are sexually mature. This procedure constitutes a "conjugation test" which is carried out at weekly intervals.

The number of divisions per day in each of the five lines of a series is recorded; also the sum of the daily divisions, giving a total number of generations to date in each line. The daily records of all lines of a series may then be averaged for successive periods of ten days each, such averages giving a convenient and accurate representation of the relative metabolic activity at different periods of the life cycle.

From time to time, individuals that have undergone conjugation in the weekly conjugation tests, may be isolated to form the beginnings of filial series which are maintained in isolation cultures exactly as in the original series. From the methods employed in these tests, it is evident that such conjugations take place between rather closely-related individuals which are of the same age. The protoplasm affected by such conjugation, also, has had the same history and the same daily treatment throughout, as that maintained in the isolation cultures. After conjugation, a filial series is continued in isolation cultures, transferred daily to the same medium as that used for the parent series, and parallel records are kept in the same way. A filial series, therefore, represents the same original protoplasm as that represented by isolation cultures of the parent series. Any difference in the same calendar periods between the records of the parent series, and those of the filial series, must, therefore, be attributed to the conjugation that has taken place between the closely-related cells of the parent series.

An ideal culture medium for Uroleptus, was found to be an infusion made by boiling a small quantity of flour and a small quantity of fine-cut hay in spring water. This infusion should be used only after twenty-four hours exposure to the air. After the first fifty days of experimenting, this culture medium, made fresh each day, has been used without any alterations.

I have found that the process of 'endomixis' or asexual reorganization with restoration of vitality, occurs in Uroleptus mobilis while encysted. Such encysted stages persist for long periods, and I find that the organisms cannot

be recovered from the cysts until they have been dried for some weeks. The process of reorganization, therefore, cannot escape observation, and it has never occurred in the isolation cultures. The uniformity of results in all of the series outlined below, is sufficient evidence that no other method of asexual reorganization, or parthenogenesis, has occurred in any isolation culture.

A single ex-conjugant from a pair of 'wild' conjugating individuals was isolated on Nov. 24, 1917. It formed the parental series, or, as I shall call it, the '*A* series' and I would explicitly state again, that all results described here, have been obtained with lineal descendants, by division, of this one ancestral cell, and without change of the culture medium.* Five lines were established at the third division and the relative vitality, at successive ten-day periods, is shown in column 1 of table 1. The series ran through 313 generations by division and died out after a long period of progressively reduced vitality, on September 18, 1918.

Conjugation tests were made every week and gave constant epidemics of pairing after the first six weeks. A single pair of conjugating individuals was isolated on four different occasions. In each case the pair was watched until the two individuals had separated. One of the ex-conjugants on each occasion, was then isolated as the starting individual of a filial series. The first of these ex-conjugants formed the filial C series and came from a pair which were in the 78th generation of the parent A series, on February 4, 1918. The second, formed the filial D series which came from a pair of the A series conjugating on March 8, 1918, in the 137th generation. The third, formed the filial H series, which came from a pair conjugating on May 17, 1918, in the 237th generation of the parent A series. The fourth formed the filial J series which came from a pair conjugating on August 12, 1918, in the 311th generation of the parent A series. The parent A series died from exhaustion in the 313th generation, hence these four filial series were taken off at different periods of waning vitality of the parent protoplasm. Each was maintained in five lines and treated exactly like the parent isolation series. Their histories, in successive ten-day periods, are shown in columns 2, 3, 4 and 5.

The history of the C series was similar to that of the parent A series. After a long period of progressively reduced vitality it died on December 31, 1918, in the 348th generation. The D series outlived the parent series but did not live as long, dying after 230 days in the 271st generation. The H series is still alive and is now (January 8, 1919) in the 277th generation. The J series was taken from the parent A series when vitality of the latter was very low, each individual of the parent series dividing only 2.2 times in ten days. The effect of conjugation, as shown by the J series was to increase the division rate to 17.2 times in ten days, while, for the same calendar period, the parent series was dividing at the rate of two-tenths of one division in ten days. The J series is still dividing actively in the 236th generation.

* At the present time (March 15) descendants are still living with unimpaired vigor.

TABLE 1

AVERAGE DIVISION RATE PER INDIVIDUAL, IN TEN-DAY PERIODS

10-DAY PERIODS	A SERIES	C SERIES	D SERIES	H SERIES	J SERIES	F SERIES	I SERIES	L SERIES	PARENTAGE
1	9.8								C series from A78th generation
2	6.6								D series from A137th generation
3	4.8								H series from A237th generation
4	5.4								J series from A311th generation
5	7.8								F series from C86th generation
6	21.0								I series from F143d generation
7	18.6								L series from I199th generation
8	20.8	18.6							
9	18.0	18.8							
10	12.6	16.2							
11	13.2	16.8	16.0						
12	14.8	17.2	18.2						
13	14.6	15.6	16.0			13.4			
14	13.2	14.2	17.0			13.8			
15	15.0	16.4	17.6			16.4			
16	13.6	15.6	18.4			19.8			
17	17.0	17.4	19.0			20.2			
18	12.8	18.0	20.0	19.6		19.6			
19	7.4	13.4	15.2	16.8		15.8			
20	11.0	14.4	16.6	17.8		16.8			
21	7.0	12.8	14.4	15.8		14.0			
22	9.0	10.4	13.4	14.8		12.8			
23	7.6	11.4	14.6	19.1		14.2	18.4		
24	11.2	17.6	18.4	21.4		19.8	22.6		
25	4.8	10.6	12.6	16.4		13.8	19.4		
26	2.2	9.0	10.4	18.0		15.2	11.8		
27	0.2	9.8	9.4	15.6	17.2	12.0	15.0		
28	0.6	8.6	3.4	15.6	16.6	11.8	15.8		
29	0.2	7.6	1.4	15.4	18.6	10.8	14.8		
30	0.0	6.0	0.8	17.8	17.4	12.0	17.6		
31	*Dead*	6.8	0.4	17.8	20.4	14.6	18.0		
32		4.8	0.2	11.0	17.2	9.8	11.6		
33		2.4	0.0	15.4	15.8	9.6	16.4		
34		0.6	0.0	12.6	17.2	12.0	17.4		
35		0.0	*Dead*	7.4	14.6	8.0	14.0	16.0	
36		0.4		2.4	12.4	5.8	12.8	13.2	
37		0.4		1.6	14.4	3.8	13.8	16.2	
38		0.0		1.6	20.6	2.8	20.8	23.2	
39		0.0		0.4	17.0	0.0	15.8	18.6	
40		*Dead*		1.2	12.6	*Dead*	13.0	16.0	

The F series was started with an ex-conjugant of the C series in the 86th generation, and died December 21 in the 317th generation, ten days earlier than the parent series. The I series was taken from the F series in the 143d generation and is still living actively in the 305th generation. The L series came from this I series in the 199th generation and is actively dividing in the 118th generation. An N series (not included in the table) has recently been started from the J series in the 188th generation.

While table 1 by itself, shows clearly enough that conjugation restores vitality to an optimum, the results may be shown still more strikingly by a comparison of longer periods of time whereby minor fluctuations are less conspicuous. I find from the records of the conjugation tests of all series, that conjugation does not begin to take place until from fifty to seventy days from the start of a series. I have chosen the period of the first sixty days, therefore, as representing the period of sexual immaturity. The records show, furthermore, that this is also the period of optimum metabolic activity.

Comparing the mean division rate of a filial series in this first sixty day period, with the division rate of the parent series for the same calendar sixty days, shows the extent, in division activity, to which conjugation has restored vitality to the parent protoplasm. Thus, during the first sixty days of the C series, the five lines had a mean daily division rate of 8.6333, or each individual averaged 1.726 divisions per day, or in ten days, 17.2666 divisions. In the same sixty days, the parent A series, starting at the 78th generation, had a mean division rate of 7.8666 daily, each individual averaging 15.7333 divisions in ten days. The difference, 1.53, indicates the average increase, in number of divisions in a ten-day period, of each individual of the C series, over each individual of the parent A series. In this case the filial generation was taken from the parent when vitality of the latter was near its optimum, and a small discrepancy between parent and offspring is to be expected. With increasing age of the parent, and with corresponding reduction in vitality of its protoplasm, one might reasonably expect that conjugation between two such weakened individuals, would result in a filial generation in which the discrepancy between parent and offspring would remain practically the same as above. The results, however, do not support this expectation; on the contrary, the discrepancy increases with age of the parent protoplasm, as shown in table 2.

It is evident, from the foregoing, that conjugation results in the complete restoration of vitality regardless of the age or the weakened condition of the parent protoplasm, although both parental and filial series are fed at the same times on exactly the same culture medium. This is particularly striking in the case of the J series.

Table 2 also shows that all filial series return to a certain optimum of metabolic vigor as a result of conjugation, a vigor represented by from 17.1 to 17.9 divisions per individual in ten days. From this optimum there is a gradual loss of vitality which is common to all series and which finally leads to death from old age. This is clearly shown in table 3.

TABLE 2

COMPARISON OF DIVISION RATES OF PARENT AND OFFSPRING DURING FIRST SIXTY DAYS OF LATTER

	C FROM A		D FROM A		H FROM A		J FROM A		F FROM C		I FROM F	
	SERIES	SERIES	SERIES	SERIES	SERIES	SERIES	SERIES	SERIES	SERIES	SERIES	SERIES	SERIES
Dates	2/7 — 4/8		3/8 — 5/7		5/22 — 7/21		8/16 — 10/15		3/28 — 5/27		7/7 — 9/5	
Parent's age	78 generations		137 generations		237 generations		311 generations		86 generations		143 generations	
Mean division rate per day, all 5 lines	8.6333	7.8667	8.5833	7.0666	8.6666	6.2666	8.9666	0.125	8.6000	8.1000	8.5833	6.5666
Average division rate per individual per ten days of period.	17.2666	15.7334	17.1666	14.1333	17.3332	12.5332	17.9332	0.2500	17.2000	16.2000	17.1666	13.1332
Increase per individual per ten days of period.	1.5332		3.0333		4.8000		17.6832		1.0000		4.0334	

TABLE 3

DIVISION RATES IN SUCCESSIVE SIXTY-DAY PERIODS, ALL SERIES

SERIES AND ORIGIN	FIRST 60 DAYS			SECOND 60 DAYS			THIRD 60 DAYS			FOURTH 60 DAYS			FIFTH 60 DAYS			SIXTH 60 DAYS
	Mean division rate per day, all 5 lines	Division rate per individual per 10 days of period	Date	Mean division rate per day, all 5 lines	Division rate per individual per 10 days of period	Date	Mean division rate per day, all 5 lines	Division rate per individual per 10 days of period	Date	Mean division rate per day, all 5 lines	Division rate per individual per 10 days of period	Date	Mean division rate per day, all 5 lines	Division rate per individual per 10 days of period	Date	
A "Wild" exconjugant 11/15/'17	*	*	11/28 to 1/27	8.183 ±0.1985	16.366	1/28 to 3/29	7.183 ±0.1645	14.366	3/29 to 5/28	4.450 ±0.1822	8.900	5/28 to 7/27	0.6667 ±0.1300	1.333	7/27 to 9/25	Dead 9/18
C From A78 2/4/'18	8.6333 ±0.2185	17.266	2/ 9 to 4/ 8	7.9167 ±0.1809	15.833	4/ 9 to 6/ 8	6.4166 ±0.1945	12.833	6/ 8 to 8/ 5	3.9834 ±0.1760	7.966	8/ 5 to 10/ 4	0.7166 ±0.0593	1.433	10/ 4 to 12/ 4	Dead 12/31
D From A137 3/8/'18	8.5833 ±0.1349	17.166	3/ 8 to 5/ 7	8.1666 ±0.2475	16.333	5/ 8 to 7/ 7	5.7834 ±0.2374	11.566	7/ 7 to 9/ 5	0.3500 ±0.0737 (40 days	0.700 only)	9/ 5 to 10/15	Dead 10/15			
H From A237 5/17/'18	8.6666 ±0.1892	17.333	5/22 to 7/21	8.5333 ±0.2165	17.066	7/16 to 9/14	7.0000 ±0.2495	14.000	9/15 to 11/14	Not com	pleted.					
J From A311 8/12/'18	8.9666 ±0.2252	17.933	8/16 to 10/15	7.9167 ±0.1956	15.833	10/15 to 12/14	Not com	pleted.								
F From C86 3/25/'18	8.6000 ±0.2320	17.200	3/28 to 5/27	7.7834 ±0.2313	15.566	5/27 to 7/26	6.3333 ±0.1690	12.666	7/26 to 9/24	4.8833 ±0.2100	9.766	9/24 to 11/23	Dead 12/21			
I From F143 7/1/'18	8.5833 ±0.2468	17.166	7/ 7 to 9/ 5	7.9834 ±0.2320	15.966	9/ 5 to 11/ 4	7.5166 ±0.2256	15.0332	11/ 4 to 1/ 3	Not com	pleted.					
L From I199 11/2/'18	8.7000 ±0.2986	17.400	11/ 4 to 1/ 3	Not com	pleted.											

* Culture medium not established until 55th day.

From this table it is evident that all series follow the same general history, and that, at corresponding periods of the life cycle, all have about the same vigor, although the actual dates may range through all twelve months of the year.

The experiments thus show not only that waning vitality leading to old age and natural death is manifested by Uroleptus mobilis, but also, that conjugation between two individuals at any stage of waning vitality, leads to a complete restoration of vitality.

FALKLANDIA

By J. M. Clarke

State Museum, Albany, New York

Communicated, January 29, 1919

Falklandia is a name herewith applied to a continental land which, during the Devonian period in the occidental parts of the Southern Hemisphere, preceded Gondwana-Land and Antarctis. The history of Gondwana-Land is well established (Neumayr, Suess); its supposed earliest outlines have been approximately determined by the study of its land flora (D. White). The conception of Gondwana-Land is that of a great east-west southern continent which escaped the turmoil of world-wide postcarboniferous deformations and which continued its existence as a continental asylum for land and stream life till late in the Mesozoic time (Cretaceous) when incursions of the sea began which led to its breakdown and demolition in the Tertiary. Eastern Brazil into Sao Paulo, southern Argentine and the north half of the Falkland Islands constitute its western fragments; South Africa, the lost Lemuria (from Madagascar to Ceylon), India and Australia indicate its western extent. Those who have been responsible for the determination of this continent and especially Suess, who has discussed it in much detail, have not recorded its existence prior to the Carboniferous. Antarctis likewise, another Southern 'asylum,' defined on the basis of its terrestrial life and never accurately delimited by its proponents as to the date of its origin, gives proof of like beginning of stabilization and perhaps also of length of endurance. The fossil woods discovered in the Beacon sandstone of South Victoria-Land by James Eights ninety years ago, and the fossils brought home in recent years by Andersson, Nordenskiold, Amundsen, Shackelton and the men of Scott, tend to indicate that it was coexistent in time with Gondwana-Land.

Asylums, thought Suess, were to be defined by continuity in the succession of terrestrial life; it must be added, however, that security of such determinations can be given by the character of the life of the sea which washed the shores of such asylums. Gondwana-Land and Antarctis had a parallel existence in time, though a distinct one. Osborn's observations indicate the

breakdown of Antarctis in the Tertiary. Both Gondwana-Land and Antarctis had a far longer duration than any of the continents of today.

In the period immediately preceding the isolation of these continental masses they were united at the west; that is, in the occidental South Atlantic, the south polar land extended continuously into the land regions of the Gondwana Continent. This we know from the determinations of the Devonian strand lines in southern South America, the Falkland Islands and South Africa.

The Devonian of these latitudes is a unit both in life and in sedimentation. In this regard it is wholly unlike the Devonian of Eria, the east-west continent of the North, and it is a conclusion that is irrefragable on the basis of the intimate and refined analysis that such determinations require and have received. The haphazard observer may be blind to these radical distinctions, especially when basing interpretations upon a knowledge of the strand faunas of Eria. The known extent of these Southern Devonian shore faunas, as pointed out by the writer (Fosseis Devonianos do Parana; Monographias, Vol. I, Servico Geol. e Mineral do Brazil, 1913), indicates the union of the Gondwana and Antarctis continents throughout the Devonian. The extent of this Devonian land bridge across the Atlantic is clearly shown by the unity of shore faunas in South Africa, Sao Paulo, Argentine and Bolivia, and the indication is of a land composed of Paleozoic strata of still earlier date. This is Falklandia, the parent land asylum out of which, in Postcarboniferous time, western Gondwana and Antarctis were carved. The name is appropriately taken, for on the Falkland Islands the Devonian marine strata border the Gangamopteris (Glossopteris) beds of Gondwana-Land.

Other names which have been suggested for these pre-Gondwana austral lands have been founded on inadequate evidence. The "South Atlantic Island" of Frech indicated a Devonian land which had no connection with South Africa; Katzer imagined a north-south Devonian Atlantis running along the axis of the ocean, and Schwarz drew, with somewhat freer hand, his "Flabellites Land," as an undivided land mass along the Atlantic axis reaching from the north, and at the south spreading west and east to join Frech's "South Atlantic Island."

ON THE REAL FOLDS OF ABELIAN VARIETIES

By S. Lefschetz

Department of Mathematics, University of Kansas

Communicated by E. H. Moore, January 30, 1919

1. In this note we propose to find the number of real folds that an abelian variety of rank one may have and to establish some simple properties regarding their connectivity.

An abelian variety of genus p, V_p, belonging to an S_n, $(n > p)$, is defined by equations

$$x_j = f(u_1, u_2, \ldots u_n), \; (j = 1, 2, \ldots n) \tag{1}$$

where the (f)'s are $2p$-ply periodic meromorphic functions of the (u)'s, the periods forming a matrix Ω of a type called, after Scorza, a Riemann matrix. The rank of V_p is the number of distinct systems of values of the (u)'s, modulo Ω, corresponding to an arbitrary point of V_p. It is easy to show that to any Ω corresponds a V_p of rank one. Suppose indeed the (f)'s so chosen that any other periodic meromorphic function φ belonging to Ω be a rational function of them. If V_p were of rank > 1 then to every set of values $(u_1, u_2, \ldots u_p)$, would correspond another $(u'_1, u'_2, \ldots u'_p)$, distinct modulo Ω, such that $\varphi\,(u) = \varphi\,(u')$ whatever φ. Choosing p such functions with non zero jacobian we find that the (u')'s are functions of the (u)'s. But $\varphi\,(u'_1 - \alpha_1, \ldots u'_p - \alpha_p) = \varphi\,(u_1 - \alpha_1, \ldots u_p - \alpha_p)$ whatever the constants α, since the function at the right is of same type as φ. Hence at once $du'_k = du_k$, $u'_k - u_k = \beta_k$. The (β)'s are constants which obviously form a set of periods of the (f)'s, and therefore V_p is effectively of rank one.—All abelian varieties of rank one belonging to Ω are birationally equivalent.

2. Denoting by $\bar{m}$ the complex conjugate of any number m, a real variety of S_m is defined by the condition that when it contains one of the points, said to be conjugate, (x), $(\bar{x})$, it contains the other. Let then Ω be a Riemann matrix with a corresponding real V_p of rank one. Any simple integral of the first kind of V_p is of the form $u = \int\Sigma\, R_j\, dx_j$, where the (R)'s are rational in the (x)'s. Replacing in them all coefficients by their conjugates we obtain a new integral of the first kind of V_p, say (u), and u is a linear combination of the two integrals $u + (u), -i\,(u - (u))$, which are of real form. Hence V_p possess p independent integrals of real form, $u_1, u_2, \ldots u_p$. If V_p has a real point and we take it for lower limit of integration, our integrals will assume conjugate values at conjugate points of V_p.

Let now γ be any linear cycle of V_p and $\bar{\gamma}$ its transformed by T, transformation of the variety permuting each point with its conjugate. T transforms $\gamma + \bar{\gamma}$ into itself and $\gamma - \bar{\gamma}$ into its opposite. As $2\,\gamma, = (\gamma + \bar{\gamma}) + (\gamma - \bar{\gamma})$, the double of any cycle is the sum of two others transformed by T the one into itself and the other into its opposite. The periods of u_k with respect to cycles of the first type are real, and with respect to those of the other type they are pure complex. If q is the number of cycles of one type $2p - q$ is that of the cycles of the other type. As the real and complex parts of the periods of p independent integrals of the first kind with respect to $2p$ independent cycles form a non zero determinant, we must have $q = 2p - q$, or $q = p$. Finally we may single out $2p$ cycles $\gamma_1, \gamma_2, \ldots \gamma_{2p}$, such that $T.\gamma_k = \gamma_k$, $T.\gamma_{p+k} = -\gamma_{p+k}$, $h \leq p$ while the cycles $m_1\gamma_1 + m_2\gamma_2 + \ldots + m_{2p}\gamma_{2p}$, ($m_k$ integer), include the double of any cycle of V_p. The corresponding period matrix for the (u)'s is of the type

$$\| \omega_{h,1}, \omega_{h,2}, \ldots \omega_{h,p}; i\omega_{h,p+1}, i\omega_{h,p+2}, \ldots i\omega_{h,2p} \|, \qquad (2)$$
$$(h = 1, 2, \ldots 2p)$$

the (ω)'s being real.

Remark.—In all this V_p could be replaced by any real irreducible algebraic variety of irregularity p.

3. Conversely if to Ω corresponds a V_p of rank one with $2p$ linear cycles $\gamma_1, \gamma_2, \ldots \gamma_{2p}$, such that the cycles $m_1\gamma_1 + m_2\gamma_2 + \ldots + m_{2p}\gamma_{2p}$ include the double of any other, while p independent integrals of the first kind u_1, $u_2, \ldots u_p$, have with respect to them a period matrix of type (2), V_p is birationally equivalent to a real abelian variety. Indeed as a consequence of the assumptions made, if the equations (1) represent V_p, and if $(\omega_1, \omega_2, \ldots \omega_p)$ are a set of periods of the (f)'s so are $(\bar{\omega}_1, \bar{\omega}_2, \ldots \bar{\omega}_p)$. Moreover the (f)'s are real meromorphic functions of the (u)'s and of a finite number of constants. If we replace these constants by their conjugates we obtain new functions $\bar{f}(u_1, u_2, \ldots u_p)$ with the same periods as the (f)'s the equations

$$x'_j = f_j + \bar{f}_j,\ x''_j = -i\,(f_j - \bar{f}_j),\ (j = 1, 2, \ldots n)$$

represent in an S_{2n} a real abelian variety birationally equivalent to V_p. This real abelian variety has ∞^p real points.

4. Assuming then V_p real with real folds, to determine their number we remark that at two conjuguate points the integrals u_k of No. 2 take conjuguate values $u'_k + iu''_k$, $u'_k - iu''_k$, modulo Ω. At the real folds the (u)'s are given by $u'_k + \frac{i}{2}\sum_1^p{}^{\mu}\, r_\mu\, \omega_{k,p+\mu}$, ($u'_k$ arbitrary, $r_\mu = 0$ or 1; $k = 1, 2, \ldots p$). There are in general $p - s$ periods of this type such that no linear combination of them with integral coefficients is of the form

$$\sum_1^p{}^{\mu} (m_\mu\, \omega_{k,\mu} + im'_\mu\, \omega_{k,p+\mu}),\ (k = 1, 2, \ldots p).$$

Hence $p - s$ of the integers r can be made equal to zero. Taking the others equal to zero or one yields 2^s distinct real folds.—That there are varieties having that number of real folds whatever $s \leqq p$ is shown by the canonical matrix

$$\left\| \begin{matrix} 1, 0 \ldots 0, a_{11}, a_{12}, \ldots a_{1p} \\ 0, 1 \ldots 0, a_{21}, a_{22}, \ldots a_{2p} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ 0, 0 \ldots 1, a_{p1}, \ldots\ldots a_{pp} \end{matrix} \right\|$$

($a_{\mu\nu} = a_{\nu\mu} = \frac{1}{2}\, m_{\mu\nu} + i\, a''_{\mu\nu}$; $m_{\mu\nu}$ integers forming a matrix of rank s modulo 2; the quadratic form $\Sigma\, a''_{\mu\nu}\, x_\mu\, x_\nu$ is definite, positive.)

Hence a real abelian variety can have any number of real folds given by 2^s, $0 \leqq s \leqq p$.

5. Any real fold of the variety can be transformed into any other by a birational transformation belonging to the continuous group $(u_k, u_k + c_k)$. Hence

the real folds form equivalent p dimensional cycles, and the $q \leqq p$ dimensional ones which one of the folds contains are equivalent to some cycle, on any other. Moreover the real folds form homeomorphic manifolds. One of them is defined by the relations $u_k = \sum_1^p t_\mu \omega_{k\mu}$, where the (t)'s are real parameters varying from zero to one. It follows that any one of the folds corresponds point by point to the interior of a p-dimensional cube in the same manner as the points of a ring to those within a square. From this follows readily that the real folds are two sided manifolds of j-th index of connection equal to (p_j); this being also the number of independent real cycles of V_p. We may remark finally that each real fold is transformed into itself by the two sets of birational transformations, $(u_k, u_k + c_k)$, $(u_k, - u_k + c_k)$, where the (c)'s are real constants.

6. It is of interest to consider the types of real Jacobi varieties.—The Jacobi variety V_p of a curve of genus p, C_p, is an abelian variety of rank one belonging to the period matrix of p independent integrals of the first kind of C_p. Let C_p be real and its integrals of the first kind $v_1, v_2 \dots v_p$ real in form with a real point M_0 for lower limit of integration. We assume that C_p has $q > 0$ real branches. We may then take for V_p a representation (1), the (f)'s being real functions of the variables u, now defined by $u_k = \sum_1^p \int_{M_0}^{M_j} dv_k$, the points $M_1, M_2, \dots M_p$, being alone variable on C_p. In order that the point which corresponds to them on V_p be real it is necessary and sufficient that the set of the (M)'s coincide with that of their conjuguates,—a corollary of the fact that when a point of V_p moves on a real fold the increments of the (u)'s are real.

Let $B_1, B_2, \dots B_q$, be the real branches of C_p. If we constrain α_1 of the (M)'s to be on B_1, α_2 on $B_2, \dots \alpha_q$ on B_q, the corresponding point of V_p will be on a definite real fold F_1 of the variety, provided of course that $p - \Sigma \alpha_i$ be $\geqq 0$ and even. Assume one of the (α)'s, for example α_1, to be greater than one. I say that the fold F_2 corresponding to the integers $\alpha_1 - 2, \alpha_2, \dots \alpha_q$, coincides with F_1. Indeed a ready corollary from our previous discussion is that two distinct real folds of an abelian variety have no common points, while F_1 and F_2 intersect along the $p - 1$ dimensional manifold obtained when α_1 of the (M)'s of which two coincide are on B_1, the $p - \alpha_1$ remaining being disposed as previously. Hence if $q \leqq p + 1$, V_p has as many real folds as there are independent solutions of the congruence $\Sigma\alpha_i - p \equiv 0$, modulo 2, that is 2^{q-1}. If $q > p + 1$, V_p would have more than 2^p real folds. As this is impossible, $q \leqq p + 1$ and we have here a new proof of Harnack's theorem that a curve of genus p has at most $p + 1$ real branches.

We see then that a Jacobi variety can have any number of real folds given by 2^s, s being an integer at most equal to the genus p.

COVARIANTS OF BINARY MODULAR GROUPS

By Oliver Edmunds Glenn

Department of Mathematics, University of Pennsylvania

Communicated by E. H. Moore, January 30, 1919

To transform the general case of a binary quantic

$$f_m = (a_0, a_1, \ldots, a_m \between x_1, x_2)^m,$$

by substituting for its variables

$$x_1 = \lambda_1 x_1' + \mu_1 x_2',\ x_2 = \lambda_2 x_1' + \mu_2 x_2',$$

where the coefficients of the transformation are least positive residues of a prime number p, is to generate an infinitude of binary forms associated with f_m, which are invariants of the linear modular group G of order $(p^2 - p) \times (p^2 - 1)$, on the variables x_1, x_2.

Much difficulty has been encountered by those who have investigated these invariants, both as to methods of generating complete systems and as to proofs of their finiteness. This paper is in the form of a summary of the present writer's research on this problem. It contains an outline of a method of construction of the formal modular covariants which has a measure of generality, —and which has proved to be definitive,—at least for particular moduli. Secondly we give, in explicit form, the finite modular systems of the cubic (mod 2) and of the quadratic (mod 3). Phases of the formal modular theory not mentioned in this paper are treated in articles by the present writer, referred to at the end of this paper, and in Dickson's Madison Colloquium lectures.

1. Modular convolution.—If $S(a_0, a_1, \ldots)$ is any modular seminvariant of f_m which satisfies a certain pair of conditions,[5c] and $a_0', a_1', \ldots$ are the coefficients of the transformed of f_m by means of

$$x_1 = x_1 + tx_2',\ x_2 = x_2';\ (t \text{ any residue mod } p),$$

then, S' being the conjugate to S under the substitution $(a_0a_2)(a_1)$,

$$S'(a_0', a_1', \ldots) \equiv S(a_0, \ldots)t^{p-1} + S_1 t^{p-2} + \ldots + S_{p-1} \pmod{p}, \quad (1)$$

written with homogeneous variables x_1, x_2, instead of t, is a formal covariant modulo p. When this principle is applied in the case of the seminvariant leading coefficient of the covariant K_ν:

$$K_\nu = C_0 x_1^\nu + C_1 x_1^{\nu-1} x_2 + \ldots + C_\nu x_2^\nu,$$

where ν is a number of the form

$$\nu = (sp - \nu')(p - 1) = \sigma(p - 1),$$

there results a covariant[6]

$$[K_\nu] = [C_0 + (K_\nu)]\,x_1^{p-1} + \sum_{r=0}^{p-2}\sum_{s=0}^{\sigma-1} C_{\nu-s(p-1)-r}\,x_1^{p-r-1}x_2^r, \qquad (2)$$

where (K_ν) is the pure invariant

$$(K_\nu) = C_{p-1} + C_{2(p-1)} + \ldots + C_{(\sigma-1)(p-1)}. \qquad (3)$$

Now if we form the product of any two binary forms, as of f_m and

$$g_n = (b_0, b_1, \ldots, b_n \between x_1, x_2)^n \ (m \geqq n),$$

where $m + n$ is a number of the form $\sigma\,(p-1)$, and construct the formulas analogous to (2), (3) for the result, we get

$$(f_m g_n) = \sum_{i=0}^{j(p-1)}\sum_{j=1}^{\sigma-1} a_i b_{j(p-1)-i},$$

$$[f_m g_n] = [a_0 b_0 + (f_m g_n)]\,x_1^{p-1} + \sum_{r=0}^{p-2}\sum_{s=0}^{\sigma-1}\sum_{i=0}^{t} a_i b_{(\sigma-s)(p-1)-r-i}\,x_1^{p-r-1}x_2^r$$

$$\{t = (\sigma - s)(p-1) - r\}.$$

This process of constructing the concomitants $(f_m g_n)$, $[f_m g_n]$ from the product $f_m \cdot g_n$ is analogous to transvection and symbolical convolution in the theory of algebraic concomitants.

An example of the covariant (1) is obtained from the following seminvariant[3] of a quadratic form f_2:

$$S = a_0^{p-1}a_1 - a_1^p.$$

This seminvariant satisfies the two necessary and sufficient conditions that it may be the leading coefficient of a covariant, viz., of [5c]

$$C_1 = (a_0x_1 + a_1x_2)^p - (a_0x_1 + a_1x_2)(a_0x_1^2 + 2a_1x_1x_2 + a_2x_2^2)^{p-1}, \qquad (4)$$

where powers of x_1, x_2 are to be reduced by Fermat's theorem.

2. *Concomitant scales.*—If K is any modular covariant of order $(rp - \nu) \times (p-1) = m$, then by application, to K, of the latter of the two modular invariantive operators [5b]

$$E = a_0^p\frac{\delta}{\delta a_0} + a_1^p\frac{\delta}{\delta a_1} + \ldots + a_m^p\frac{\delta}{\delta a_m},$$

$$\omega = x_1^p\frac{\delta}{\delta x_1} + x_2^p\frac{\delta}{\delta x_2},$$

there is obtained a set or scale of $\mu = p + \nu + 2$ concomitants

$$(K),\ \omega^\lambda[K]\ (\lambda = 0, 1, \ldots, p-1),\ \omega^t K\ (t = 0, 1, \ldots, \nu). \qquad (5)$$

This set, which we call a *μ-adic* scale[6] for K, is analogous to the set obtainable by convolution from a symbolical algebraic covariant

$$\Pi\,(ab)\,a_x^p\,b_x^q\,c_x^r \cdots.$$

A scale of modular concomitants is said to be *complete* when it contains all invariants and covariants of the first degree in the coefficients of K which can be obtained from K by empirical or modular invariantive processes, i.e. a *μ-adic* scale is complete when it constitutes a fundamental system of first degree concomitants of K. Thus we find that we are able to construct a complete modular system of a form f_m by a process of passing from a complete scale for f_m of the first degree to scales derived from covariants of f_m of degree > 1.

3. *A complete system of the quadratic, modulo 3.*—I have proved that the following eighteen quantics constitute a complete formal system modulo 3 of f_2; the proof resting primarily upon the fact that, when $p = 3$, every covariant is of even order, and hence the *μ-adic* scale (5) is complete:

$$\begin{gathered} f_2,\ C_1,\ [f_2^2],\ [f_2C_1],\ [f_2^2C_1],\ Ef_2, \\ \omega f_2,\ \omega C_1,\ \omega[f_2^2],\ \omega Ef_2,\ \omega^2 f_2,\ \omega^2 C_1, \\ L,\ Q,\ (f_2^2),\ ([f_2^2]^2),\ (C_1Ef_2),\ \Gamma; \end{gathered} \tag{6}$$

where[3] $\Gamma = (a_0 + a_2)(2a_0 + 2a_1 + a_2)(2a_0 + a_1 + a_2)$, and L, Q are the functions to which the universal covariants of the group $G_{(p^2-p)(p^2-1)}$ reduce when $p = 3$. These covariants are[3]

$$L = x_1^p x_2 - x_1 x_2^p,\ Q = (x_1^{p^2} x_2 - x_1 x_2^{p^2})/L.$$

The last four quantics in the system (6) are pure invariants and constitute a complete system of invariants of f_2 modulo 3. This set of invariants was first derived by Dickson. The orders of the forms (6) range from 0 to 6 and the degrees from 0 to 6.

4. *A complete system of the cubic, modulo 2.*—When $p = 2$ the μ concomitants (5) do not form a complete scale. Every form of order > 3 is reducible[5e] modulo 2 in terms of invariants of the first degree in its coefficients and of its covariants of orders 1, 2 and 3. If K_ν is the covariant shown in §1 we have ($p = 2$),

$$\begin{aligned} (K_\nu) &= C_1 + \ldots + C_{\nu-1}, \\ [K_\nu] &= [C_0 + (K_\nu)]x_1 + [(K_\nu) + C_\nu]x_2. \end{aligned}$$

These, and the additional covariant

$$\{K_\nu\} = C_0 x_1^2 + (K_\nu)x_1x_2 + C_\nu x_2^2,$$

exist for all orders ν. When ν is odd there exists also a cubic covariant[5c]

$$\{\overline{K_\nu}\} = C_0 x_1^3 + I_1 x_1^2 x_2 + I_2 x_1 x_2^2 + C_\nu x_2^3,$$

where I_1, I_2 are definite linear expressions in $C_1, \ldots, C_{\nu-1}$ such that[5d]

$$I_1 + I_2 \equiv (K_\nu)\ (\text{mod } 2).$$

These concomitants, and their polars by $\omega = x_1^2 \frac{\delta}{\delta x_1} + x_2^2 \frac{\delta}{\delta x_2}$, form a complete scale for K_ν.

A fundamental system of formal covariants of f_3 modulo 2 consists of the following twenty quantics;[5d] where we abbreviate by H the algebraic hessian of f_3, and

$$t = H + (f_3)\{f_3\},\ l = Q[H] + f_3\,(f_3),$$

and in which L, Q are the universal covariants of the group G_6:

$$\begin{gathered} f_3, H, [f_3], \{f_3\}, Ef_3, E[f_3], \{f_3t\}, \{Pt\}, [f_3t], \\ [Ef_3 \,.\, t], \{\overline{f_3t}\}, \{\overline{tEf_3}\}, [Ql], L, Q, \\ (f_3), (H), (tf_3), (tEf_3), I. \end{gathered}$$

The invariant I, which is not represented as belonging to any scale, is

$$I = a_0^2 + a_0a_3 + a_3^2 + (a_0 + a_3)(f_3).$$

The orders of the forms in this system range from 0 to 3, and their degrees from 0 to 4.

The proofs of the existence of $\{\overline{K_\nu}\}$ and of $[K_\nu]$ involve certain remarkable congruential properties of the binomial coefficients.[6]

Reduction methods rest largely upon the fact that two modular covariants of the same order and led by the same seminvariant, while not identical as a rule, are necessarily congruent to each other moduli $L\,(= x_1^px_2 - x_1x_2^p)$, and p. Thus a covariant is not uniquely determined by its seminvariant leading coefficient as is the case with algebraic covariants. Another noteworthy general fact is that of the existence of covariants whose leading coefficients are modular invariants.

[1] Hurwitz, A., *Archiv. Math. Phys., Leipzig*, (Ser. 3), **5**, 1903, (17).

[2] Dickson, L. E., *Trans. Amer. Math. Soc.*, **10**, 1909, (123); **12**, 1911, (75); **14**, 1913, (290); and **15**, 1914, (497); *Amer. J. Math., Baltimore*, **31**, 1909; etc.

[3] Dickson, *Madison Colloquium Lectures*, 1913.

[4] Miss Sanderson, *Trans. Amer. Math. Soc.*, **14**, 1913.

[5] Glenn, O. E., (a) *Amer. J. Math.*, **37**, 1915, (73); (b) *Bull. Amer. Math. Soc., New York*, **21**, 1915, (167); (c) *Trans. Amer. Math. Soc.*, **17**, 1916, (545); (d) **19**, 1918, (109); (e) *Ann. Math., Princeton*, **19**, 1918, (201).

[6] A paper containing a portion of the general theory here mentioned and the derivation of the system of the quadratic modulo 3, as well as certain other developments, is to appear in *Trans. Amer. Math. Soc.*

THE GENERAL SOLUTION OF THE INDETERMINATE EQUATION: $Ax + By + Cz + \ldots\ldots = r$.

By D. N. Lehmer

Department of Mathematics, University of California

Communicated by E. H. Moore, January 30, 1919

Little has been done since Jacobi (*Werke*, **6**, 355) in connection with the solution of the general linear indeterminate equation. Jacobi has given no less than four methods all involving the reduction of the equation by means of a set of auxiliary equations to the solution of an equation in two variables the solution of which is immediately obtained from the theory of the ordinary continued fraction. The solution here presented treats the general equation in the same non-tentative way that is found in the continued fraction solution for two variables. The method applies equally well when the right hand member is zero and gives a perfectly general solution from which all other special solutions may be obtained.

Consider the set of positive or negative, non-zero integers $a_1, b_1, c_1, \ldots, k_1, l_1$ and let m be the number of terms in the set. We derive from this first set a second set $a_2, b_2, c_2, \ldots k_2, l_2$ by means of the equations:

$$\begin{aligned} a_2 &= b_1 - \alpha_1 a_1, \\ b_2 &= c_1 - \beta_1 a_1 \\ &\ldots\ldots\ldots \\ k_2 &= l_1 - \kappa_1 a_1, \\ l_2 &= a_1 \end{aligned}$$

where if a_1 is different from unity the numbers $\alpha_1, \beta_1, \ldots \kappa_1$ are so taken that $a_2, b_2, c_2, \ldots k_2$ are the smallest positive residues, not zero, of the numbers $b_1, c_1, \ldots k_1, l_1$ respectively with respect to the modulus a_1. If a_1 is unity then $\alpha_1, \beta_1 \ldots \kappa_1$ are taken equal to $b_1, c_1, \ldots l_1$ respectively, so that in this case the second set consists of zeros with the exception of the last term, l_2 which is unity.

We derive in the same way a third set from the second by means of the equations:

$$\begin{aligned} a_3 &= b_2 - \alpha_2 a_2 \\ b_3 &= c_2 - \beta_2 a_2 \\ &\ldots\ldots\ldots \\ k_3 &= l_2 - \kappa_2 a_2 \\ l_3 &= a_2 \end{aligned}$$

As in the preceding set, if a_2 is different from unity, we take the numbers $\alpha_2, \beta_2, \ldots \kappa_2$ so that $a_3, b_3, \ldots k_3$ are the smallest positive, non-zero residues of $b_2, c_2, \ldots l_2$ respectively, modulo a_2. If a_2 is unity, $\alpha_2, \beta_2, \ldots \kappa_2$ are taken equal respectively to $b_2, c_2, \ldots l_2$, and in this case the third set consists of zeros with the exception of the last, l_3, which is unity.

Continuing this process, if the original set had no common factor other than unity, we must arrive at a set in which the first number is unity. For it is clear that $a_{n+1} < a_n$ except when b_n is divisible by a_n in which case $a_{n+1} = a_n$. Further $a_{n+2} < a_n$ except when both b_n and c_n are divisible by a_n, in which case $a_{n+2} = a_{n+1} = a_n$, and so on. If now the original set had the greatest common divisor unity so will also the set $a_n, b_n, c_n, \ldots k_n, l_n$, and so not all the numbers $b_n, c_n, \ldots k_n, l_n$ can be divisible by a_n. After a number of steps in the process at most equal to $n - 1$ an a_λ must appear which is less than a_n and not zero. In the same way another set must appear in which the first number is less than a_λ and so on. This process must then lead to a set in which the first number is unity. By taking one more step a set is then obtained in which the numbers are all zeros except the last which is unity.

We proceed now to reverse the above process, and taking the numbers $\alpha_1, \beta_1, \ldots \kappa_1 : \alpha_2, \beta_2, \ldots \kappa_2 : \ldots$ as given we will show how to reconstruct the original set $a_1, b_1, c_1, \ldots k_1, l_1$ from them. We construct first a determinant of order m in which the element of the prinicipal diagonal are all units, and all the other elements are zero. Using the first set of numbers $\alpha_1, \beta_1, \ldots \kappa_1$, which we will call the first 'partial quotient set' we construct what we will call the 'first determinant' as follows: The top row of the above determinant is erased and another row is added at the bottom which has for its elements $1, \alpha_1, \beta_1, \ldots \kappa_1$. This bottom row we will call the first 'convergent set' and for uniformity of notation we write it $A_1, B_1, C_1, \ldots K_1, L_1$. It is seen that the value of the first determinant is unity.

Using the second partial quotient set, $\alpha_2, \beta_2, \ldots \kappa_2$. we obtain from the first determinant a second determinant by erasing the top row and adding for the bottom row the second convergent set $A_2, B_2, C_2, \ldots K_2, L_2$, the elements of which are obtained by adding the columns of the first determinant after multiplying the first row by 1, the second by α_2, the third by β_2, etc. and the bottom row by κ_2. In the same way we get the third determinant and the third convergent set, using the third partial quotient set $\alpha_3, \beta_3, \ldots \kappa_3$. The nth convergent set, which is the bottom row of the nth determinant is related to the preceding sets by the recursion formulae:

$$A_n = \kappa_n A_{n-1} + \ldots \alpha_n A_{n-m+1} + A_{n-m}$$

with similar formulae for the B_n, C_n, etc. It is clear from the way the determinants are derived from each other and from the original determinant that the value of each is unity.

We state now the remarkable theorem that the last convergent set is identical with the original set $a_1, b_1, c_1 \ldots k_1, l_1$, from which the successive partial quotient sets were derived. This theorem comes out of the general theory of continued fractions, of multiplicity m, but without any appeal to that theory Professor Frank Irwin has derived it very simply from the following equations which are easily established by complete induction:

$$
\begin{array}{lll}
a_1 = a_nA_{n-m} + b_nA_{n-m+1} + \ldots k_nA_{n-1} + l_nA_n & & n = 1, 2, \ldots \\
b_1 = a_nB_{n-m} + b_nB_{n-m+1} + \ldots k_nB_{n-1} + l_nB_n & & A_0 = A_{-1} = A_{-2} = \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & \ldots = A_{-m+1} = 0, \\
l_1 = a_nL_{n-m} + b_nL_{n-m+1} + \ldots k_nL_{n-1} + l_nL_n & & A_{-m} = 1, \text{etc.}
\end{array}
$$

Now, as we have seen, in the last set a_n, b_n, c_n, k_n are all zero, and l_n is equal to unity. This gives the theorem.

This theorem throws into our hands a straightforward method of writing down a determinant equal to unity whose bottom row is any given set of positive or negative non-zero integers, with greatest common divisor unity. Let this last row be A_n, B_n, C_n, $\ldots K_n$, L_n, and let the co-factors of these elements in the last determinant be A'_n, B'_n, C'_n, $\ldots K'_n$, L'_n. Then these co-factors furnish a set of values for the unknown in the indeterminate equation $A_nx + B_ny + \ldots L_nv = 1$, and by multiplying these values through by r we get a solution of the equation when the right member is equal to r. Moreover since $A_nA'_\rho + B_nB'_\rho + C_nC'_\rho + \ldots K_nK'_\rho + L_nL'_\rho = 0$ for $\rho = n - 1, n - 2 \ldots n - m + 1$ we may write for the most general value of the variables:

$$
\begin{array}{ll}
x = rA'_n + sA'_{n-1} + tA'_{n-2} + \ldots & +pA'_{n-m+1} + qA_{n-m} \\
y = rB'_n + sB'_{n-1} + tB'_{n-2} + & pB'_{n-m+1} + qB'_{n-m} \\
z = rC'_n + sC'_{n-1} + tC'_{n-2} + & +pC'_{n-m+1} + qC'_{n-m} \\
\ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots\ldots\ldots
\end{array}
$$

That this is the most general form of the solution follows from the fact since the determinant of the co-factors is unity a set of integer values of $r, s, t, \ldots p, q$ can always be found for any given set of values of $x, y, z, \ldots$

We give as a numerical example the problem of determining the most general solution of the indeterminate equation:

$$33x + 55y + 79z - 99w = r.$$

The following is a convenient arrangement of the work of computing the sets a_1, b_1, c_1, d_1, α_1, β_1, γ_1, etc.

a_n	b_n	c_n	d_n	α_n	β_n	γ_n
33	55	79	−99	1	2	−4
22	13	33	33	0	1	1
13	11	11	22	0	0	1
11	11	9	13	0	0	1
11	9	2	11	0	0	0
9	2	11	11	0	1	1
2	2	2	9	0	0	4
2	2	1	2	0	0	0
2	1	2	2	0	0	0
1	2	2	2	2	2	2
0	0	0	1			

With these values of $\alpha_n, \beta_n, \gamma_n$ we proceed to compute the successive convergent sets and the corresponding determinants. The following is a convenient arrangement of the work:

α_n	β_n	γ_n	A_n	B_n	C_n	D_n
			1	0	0	0
			0	1	0	0
			0	0	1	0
			0	0	0	1
1	2	−4	1	1	2	−4
0	1	1	1	2	2	−3
0	0	1	1	2	3	−3
0	0	1	1	2	3	−2
0	0	0	1	1	2	−4
0	1	1	3	5	7	−9
0	0	4	13	22	31	−39
0	0	0	1	2	3	−2
0	0	0	1	1	2	−4
2	2	2	33	55	79	−99

The last determinant yields us the solution of the proposed problem. Computing the minors we easily get the following general values for the unknowns in the equation:

$$x = -29r - 57s + 54t - 65u$$
$$y = 3r + 4s - 4t + 7u$$
$$z = 5r + 11s - 11t + 11u$$
$$w = -4r - 8s + 7t - 9u$$

. . . .

THE ARCHAEOLOGY OF THE SOUTHWEST: A PRELIMINARY REPORT

By N. C. Nelson

American Museum of Natural History, New York

Communicated by Henry F. Osborn, February 6, 1919

Introductory.—Since my last communication to the Proceedings of the National Academy (Vol. 3, pp. 192–195) two years ago, concerning the time distribution of aboriginal traits in the Mammoth Cave region of Kentucky, it has become possible to report upon the stratigraphic conditions in two additional North American culture centers. One of these new centers, viz., the Florida-Georgia characterization area, directly adjoins that in which the Mammoth Cave occurs, in fact is in a large sense one with it and need not therefore be specifically discussed. The other center, of special concern here is sepa-

rated from the preceding by the totally different Plains culture area and lies in the semi-arid plateau region known culturally as the Pueblo area and geographically as the Southwest. The mentioning of these three distinct and partly separated centers together in this place is made possible and necessary by the fact that our archaeological findings in them are identical in their general import, as will be brought out at the end of the discussion.

The Pueblo culture area is from several standpoints our most profitable field for the study of primitive man. It is to us what Egypt is to the Old World and more. For not only have we here an abundance of ruins more or less ancient but we have the relatively unspoiled descendants of the builders still surviving, and we have documentary data concerning them reaching back nearly four hundred years. It is a field therefore in which the ethnologist, the historian, and the archaeologist can work hand in hand; and, needless to say, they have done so. Each mode of approach has, however, its particular limitations; and it has of late become evident that an adequate solution of the problem presented lies in a coordination of effort. The ethnologist, e.g., sees the problem clearly only in its spatial dimension; the historian makes a brief beginning with the time projection but to complete his work he must of necessity appeal to the archaeologist. The archaeologist, on the other hand, while he may see the problem in both dimensions, unless already an ethnologist, is obliged to call upon such a specialist to assist him in the interpretation of his findings. The failure to fully appreciate these interrelations will probably in a large measure explain the history of anthropological investigation in the Southwest.

History of Archaeological Investigation.—The Pueblo culture area has been under consideration for seventy-three years and something like three stages are discernible in the process. Of the numerous reports now available on the antiquities, those of the first thirty years were written by staff members of various governmental expeditions and surveys and were of a general descriptive character. About 1880 the investigation became institutionalized, so to speak. Specialists in history, ethnology, and archaeology entered the field and all have delivered more or less convincing reports on the problem from their particular points of view. But, naturally enough, only the historian has in any sense finished his task. The ethnologist has his work well under way; while as for the archaeologist—in spite of all he has done and written—his results have not until lately been carried much beyond the analytic stage. During the last few years finally there has been a distinct effort on the part of several investigators to reach the synthetic level or in other words to get beyond the descriptive and classificatory routine work to really interpretative results. It is pleasant in this connection to be able to say that the American Museum's archaeological work, prolonged now for seven years and taken part in by Messrs. Leslie Spier and Earl H. Morris as well as the writer, has been primarily of this interpretative character. We have entered the field not so much to recover specimens as to solve problems. Owing to the immensity of the

field we have, however, like other investigators, been compelled to begin with merely local problems; but the solution of these local problems has—so we believe—led to the discovery of simple methods applicable to the Pueblo problem in its entirety.

The Field Data.—The Pueblo Indians, in the light of the ethnological and historical information we have concerning them, may be defined as a group of sedentary tribes who build substantial rectangular houses of a more or less compact and communistic type, who commonly construct semi-subterranean ceremonial chambers, either round or rectangular, who build reservoirs and irrigation ditches, who grow corn, beans, and squashes for food as well as cotton for clothing, who grind their corn on a metate, who use the curved rabbit-stick and the tubular pipe, who possess a special type (or types) of grooved ax, who work turquoise and who make pottery having very striking local peculiarities. The ethnologist would indicate additional characteristics of a linguistic, social, and religious nature; but these because they have received less definite or permanent material expression are of secondary importance to the arachaeologist as working data. Some of the cited traits, like maize growing, the Pueblo share in common with other and even very distant tribes of North and South America; certain other traits like the round ceremonial chamber are only partially diffused over their own territory; and still other traits, like cotton growing and the stone ax, have completely disappeared in modern times. Perhaps the most characteristic of the surviving traits, considered both as geographical and as historical phenomena, are architecture and ceramics.

The tribes who today exhibit the above characters are domiciled in about thirty villages specifically known as *pueblos* in contradistinction, first, to the less local, rather loosely constructed and only semi-permanent type of villages known as *rancherias* and, second, to the still more widespread temporary settlements known as *camps*. Their numerical strength, according to the last census is about 11,000 souls; and their territorial possessions, in the form of land grants and reservations, are officially placed at about 5,000 square miles. These artificial boundaries are not exactly conterminous with the actual range of the tribes but the figures give a fair idea of the amount of territory from which they draw sustenance.

In 1540 or thereabouts when the Spanish explorers first entered the country the Pueblo appear to have inhabited all of seventy villages and to have numbered about 20,000, a few more or less. Their territorial range was about 13,000 square miles, or more than twice what it is today, a large section having been vacated, e.g., on the southeast. That much is determined for us in part by the historian; but from this point on for another short stretch only the ethnologist can accompany us.

After several decades of more or less desultory work we are now in position to say that the Pueblo in prehistoric times ranged over a territory little short of 140,000 square miles in extent, throughout which they have left ruins and other characteristic remains very similar to those found in the territory still

occupied. Beyond this territory, finally, there is a wide marginal zone, which taken together with the central area is little short of 1,000,000 square miles in extent, and throughout which appear ruins and remains in some respects like those of the Pueblo area proper and in other respects like the remains characteristic of Nomadic regions, i.e., essentially 'rancherias.' In short, the geographic distribution of Pueblo traits takes the form of a *center* of high and unalloyed development and a *marginal zone* different segments of which have been more or less affected by influences from other adjacent culture centers. The bearers of the pure Pueblo culture still reside in the heart of the old center; and several tribal groups of the hybrid Pueblo-Nomad type also continue to reside in the western and southern portions of the marginal zone. Lastly, several slightly modified but vigorous strains of nomadic cultures are present on the north and southeast, not only in the marginal zone but in considerable stretches of what was once pure Pueblo domain. The spatial condition of affairs is diagrammatically represented in figure 1.

On the face of it the diagram indicates two things: first, a tremendous geographical concentration of the pure Pueblo culture or as we might just as well say, a falling off in Pueblo influence; and, second, and what amounts to the same thing, several great nomadic invasions which have swallowed up or pressed back the Pueblo traits on the north and southeast and all but detached several small marginal centers on the south and west. After several years of contact with the facts of this situation the writer is unable to escape the conclusion that they are all very intimately connected and that either set of them largely explains the other. At this point, then, we have to leave the ethnologist behind: if we wish to know anything further we have to dig below the surface. Given the spatial phenomenon presented in our diagram, it is for the archaeologist to present the same set of facts as a time phenomenon, in other words, to arrange the data in their proper historical sequence. And to do this he must devise adequate methods of his own.

Concerning Methods.—One of the difficulties that the archaeologist has labored under has been the mass of the data confronting him. In his effort to master details he has lost touch with the problem as a whole. No one man has yet seen the entire field with his own eyes. Dr. J. W. Fewkes, who may be said to have initiated the last phase of the investigation, has, e.g., done his work largely in the western sector and has generalized on the problem from that point of view, using mainly architectural traits as a basis. Dr. A. V. Kidder has worked mostly in the northern sector and has generalized mainly on the basis of ceramic traits. Both employed the method of direct comparison. The writer finally has done his work chiefly in the southeastern sector, using ceramics as a medium and employing as a method, in addition to that of direct comparison, the simple principle of stratification. Resort to this medium and to this principle came about in a very natural manner.

The data we have to deal with consist of some six or seven hundred major, i.e., pueblo, ruins of various types, located within our center of high develop-

ment; and, in addition, an untold number of minor ruins, partly of the rancheria type, scattered over both the Central area and the great Marginal zone. The immediate problem confronting the archaeologist has been to decide whether these ruins were chronologically separable. Until recently the general verdict has been that they were not, except of course within the narrow limits of 'historic' and 'prehistoric.' A third and intermediary group we may perhaps concede to have been detached on the basis of traditional references to it, but that is all. Our prehistoric ruins were simply 'prehistoric,' that is, roughly speaking, they were of the same age: and the result has been some extraordinary speculations about great numbers of peoples and their mysterious disappearance, which has finally been credited by some to a fancied 'change of climate.'

Unfortunately, the appearance of the ruins was no criterion of age. Neither did the achitecture as architecture give away in a really clear manner the order of development, doubtless partly because architecture is in a large measure determined by environmental conditions. Pottery—an ever present accompaniment of the ruins—is on the other hand, an exceedingly plastic phenomenon, varying from place to place and from time to time, in response to the inventive faculty, far more readily than does architecture. We may therefore decide the relative age of any given ruin by determining the age of the particular style of pottery which it exhibits; and this latter feat is easily accomplished. We have but to find the stratigraphic position of this particular style in the total series of styles as they occur in refuse heaps or in actually superposed ruins. Sometimes the stratigraphic position has to be determined without digging into the refuse heaps—there being none, but the principle involved is the same: we have to begin with the style of ware lying on top or still in use and must work back or down through the series until we arrive at the bottommost style, which is the oldest. We may indicate our general procedure therefore by saying that instead of as formerly hunting out the most picturesque ruins for excavation or at any rate the ruins giving promise of the finest outlay of specimens, we have begun commonly with the despised ruins of historic date situated in the heart of the present Pueblo habitat.

Results to Date.—Our method has not yet been applied to the entire Pueblo area and consequently our results are not complete in respect to numerous details. Nevertheless, the general outline of things, i.e., the chronological disposition of the ruins, is already tolerably clear and has been diagrammatically represented in figure 2 which is, as it were, a section in the A-B line of figure 1. To date we have, so to speak, dug down through the superposed culture levels found in both the Zuni and the Tewa circles, in fact through several additional but more ancient circles not presented in the diagram. By thus discovering the time order of things we have with the same effort discovered the key to the whole spatial arrangement. For as we dig down through the vertical series we pass gradually from Pueblo to Rancheria traits and from Rancheria to Nomadic traits exactly as if we were traveling from the present center of Pueblo life out over the various zones to the Nomadic border. Briefly, it is archaeolog-

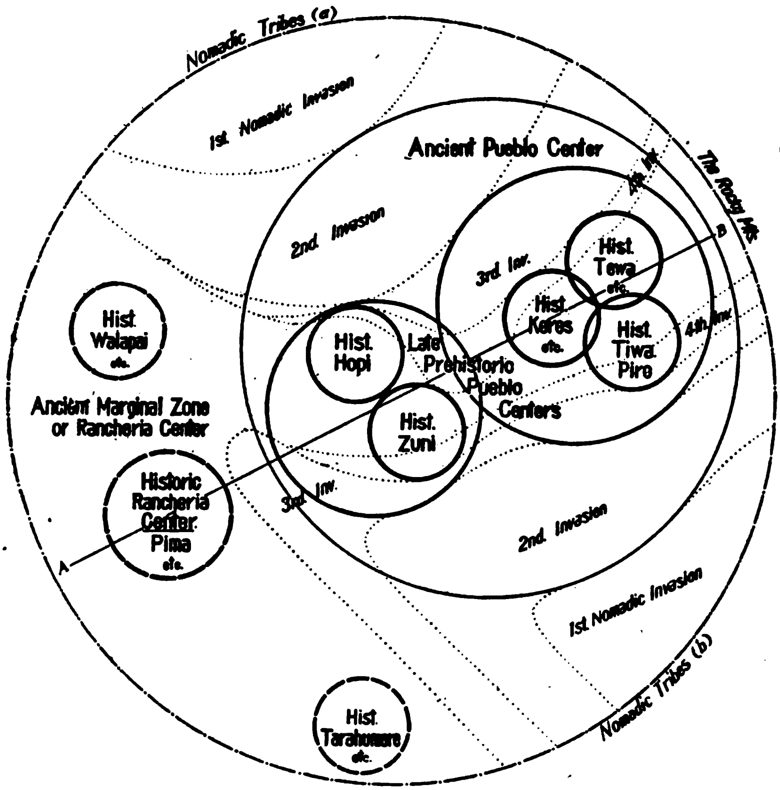


Fig 1.

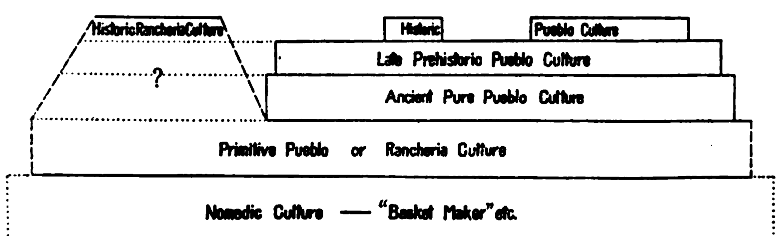


Fig 2.

FIG. 1. SPATIAL DISTRIBUTION OF PUEBLO TRAIT GROUPS

FIG. 2. TIME DISTRIBUTION OF PUEBLO TRAIT GROUPS, IN A-B LINE OF FIGURE 1

ically demonstrable, as perhaps many will say they knew long ago, that the Pueblo culture grew out of a Nomadic one.

Conclusions.—There is no space here for the discussion of details. There remains to add however that it is not clear whether the Pueblo phenomenon is something altogether special in culture or whether in reality it is a fine illustra-

tion of a general principle. That is, I am not certain whether the accident of Nomadic invasion produced the outstanding Pueblo traits or whether in the absence of such pressure we should have observed the same phenomenon in all its essentials. In short, does or does not the 'age and area' hypothesis, commonly subscribed to by students of organic life, hold true also in the province of human culture? But, passing over that detail, one thing seems fairly established both here in the Southwest and in several of our eastern culture areas. It is that North America north of Mexico, before it became settled by sedentary agricultural tribes who developed many of the traits common to that type of life the world over, was settled by a generally more primitive nomadic type of peoples subsisting mainly by hunting, such as still persist over all of the northern and northwestern portions of the continent. The above summary account is based upon data from the Archer M. Huntington Survey of Southwestern United States conducted by the American Museum of Natural History. The full report upon this survey will be published by the Museum.

AN ADJUSTMENT IN RELATION TO THE FRESNEL COEFFICIENT[1]

By Carl Barus

Department of Physics, Brown University

Communicated, February 10, 1919

1. Apparatus. One internal reflection.—The specific part of the apparatus is the glass cylinder, G, figure 1, with a carefully polished mantel, capable of rotating around an axis, A, normal to the ray-plane of the interferometer.

If micrometer facilities are to be dispensed with, and that is permissible in the present experiment,[2] the interferometer may be designed as in figure 1. The white light L from the collimator takes the respective paths $dCC'd'b$ and $bd'C'Cd$, the plate N being half silvered and N' an opaque mirror. The telescope or spectro-telescope is at T. The glass face at N may be turned either way.

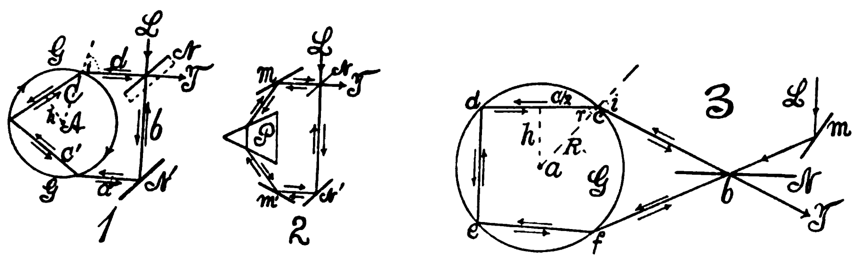
Such an interferometer is self adjusting (cf. preceding paper). In the form, figure 1, two reversed spectra will be visible in the telescope, which if superimposed by rotating N or N' on a vertical axis, will show the linear phenomenon at once, in any color at pleasure. The fringes may be enlarged by rotating N or N' on a horizontal axis and they are symmetrically equal in size on the two sides of the adjustment for infinitely large fringes.

If the achromatics are wanted, a prism must be inserted into the rays b (preferably) between N and N', with a prism angle and other conditions selected to counteract the refraction of the cylinder G.

2. Apparatus. Two internal reflections.—As the fringes were found without much difficulty (§5) in case of one internal reflection, it seemed desirable

to ascertain whether this would still be feasible in the apparently more favorable, but also more difficult case of two internal reflections. In figure 3, white light arrives from a collimator at L and strikes an auxiliary mirror m, before reaching the half silver N. If m is capable of rotating both on a horizontal and vertical axis as well as sliding right and left in the diagram, it greatly facilitates adjustments of angle and location of rays. The two beams $bcdef$ and $bfedc$ reunite at b after passing the glass cylinder G (rotating around the axis a) and are observed by the telescope at T. As the spectra (after refraction at c and f) are reflected 3 and 2 times respectively, the fringes of non-reversed spectra will be obtained covering the whole length of spectrum. A glass G of low index of refraction will here usually be preferable.

In case of a half silver mirror at a small glancing angle there are usually two pairs of bright spectrum images, and one fainter pair, apart from very faint ones. One bright and one faint pair carry identical fringes and the spectrum images may be small enough to be separated. In case of clear



glass, however, there is practically but one pair of bright images, and they carry fringes when properly superposed.

3. Equations.—The first question to be elucidated is the nature of the conditions of refraction. From the figure, in view of the symmetry of the arrangement, if b is the breadth of the ray parallelogram and R the radius of the cylinder, μ its index of refraction, h the distance of the chord C from the axis A, i and r the angles of incidence and refraction of the rays dC or $d'C'$:

$$\sin i = \sin 2r = b/2R \tag{1}$$
$$\sin r = h/R \tag{2}$$
$$\mu = 2\cos r = b/2h \tag{3}$$

The relations remain the same if $b/2r$ is constant. If the (small) value $b =$ 10 cm. is inserted into the equation, the conditions may best be shown by a graph for i and μ. It will then be seen that for diameter $2r$ between 10 and 11 cm., the available indices of refraction of the glass would increase from 1.4 to 1.7 roughly, while the angle i falls from 90° to about 65°. Hence the experiment requires the interfering rays to impinge near the outer limits of the cylinder.

It is next in order to consider the possibly observable conditions of the (apparent) ether drag. The velocity within the refracting medium of index μ is usually written (or follows from the theory[3] of relativity) in the form

$$c/\mu \pm v\,(1 - 1/\mu^2) \tag{4}$$

where v is the velocity of the medium in the direction, or contrary to the direction of the velocity of light c. It remains to determine the average speed of the beam along the chord C of figure 1. From the figure

$$C = \mu\, R \text{ and } b = 2\,\mu\, h \tag{5}$$

whence

$$b = 2\,\mu\, R\, \sqrt{1 - \mu^2/4} \tag{6}$$

In figure 1 let ω be the angular velocity of the cylinder G and dx an element of the chord C at a distance ρ from the axis A. Let the minimum distance of this chord from A be h and θ its angle with ρ.
Then

$$dx = \rho^2\, \omega\, dt/h,$$

if dx is described in the time dt. Hence

$$dx/dt = \omega\, (h^2 + x^2)/h \tag{7}$$

To find the mean speed v along C, we may multiply dx/dt by dx, integrate between 0 and $C/2$ and divide the result by $C/2$. Thus

$$v = \omega\, (h + C^2/12h) \tag{8}$$

Reducing this equation by (1), (2), (5), eventually

$$v = R\omega\, \frac{1 - \mu^2/6}{\sqrt{1 - \mu^2/4}} \tag{9}$$

or the mean speed along C may be expressed in terms of R, ω, μ, while v is naturally proportional to R and ω

The ratio of the speed in equation 9 (seeing that it is respectively + and − for the two interfering rays) to the velocity of light is thus $2v/c$. Since these rays traverse a path $2C$ in the rotating cylinder in opposite directions the path difference resulting will be

$$\Delta P' = (2v/C)\, 2C = 4Cv/c = \frac{4\mu\omega R^2}{c}\, \frac{1 - \mu^2/6}{\sqrt{1 - \mu^2/4}} \tag{10}$$

so that the path difference for a given μ and ω increases with the square of the radius, R, of the cylinder or disc.

But the equation (4) introduces another factor $(1 - 1/\mu^2)$ so that finally the path difference is

$$\Delta P = \frac{4\omega R^2}{c} \frac{\mu(1-\mu^2/6)(1-1/\mu^2)}{\sqrt{1-\mu^2/4}} \tag{11}$$

We may now take the above case (b = 10 cm.) from the graph i, μ, for a small cylinder, making 100 turns per second.

$$R = 5.3 \text{ cm}; \mu = 1.63; \omega = 628; i = 70.6°; r = 35.3°; b = 10 \text{ cm}.$$

In accordance with equation (10), therefore, the uncorrected path difference is

$$\Delta P' = 2.95 \times 10^{-6} \text{ cm}.$$

and the corrected path difference finally,

$$\Delta P = 1.84 \times 10^{-6} \text{ cm}.$$

The fringes which appear in the above interferometer are primarily those of reversed spectra. If the yellow parts of the spectra ($\lambda = 60 \times 10^{-6}$) are superposed, 0.031 of a fringe would pass for the given radius of cylinder (R = 5.3 cm.) at 100 turns. A cylinder 30 cm. in diameter (about a foot) would therefore show .28 fringe, and since this may be doubled by reversing the rotation of the cylinder, (by which strains due to centrifugal force are also eliminated) something short of $\frac{2}{3}$ of a fringe should be observed.

With an ocular micrometer divided in 1/10 millimeter, it should be possible to secure fringes as much as 3 mm. apart, so that a displacement of 20 scale parts may be expected, ten for each of the directions of rotation.

4. *Equations. Two reflections.*—The equations for this case are somewhat more involved than the preceding; but it suffices to accept for the angle of incidence i at the cylinder G, figure 3, the value given by the old-fashioned theory of the rainbow; viz.,

$$8 \cos^2 i = \mu^2 - 1 \tag{12}$$

The chord C from c to d, etc., and its distance h from the axis a will be, as before, $C = 2R \cos r$, $h = R \sin r$, where r is the angle of refraction and R the radius of the cylinder. Finally equation (8), for the average speed v along a chord, also applies. Hence with the inclusion of equation (4), the path difference on rotation may be written, c being the velocity of light,

$$3 \times 2\, C\, (v/c)\, (1 - 1/\mu^2) \tag{13}$$

since there are three chords, C, in sequence. This expression may be reduced by the equations for C, h, v, and equation (12), eventually to a form convenient for computation.

$$(9R^2\omega/c)\,(3/\mu^2 + 1)\,\sqrt{(1-1/\mu^2)/(9/\mu^2-1)}$$

Data similar to the above may now be inserted; viz., for a small cylinder of water (to be used in the experiments below)

$$R = 5 \text{ cm.}; \mu = 1.33; \omega = 628; c = 3 \times 10^{10}$$

whence the path difference 1.82×10^{-6} cm. results.

This, curiously enough, is about the same value which was obtained in case of equation (11), so that identical deductions apply. The conditions are somewhat more favorable for larger values of μ. Thus in the limiting case $\mu^2 = 3$, the path difference would be about doubled.

5. *Experiments.*—To carry out these experiments at the present time is of course out of the question; but a number of contributory observations may be made with advantage. The case of figure 2 is similar to figure 1, where the dispersion of the cylinder G in the former case is simulated by the prism P and the auxiliary mirrors m, m', of the latter. If the slit of the collimator at L is not too coarse, two reversed spectra will be seen in the telescope at T, which on being superposed by rotating m or N on a vertical axis, will show a vivid linear phenomenon in the line of symmetry of the two superposed spectra. On rotating m or N on a horizontal axis, the distance apart of the fringe dots along this line may be given any reasonable value, at pleasure. These displacements are at once referred to the definite wave length in which the linear phenomenon is put. The dispersion of the prism has no bearing on the clearness of the phenomenon: 30° and 60° prism were tested with like results.

To obtain the achromatics and increased luminosity in the spectrum fringes (now to be horizontal bands throughout from red to blue), the rays of the spectrum will have to be reassembled and that may be done by inserting a second prism say P', between L and N', in a way to counteract the effect of the first. If the achromatics are to be obtained, the glass paths of the two rays in P and P', respectively, must be coincident. Hence, the axis of the collimator at L must be inclined to accommodate the angle of minimum deviation of the identical prisms P, P'; and while N and m are parallel, N' and m' normal to each other, L and T have their axes symmetric to N. The adjustment is not difficult as they need not be perfect to secure good achromatics; but if it is not made the fringes are numerous, colored, and unsatisfactory.

The experiments, figure 2, differ from the case figure 1, because the rays are parallel in the former case and condensed to a caustic by the eccentric refraction of the cylinder in the latter. Hence with these a short range telescope with strong objective is necessary; but as has been stated, the lines of the solar spectrum nevertheless come out clearly. Experiments were therefore made by simulating the glass cylinder GG by a thin cylindrical glass shell, closed below and above and containing a solution of mercury potassic iodide with an index at pleasure between 1.5 and 1.7. It was not difficult to meet the conditions of figure 1 so far as mere refraction is concerned, and certain incidental results obtained in this work have been given elsewhere.

The active slit in this experiment is the image within the cylinder, of the slit of the collimator and the former is sufficiently fine to show the Fraunhofer lines, even when the latter is a millimeter broad, so that there is no deficiency of light.

But in relation to the detection of the interferences, the two reversed spectra, strongly divergent in their homogeneous rays, introduce certain grave difficulties. For it will appear that the spectrum issuing at d', figure 1, passes over the distance b further than the spectrum issuing at d, before they reach the telescope together. The result is that the apices of two spectra lie in different focal planes, unless the telescope T is very remote. This makes the adjustment difficult.

To obviate this annoyance a symmetrical adjustment, with an additional mirror at d, figure 1, corresponding symmetrically to N and a symmetrically placed cylinder G, is here preferable. In such a case the spectra lie in the same focal plane, and since they have undergone 2 and 3 reflections, respectively, before reaching T, the interferences of non-reversed spectra are obtained without much difficulty. In my experiments, owing to the irregularity of the glass cylinder used, the fringes were correspondingly irregular; but otherwise clear and strong, as a wide slit is admissible.

The case of two internal reflections is complicated by the occurrence of multiple images from N, figure 3, even when one side is half silvered. This is particularly the case when the cylinder G contains water, as in my first experiments; for the glancing angle at b is then but 25°. There is an advantage, however, inasmuch as N may be placed at a correspondingly large distance from G. In spite of the duplicated images, the fringes were found more easily and were less irregular than anticipated. They are liable to be reproduced usually in a different size and orientation in each of the images. They will be found in the colored edge and even in the white glare (caustic) which emanates from the cylindrical surfaces. They could be made quite large, clear and strong, moreover, although the cylinder used was (as above) an ordinary glass shade. As in case of the triangular interferometer, the fringes rotate when N is displaced parallel to itself on the micrometer screw. To control their size N is to be rotated on a horizontal axis.

[1] Advance Report from the Carnegie Publications of Washington, D. C.

[2] With regard to the symmetrical interferometer form, cf. Michelson and Morley, *Amer. J. Sci., New Haven*, 31, 1886, (377), also Zeeman, below.

[3] The insufficiency of this equation has been shown by Zeeman, *Proc., Amsterdam Acad.*, September 1914, and September 1915. But an estimate only is above in question.

PIEBALD RATS AND THE THEORY OF GENES

By W. E. Castle.

Bussey Institution, Forest Hills, Boston, Mass.

Communicated February 26, 1919

The study of heredity as an exact science dates from the rediscovery of Mendel's law in 1900. After the validity of the law had been established by abundant and conclusive evidence, the question arose, are the gametes pure. Is a character which disappears in crosses, and then reappears a generation later in 25% of the offspring, subject to contamination or modification during the process? The idea of gametic purity was at first looked on with favor. Bateson[1] although he never gave unqualified adhesion to this view, formulated it very clearly, thus. "The pure [homozygous] dominant and the pure recessive members of each generation are not merely like, but identical with the pure parents, and their descendants obtained by self-fertilization are similarly pure. If they are pure, surely the male and female elements of which they were composed must also be pure."

My own experimental studies of heredity, begun in 1902, early led me to observe characters which were unmistakably *changed* by crosses and so I have for many years advocated the view that the gametes are not pure in the sense expressed by Bateson. Moreover it was observed that characters which mendelize in crosses may, even when uncrossed, show fluctuating or graded variation in consequence of which systematic selection is able to produce very diverse races as regards a single mendelizing character, the ordinary allelomorph of which is wholly excluded from the experiment. This observation shows that characters may vary otherwise than by contamination and I was in consequence led to adopt the hypothesis that unit-characters are "inconstant" in varying degrees, but probably never perfectly constant.

This view has been repeatedly challenged, either by those who questioned the evidence cited in support of it, or by those who first substituted a different concept, 'gene,' for that of 'unit-character' and then denied that a 'gene' can vary. Dissent to the evidence for character variability has gradually disappeared as others have independently undertaken to study the visible characters of organisms as affected by crossing or systematic selection. The findings are commonly such as I have described in the case of the hooded pattern of piebald rats, which I have been studying for several years. This pattern is a simple recessive in crosses with the self pattern of wild rats, but it usually emerges from such crosses in a modified form, the amount of white in the pattern being either increased or diminished according to what stock is selected for experimental study. Even when uncrossed and bred as pure as possible, I have always found a certain amount of genetic variability to persist in a hooded race, so that selection, plus or minus is effective in changing it. The

facts as I have described them are now pretty generally accepted as correct, but two different views as to their interpretation have been suggested, both of which can not be true. These two views were outlined by Castle and Phillips[2] when the first part of the experimental data was published. We considered the evidence then in hand inconclusive as between the two interpretations and planned experiments to yield, if possible, decisive evidence for one or the other. This evidence is now complete, but before I undertake to summarize it, I wish to outline the alternative interpretations to be tested. They center about the concept of the 'gene,' to which reference has already been made. The term gene or gen was introduced by Johannsen in an attempt to simplify the ideas involved in the previously current term, unit-character. By unit-character was understood (1) any visible character of an organism which behaves as an indivisible unit in Mendelian inheritance and (2) by implication, that thing in the germ-cell which produces the visible character. Johannsen[3] pointed out that these two things were logically distinct, suggested the term gene for the hypothetical germ-cell determiner, and made it clear that it is not possible to say how many germinal determiners (genes) are involved in the production of a single visible character, but only how many are present in alternative forms (as allelomorphs). He therefore advised the entire discontinuance of the use of the term unit-character and proposed to discuss the subject of heredity exclusively in terms of genes. This is the so-called genotype theory.

Before this theory could be accepted unreservedly, it has seemed desirable to know whether all observed inheritance phenomena can be expressed satisfactorily in terms of genes, which are supposed to be to heredity what atoms are to chemistry, the ultimate, indivisible units, which constitute gametes much as atoms in combination constitute compounds. It also seemed desirable to know whether a single gene is indeed invariable like an atom (or a simple chemical compound).

Much study has in recent years been given to these questions with the result that (1) to express all heredity in terms of unvarying genes, it is necessary to suppose that besides the single gene indispensable to the production of a visible character, its gene proper, there occur also other genes whose action is subsidiary. Their action may not be indispensable to the production of a character, yet they certainly modify its visible form. These are called modifying genes. In some cases they are known to have other functions also. Thus the gene proper of one character may function also as a modifying gene for another character. But in the majority of cases the only ground for hypothecating the existence of modifying genes is the fact that characters are visibly modified.

As an alternative to the theory of modifying genes, the theory has been considered that genes may themselves be variable and if so, genes purely modifying in function might be dispensed with.

These are the alternative views that we had in mind in our experiments with the hooded character of rats. It had been established that the hooded character varied but that it gave unifactorial inheritance ratios. The question to be determined was whether the single gene plainly in evidence was or was not variable. To test the point it was necessary to make the 'residual heredity' as nearly constant as possible, whether this consisted of modifying genes or not. For the purpose of determining whether the gene proper for the hooded character had or had not varied in the course of our selection experiments, we proposed to utilize two very diverse races of hooded rats produced by many generations of selection in opposite directions. They were (1) *a plus selected race* in which the pigmented areas had been increased as much as possible by selection, and (2) *a minus selected race* in which the pigmented areas had been reduced as much as possible by selection, both having been derived at the out-

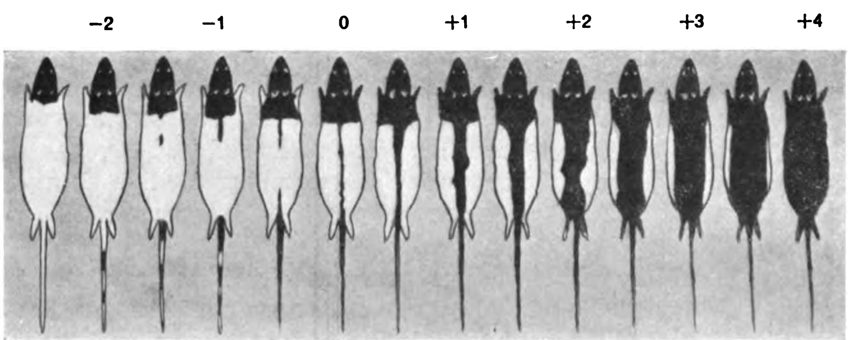


FIG. 1. A SET OF "GRADES" USED IN CLASSIFYING THE OBSERVED VARIATIONS OF THE HOODED CHARACTER OF RATS

Pictures at the extreme right and extreme left of the series show the modal conditions of the plus selected and of the minus selected races respectively.

set from a common stock. In the course of the selection they had become very different in appearance, the plus race being practically black all over as seen from above, the minus race white all over except for a black hood on head and shoulders. In classifying the young of each generation of rats an arbitrary set of grades was found useful. (See fig. 1.)

In order to compare the genetic value of the gene proper for the hooded character in one of these races with its value in the other, it was necessary first to eliminate all modifying genes or else to make them similar in the two races. To do this the plan was adopted of making repeated crosses of each race with a third race, entirely free from the hooded character, thus combining with the residual heredity of the third race the hooded character from each of the selected races. A race of wild rats was chosen as the third race and tables 1 and 2 show how crosses with this race affected the grade of the hooded character as recovered in hooded individuals in the F_2 generation.

A first cross of the plus race (table 1) lowered the grade of the hooded character from about +3.73 to +3.17. A second cross brought a slight rise in the mean grade of the extracted hooded young to +3.34, and a small group of 19 hooded young extracted from a third cross had a mean grade of +3.04. It will be observed that the hooded character was lowered not over three-fourths of a grade by three successive crosses. This fact led me to conclude provisionally in 1916[4] that the hooded gene proper had really changed in the course of our selection experiments, since after the crosses it *remained different* from what it had been originally. This view is obviously erroneous in the light of the results obtained from the minus crosses subsequently studied.

TABLE 1

RESULTS OF CROSSING THE PLUS SELECTED RACE WITH A WILD RACE

	MEAN GRADE	STANDARD DEVIATION	NUMBER OF HOODED YOUNG
Control, uncrossed plus race, generation 10	+3.73	0.36	776
Once extracted hooded F_2 young	+3.17	0.73	73
Twice extracted hooded F_2 young	+3.34	0.50	256
Thrice extracted hooded F_2 young	+3.04	0.64	19

TABLE 2

RESULTS OF CROSSING THE MINUS SELECTED RACE WITH A WILD RACE

	MEAN GRADE	STANDARD DEVIATION	NUMBER ON HOODED YOUNG
Control, uncrossed minus race, generation 16	−2.63	0.27	1,980
Once extracted hooded F_2 young	−0.38	1.25	121
Twice extracted hooded F_2 young	+1.01	0.92	49
Thrice extracted hooded F_2 young	+2.55	0.66	104

The crosses of the minus selected race were started some six generations later in the history of our selection experiments than were those with the plus selected race. They gave results much more striking than those of the plus crosses. (See table 2.) The mean grade of the minus race, when the crosses were started, was −2.63. The F_2 hooded young from a first cross with the wild race were of mean grade −.38, a change of over two grades. A second cross produced hooded F_2 young chiefly *plus* in character, mean grade +1.01. A third cross produced F_2 hooded young exclusively plus in character, mean +2.55. One family in this lot of thrice extracted hooded young, consisted of 14 hooded individuals of mean grade +3.05, almost exactly identical in mean grade with the thrice extracted hooded young of the plus series (table 1).

This result indicates that three crosses with a third race had sufficed practically to eliminate whatever differences had been produced in the minus and

plus races respectively by long continued selection in opposite directions. Those differences accordingly were based on residual heredity, not on changes in the hooded gene proper. For when the residual heredity was equalized, the hooded character appeared substantially the same in the two races. These findings harmonize with the idea that the residual heredity in question consists of several modifying genes independent of the hooded gene proper. Another point favoring that interpretation is the increased variability of the hooded character following the first cross, and its subsequent decrease following the second and third crosses. See the column, standard deviation, in tables 1 and 2.

These results favor the widely accepted view that the single gene is not subject to fluctuating variability, but is stable like a chemical compound of definite composition and changes only similarly, by definite steps (mutation in the sense of Morgan, not of DeVries). They offer no obstacles to the proposition of Johannsen (ably supported by East), that a gene terminology is adequate to express all known varieties of inheritance phenomena.

The full results of this investigation will be published by the Carnegie Institution of Washington.

[1] Bateson, W., *Report I to the Evolution Committee of the Royal Society*, 1902, p. 12.
[2] Castle, W. E., and Phillips, J. C., *Carnegie Inst. Washington, Pub.*, No. 195, 1914.
[3] Johannsen, W., *Elemente der exakten Erblichkeitslehre*, 1909.
[4] Castle, W. E., and Wright, S., *Carnegie Inst. Washington, Pub.* No. 241, 1916.

BUD VARIATION

By A. B. Stout

New York Botanical Garden, New York City

Communicated by R. A. Harper, February 26, 1919

The common experience of horticulturists and plant breeders is that propagation by buds, cuttings, layering, etc. (asexual propagation) yields a comparatively uniform progeny, while propagation by seed (sexual reproduction) and especially that which involves mating of unlike parents whether of the same or of different species or races, is likely to give decided variation among progeny. On the other hand, common experience and practice recognizes the widespread occurrence of bud variations and the importance of utilizing them in developing new types of important commercial races, or in maintaining old races at a high standard, as is well illustrated by the recent studies (Shamel and others 1918) of bud variations in the citrus fruits.

In scientific and theoretical breeding, much attention has been given to the study of heredity in sexual reproduction. In many species this is the only method that can be utilized, and a knowledge of such heredity is of great practical as well as of theoretical interest. When, however, the question arises re-

garding the constancy of characters or of assumed factors of heredity, it seems very evident that critical studies of somatic variations, especially wherever it is possible to propagate vegetatively, are of fundamental importance.

To Darwin, bud variations in plants were evidences of the very indiscriminate variability that is everywhere present in organisms. Their broad significance and range were recognized by his conclusions that they include: (1) reversions to remote ancestral characters; (2) reversions (in hybrids) to the more immediate parental qualities; and (3) cases of real spontaneous change in hereditary composition of continuous as well as of discontinuous range. Darwin did not believe in fixed hereditary units.

These same types of bud variation are, in general, recognized by de Vries (1901). He attempts, however, to assign mutational value of discontinuous rank to the spontaneous somatic variations quite as he does to seed mutations. Yet he recognizes a wide variability and irregular hereditary performance in both. For example, he describes half races, middle races and eversporting varieties in the various steps in the development of varieties from pure species involving characters frequently concerned in bud sports and ascribes the series of changes to conditions of latency, semilatency, lability or activity of hereditary units (pangens). To many critics of the mutational doctrines it is difficult to recognize such spontaneous hereditary variations as discontinuous either from the facts or explanations presented by de Vries or by other investigators. We may note further that many bud variations, such as the development of variegated branches on pure green stems were considered by de Vries as progressive mutations.

In general, Cramer's (1907) classification of bud variations follows that of de Vries for seed mutants with, however, a greater emphasis on the operation of Mendelian segregations in certain groups. He recognizes, however, quite as did Darwin and de Vries, the occurrence of a large group of bud variations in which continuous and sporadic or eversporting variability is in evidence.

The present day Mendelian studies of the seed progenies from bud variations and of characters exhibiting such variations show as a rule decidedly mixed and non-Mendelian results, or, at least, their interpretations involve subsidiary hypotheses. Some special tendencies in such interpretations may be noted: (1) the assumption that the transmission of certain characters is by the cytoplasm rather than by the nucleus; (2) the assumption that somatic variations are losses of hereditary factors accomplished by qualitative or segregative cell divisions, and (3) the claim that hereditary factors may themselves sporadically change, and the new factors come to immediate expression by dominance, or remain recessive, or exhibit various influence as modifying factors. Thus in certain attempts to analyze the heredity of seed colors in variegated corn it is assumed that factors for variegated color can change reciprocally in a series of somatic divisions and that as many as ten 'multiple allelomorphs' may be present, which, as Jennings (1917) remarks, "leap back and forth from one character to another in bewildering fashion."

A better understanding of the nature of the tissue complex in certain bud variations has been gained from the knowledge of chimeral structures. When a somewhat permanent somatic variation occurs in only a part of the cells or in a single cell of a growing point such a cell or cells may be so situated that the cell progeny form permanent layers giving periclinal chimeras, or sectors giving sectorial chimeras. There is also the possibility that irregular processes of development or the occurrence of repeated somatic variations during organogenesis may give complex mixtures or hyperchimeras. Anatomical proof of the existence of such chimeras was first presented by Baur (1909) and the experimental production or interspecific chimeras was demonstrated by Winkler (1907).

Numerous cases of albomarginate variegation are apparently of the periclinal type. The originial variation in such cases is partial, affecting a part of a growing point only. Further changes, such as return to pure green branches, may involve simply a mechanical readjustment of the elements present in the growing points (Stout 1913). Chimeral association of cells differing sporadically but more or less permanently in fundamental hereditary qualities undoubtedly accounts for much of the irregularity seen in the seed progeny of bud sports. Various types of chimeras especially periclinal and hyperchimeras, without doubt grade imperceptibly into cases of ordinary differentiation in which the cells quite alike in fundamental hereditary qualities become differentiated through their relations as parts of the whole.

In general, our available knowledge regarding somatic variations indicates that they show a wide range of variability suggesting that hereditary elements or units are themselves variable even in a series of somatic cell-divisions. The evidence is especially convincing, for here there is a most direct lineage of cell elements far more simple than that obtained in reproduction by seed progeny which involves the intricacies of periodic reduction and fertilization. Unquestionably the phenomena of bud variation involve the most fundamental questions of heredity. The intensive study of bud variants in successive generations propagated vegetatively should reveal definite facts regarding the nature, frequency, and permanence of spontaneous changes. Since 1911 the writer has studied bud variations in a variety of the variegated *Coleus* with these aims in mind. Over 1211 pedigreed plants have been grown, comprising fourteen generations (two each year), all propagated by cuttings, the first of which were taken from two similar sister plants.

The frequent variations that have appeared range from sudden changes to gradual fluctuations, and these may first become evident in a part (even a small area) or the whole of either a leaf, a bud, or a plant. The characters studied have been those of leaf form and leaf coloration.

The somatic variations found in leaf coloration involved (1) gain or loss, increase and decrease of green and yellow, (2) reversals of the relative positions of the green and yellow in leaves, (3) increase and decrease of red pigmentation, and (4) changes in the distribution of the red pigmentation, especially that

giving concentration in the epidermis of the upper surface of the leaves. Some of the variations involve very striking changes in epidermal pigmentation, superficially resembling chimeral relationships. During the first seven generations, sixteen distinct color patterns were obtained by bud variation and isolated by selection; fifteen of these were produced as marked sudden variations, and six of the types also occurred as fluctuating variations. One pattern has appeared only as a fluctuation. Between any of these types a wide range of intermediates was found from which many additional patterns might probably have been isolated. There has been reversion to parent patterns; colors that have been lost or thrown out have reappeared; yellow-green patterns have given pure green sports and later the yellow has reappeared.

A calculation of the frequency of bud variations on the basis of the estimated number of buds that developed into branches shows clearly that decrease of red occurred with about twice the frequency as did increase of red; likewise decrease of yellow occurred about twice as often as increase of yellow. Among sister lines of clonal descent there were marked differences in the ratios of frequency for any one change in coloration. The most frequent change was loss of yellow with increase of green, for which the frequency ratio was 1:2960. The change occurring with least frequency was that of increase of epidermal red pigmentation to solid red for which the ratio was 1:19,250.

Variations in leaf form were fully as striking as those of pigmentation. Deeply laciniate-leaved forms arose in thirteen instances as fluctuations affecting an entire plant, and in one case as a decided bud variation. A striking feature of the laciniate character was the marked periodicity in its development; plants of the variety having strongly laciniate leaves in winter produced entire leaves in summer, while the great bulk of the sister plants of other varieties produced only entire leaves. In general these variations in form are continuous and the extremes are in decided contrast with each other.

Selection for extremes and for intermediates has in every case given a progeny of marked constancy but with further fluctuations and sporadic variations about a new mode. The types thus arising are for purposes of propagation the equivalents of the 'Kleinarten' or 'biotypes' that commonly occur in species propagated by seed. In their bearing on the theories of continuous variation and the effects of selection, my results are quite identical with those obtained by Castle and Phillips (1914) in their study of color patterns in biparental reproduction in rats and interpreted as indicative of actual variation in the hereditary units. The results are also quite identical with those Jennings (1916) has obtained with *Difflugia*, by asexual propagation analogous to that I have used in *Coleus*.

It is quite clear that the changes seen in *Coleus* do not involve a permanent loss of definite hereditary units by vegetative segregation. There is also no opportunity for any such recombination of multiple modifying factors as is assumed to give similar variations in sexually reproduced progeny. As far as is known vegetative propagation gives the greatest possible degree of purity

in cell lineage uncontaminated by the recombinations involved in the reduction and fusion accompanying sexual reproduction. The evidence is hence very conclusive that the hereditary complex and that individual units of the complex are subject to variations that become manifest either as sudden mutations or as fluctuating variations, and that any of these may perpetuate themselves.

The facts suggest strongly that the possibilities for the development of red, green and yellow may be present in all cells as metidentical characters (in the sense used by Detto, 1907). The total production, the distribution, and the concentration of the various chemical substances concerned, however, plainly involve interactions between different cells or groups of cells and are in this sense epigenetic. The explanation suggested by the production of patterns in colloids by the Liesegang precipitation phenomena, especially as applied by Gebhardt (1912) to the markings of butterfly wings and by Kuster (1912, 1917) to the development of many types of variegation in plants including *Coleus*, seems to apply to the production of color patterns in *Coleus*. On this view colored patterns may be considered as due to the formation of localized centers for the development, diffusion and concentration of pigments.

In *Coleus* both the (1) fundamental qualities (metidentical) and the (2) processes of cellular and tissue interaction immediately involved in the development of patterns exhibit spontaneous changes that are continuous in degree and are quite constant from the first or can be made so by selection.

Colored illustrations of the principal color patterns obtained, together with the presentation of data (for the first four years of the study) have already been presented (Stout 1915). With the facts and conclusions there given, the results since obtained are fully in agreement.

Baur, E., *Zs. Ind. Abs. Vererbs.*, **1**, 1909, (330–351).

Castle, W. E., and Phillips, J. C., *Piebald rats and selection*, 1914.

Cramer, P. J. S., *Kritische Uebersicht der bekannten Falle von Knospenvariation*, 1907.

Detto, C., *Biol. Centralb.*, **27**, 1907, (80–95, 106–112, 142–160, 162–173).

Gebhardt, F. A. M. W., *Verh. Deuts. Zool. Ges.*, **22**, 1912, (179–204).

Jennings, H. S., *Genetics*, **1**, 1916, (407–534); *Amer. Nat.*, *Lancaster*, **51**, 1917, (301–306).

Küster, E., *Beiträge zur entwicklungs mechanischen Anatomie der Pflanzen*, 1913; *Flora*, **10**, 1917, (1–33).

Shamel, A. D., Scott, L. B., and Pomeroy, C. S., *Washington U.S. Dept. Agric. Bull.*, 1918, Nos. 623, 624, 697.

Stout, A. B., *Bull. Torrey Bot. Club*, **40**, 1913, (367–372); *Carnegie Inst.*, *Washington*, 1915, *Pub.* No. 218.

de Vries, H., *Die Mutationstheorie*, 1901.

Winkler, H., *Ber. Deuts. Bot. Ges.*, **25**, 1907, (568–576).

THE GROWTH RATE OF AN ANNUAL PLANT HELIANTHUS*

By H. S. Reed and R. H. Holland

Graduate School of Tropical Agriculture and Citrus Experiment Station, University of California

Communicated by R. Pearl, February 28, 1919

If we assume that growth is a dynamic process and that the organism is produced as its end-product, certain relations ought to exist between the size of the organism at any given time and the final size attained in time, T.

Growth begins at a slow rate, but as the reaction proceeds it goes on at an increasingly rapid rate until it reaches a maximum velocity, then the rate decreases until the reaction comes to a stop. This is precisely what happens in autocatalytic processes in which the reaction is catalyzed by one of its own products. It therefore becomes of interest to inquire whether the growth rate of an organism, or group of organisms, approximates the rate of autocatalysis. As will be shown in this article, the equation of autocatalysis expresses admirably the growth rate of plants studied.

Growth may be considered as a function of two variables. The first of these is the genetic constitution of the individual. The second is the resultant of all those factors that make up what is commonly called the environment of the organism. The factors of the first group are essentially *internal*; those of the second group, essentially *external*. In analyzing the growth process it is of interest to separate so far as possible the results of these two classes of factors. If the growth rate follows approximately the course of an autocatalytic reaction, it is safe to assume that it is controlled by some internal factor resident in the organism. If it departs from the theoretical course more widely and uniformly than might be expected upon the basis of pure chance, we may believe that some other, presumably external, factor is of sufficient weight to control or, at least, influence the growth rate.

The studies embodied in the present paper are based on measurements of a group of fifty-eight sunflowers, grown for the purpose on the grounds of the Citrus Experiment Station, Riverside, California. They were grown on a small piece of tolerably uniform soil to which water sufficient to maintain satisfactory soil-moisture conditions was applied every seven days. The plants grew from the middle of May to the middle of August during a time when heat and light were ample for plant growth. As soon as the plants had reached an average height of more than 10 centimeters, sixty of the normal, appearing plants were selected at random throughout the small plot and marked with suitable labels. (During the course of the observations, two plants had to be eliminated on account of accidents). Each plant was marked with india ink at a distance of 10 centimeters below the growing tip. This mark served as a point from which further measurements were made. The

sunflower was chosen for this work because of the fact that it grows without producing branches and it was thought that measurements of growth and weight represented the growth of the entire organism with a fair degree of accuracy. The adjacent plants were removed from the vicinity of those selected so that there was space of about 20 centimeters between any plant and its nearest neighbor. In short, environmental conditions were made as nearly uniform for the individuals in this small group as it was feasible to make them under field conditions. In the latter part of July the terminal buds began to develop into blossoms and coincidentally the plants ceased to elongate.

The plants used in these studies were evidently of mixed ancestry as shown by the presence of branched and unbranched individuals. The seed had been bought at a seedstore and nothing was known of its pedigree. The branching habit is regarded by Shull (1908) to be a Mendelian character.

TABLE 1

CONSTANTS FOR GROWTH AND VARIATION IN HEIGHT OF HELIANTHUS PLANTS

DAYS	MEAN HEIGHT	INCREASE OF MEAN HEIGHT	STANDARD DEVIATION	COEFFICIENT OF VARIABILITY
	cm.	cm.		
7	17.93 ± 0.14	7.93 ± 0.14	1.62 ± 1.01	9.03 ± 0.56
14	36.36 ± 0.43	18.43 ± 0.43	4.83 ± 0.30	13.28 ± 0.85
21	67.76 ± 0.78	31.40 ± 0.89	8.93 ± 0.56	13.17 ± 0.84
28	98.10 ± 1.38	30.34 ± 1.59	15.60 ± 0.98	15.90 ± 1.02
35	131.00 ± 1.73	32.90 ± 2.21	19.52 ± 1.22	14.90 ± 0.95
42	169.50 ± 2.21	38.50 ± 2.81	25.00 ± 1.56	14.75 ± 0.94
49	205.50 ± 2.92	36.00 ± 3.66	33.00 ± 2.07	16.06 ± 1.03
56	228.30 ± 3.41	22.80 ± 4.49	38.47 ± 2.41	16.84 ± 1.08
63	247.10 ± 3.80	18.80 ± 5.10	42.92 ± 2.69	17.38 ± 1.12
70	250.50 ± 3.76	3.40 ± 5.35	42.48 ± 2.66	16.95 ± 1.09
77	253.80 ± 3.99	3.30 ± 5.48	45.06 ± 2.82	17.75 ± 1.13
84	254.50 ± 3.89	0.70 ± 5.57	43.90 ± 2.75	17.25 ± 1.11

One important difference should be noted between the plants described by Shull and those in our series, viz: Shull's plants branched from the lower nodes of the stalk, while ours branched only from the upper nodes. Church (1915) regards the branched form as a mutant of the unbranched and believes that it is the oldest mutation on record.

The branched form usually produces a head on the apex of each branch, whereas the unbranched form produces one head from the apical bud of the stem and no other.

This mixture of branched and unbranched stems is not thought to affect the validity of the measurements upon which the present study is based, since only seventeen out of the fifty-eight plants were branched. The average heights of the two classes at maturity were close enough together to be within the range of the probable error, though the length of the average growing season of the branched plants was 4.6 days longer than that of the

unbranched plants. The number of heads produced by the branched plants ranged from three to thirteen.

Rate and Extent of Growth.—Fifty-eight plants were measured at seven-day intervals, from the time when each plant was marked 10 centimeters below the growing tip until no further elongation occurred. On the eighty-fourth day, when the last measurement was taken, the plants averaged 254.5 centimeters high to the upper side of the head, with a range from 164 centimeters to 339 centimeters. The mean growing period in days was 69.79 ± 2.17. The mean height of the plants is shown by figures in table 1, along with the standard deviation and coefficients of variability. The mean height of plants at seven-day intervals is shown graphically in figure 1.

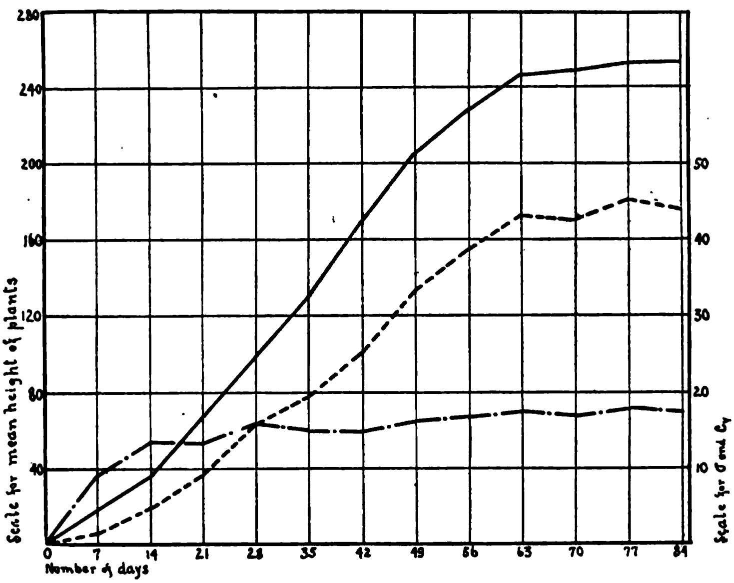


FIG. 1. GROWTH AND VARIABILITY OF HELIANTHUS

Mean height of plants ————————
Standard deviation — — — — — — — — —
Coefficient of variability ——-·—— - —— -

The data show that the plants rapidly increased in height, the maximum growth rate being exhibited between the thirty-fifth and the forty-second days, i.e., about the middle of the grand period of growth. The growth rate was smaller at the start, rapidly increasing until it reached its maximum and then declining as it approached the end of the grand period of growth.

The standard deviations of the mean values increase as the means increase but not at a proportional rate. In this case the coefficient of variability is a better measure of the variability since its size is more nearly independent of the height of the plants. This coefficient does not increase during the latter

part of the growth period. It increased very rapidly at first, but remained fairly constant after the 28th day. The variability thus appeared to reach a constant value which changed but slightly as the plants approached maturity.

The decline in the growth rate of a plant began to appear as the flower bud on the apex of the stalk began to be differentiated. As the 'head' developed, the growth of the stalk became slower, showing agreement with the condition accompanying tassel formation which Pearl and Surface (1915) found in maize. After the flowers of the composite 'head' had been pollinated there was no fur-

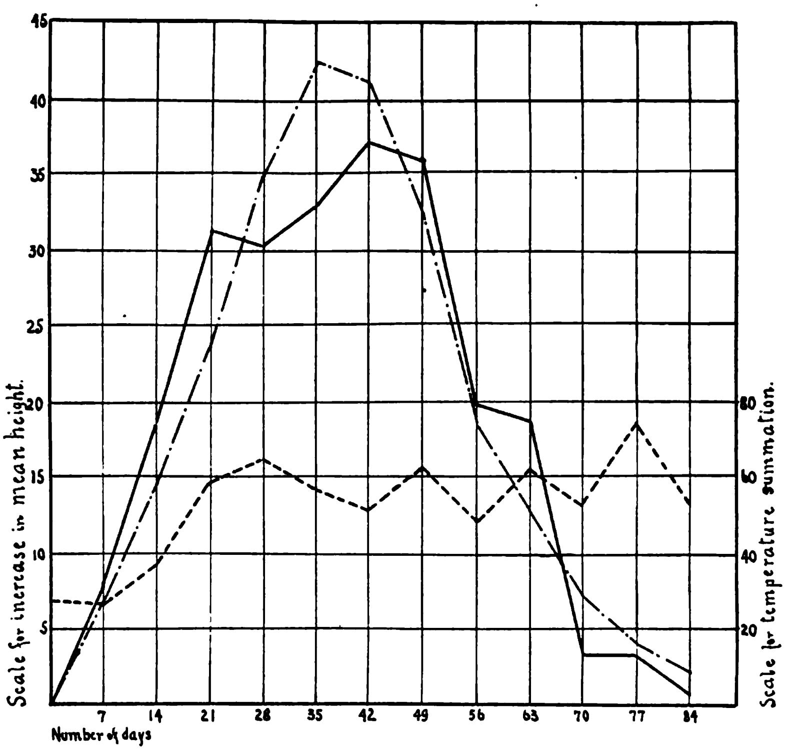


FIG. 2. INCREASES IN MEAN HEIGHT OF HELIANTHUS AT 7 DAY INTERVALS

Observed mean height ————————
Theoretical mean height —— - —— - —— -
Temperature summations — — — — — — — — —

ther elongation of the stalk. It is evident that from this time on the growth forces of the plant are devoted to seed-formation instead of stalk-elongation. Thus, variability in the time of blossoming may, and undoubtedly does, influence the grand period of growth and the total growth of this plant.

When the flowers have been pollinated the head which previously stood erect becomes pendant. The floral surface which is uppermost during the prepollination period, is lowermost in the post-pollination period.

An examination of the consecutive increases in mean height of the plants may help to give a clearer picture of the distribution of the growth increments of these plants. The mean growth increases observed at seven-day intervals have been plotted out in figure 2. The increases, starting from the day on which the plants were marked, show a general trend upward for the first forty-two days and a decline for the following forty-two days. Inspection of the graph shows that the line does not rise and fall smoothly, there being several abrupt changes. The mean height increased rapidly from the beginning to the twenty-first day. The rate fell off somewhat to the twenty-eighth day, then increased gradually until the maximum was reached on the forty-second day. From the forty-second to the forty-ninth day the rate fell off slightly and then declined abruptly to the fifty-sixth day. From the fifty-sixth to the sixty-third day there was only a slight decline in the rate, but from the sixty-third to the seventieth day there was a rapid decline followed by a halt until the seventy-seventh day, and then a descent to a point near the eighty-fourth day where growth ceased entirely.

The Correspondence Between Growth and Autocatalysis.—It may next be in order to inquire concerning the nature and action of some of the internal factors which influenced the growth of these plants. Studies on the growth of animals made by Robertson (1908, 1915) and of bacterial activity made by Miyake (1916) have shown the similarity of these processes to that of autocatalysis. In autocatalysis one of the products of the reaction catalyzes the reaction. Such reactions begin slowly, but as more of the catalyzing substance is produced the reaction goes on at an increasingly rapid rate. As the supply of reacting substances is used up, the reaction begins to slow down and comes eventually to a stop.

Brief mention will be made here to the formula used to express the course of an autocatalytic reaction. The reader who wishes more complete mathematical discussion should consult papers of Robertson (1915) and Miyake (1916). An autocatalytic reaction may be expressed by the differential equation

$$\frac{dx}{dt} = Kx\,(A - x),$$

in which A is the initial quantity of material subject to transformation, x is the amount transformed at time t, and K is a constant. The integral form of this equation is

$$\log \frac{x}{A-x} = K\,(t - t_1),$$

in which t_1 is the time at which the reaction has run half way to equilibrum; that is, the time at which $x = A/2$.

Translating these functions into terms of growth, we let A represent the final mass of the plant; x, the size of the plant at any time, t; t_1, the time at which

the mass of the plant is half the final mass, etc. Obviously it would be better to weigh the sunflower plants, but as this would require severance between the plant and the soil, it would not have been practicable to use the same plants for subsequent measurements. In the case of a straight unbranched stalk (such as most of these sunflowers were), it seems sufficiently accurate to use the height of the plant as an index of the amount of growth, at least up to the time of flowering.

In the case of these sunflowers, A is 254.5 cm., t_1 is 34.2 days, then

$$\log \frac{x}{254.5 - x} = K\ (t - 34.2).$$

Substitution of the various values of x and corresponding values of t gives the corresponding values of K which are shown in the third column of table 2.

TABLE 2

CONSTANTS FOR THE MEAN HEIGHT OF SUNFLOWERS AT SUCCESSIVE INTERVALS

t	X (OBSERVED)	K	X (CALCULATED)	θ
days	*cm.*		*cm.*	*cm.*
7	17.93	0.04128	17.05	−0.88
14	36.36	0.03851	31.43	−4.93
21	67.76	0.03341	55.35	−12.41
28	98.10	0.03274	90.09	−8.01
35	131.00	0.03250	132.21	+1.21
42	169.00	0.03794	173.06	+4.06
49	205.50	0.04196	205.64	+0.14
56	228.30	0.04312	227.01	−1.29
63	247.10	0.05295	239.74	−7.36
70	250.50	0.04997	246.87	−3.63
77	253.80	0.05892	250.56	−3.24
84	254.50	—	252.46	−2.04

The average value of K determined in this way is .0421. Using this value of K we proceed to find the values of $K\ (t - t_1)$, and from these, with the assistance of Robertson's tables, a series of calculated values of x were obtained. These were the theoretical heights of the plants at the successive intervals provided the orginial equation was a correct expression of the growth rate. The divergence, θ, between the observed and the theoretical values is shown for each interval in the last column of the table. On the whole, the correspondence between the observed and the theoretical values is very satisfactory. The observed and calculated heights of the plants are shown graphically in figure 3.

More accurate comparison of these values was made by testing the goodness of fit of the theoretical to the observed curve. Employing the method given by Elderton (1902), it was found that P = .9256, which is taken to indicate a satisfactory fit, since in approximately ninety-two cases out of one

hundred, a random sampling would give values diverging more widely from the theoretical than those actually found.

Since the observed values agree so well with the theoretical values, it seems safe to assume that the growth rate is governed by constant internal forces rather than by external forces which would be expected to be more casual in operation.

The theoretical values for the consecutive increases in the mean height of the plants give a smoother curve than the observed values give, as shown in figure 2. The sag in the observed curve near the twenty-eighth day does not appear in the theoretical curve. The summit of the theoretical curve is near

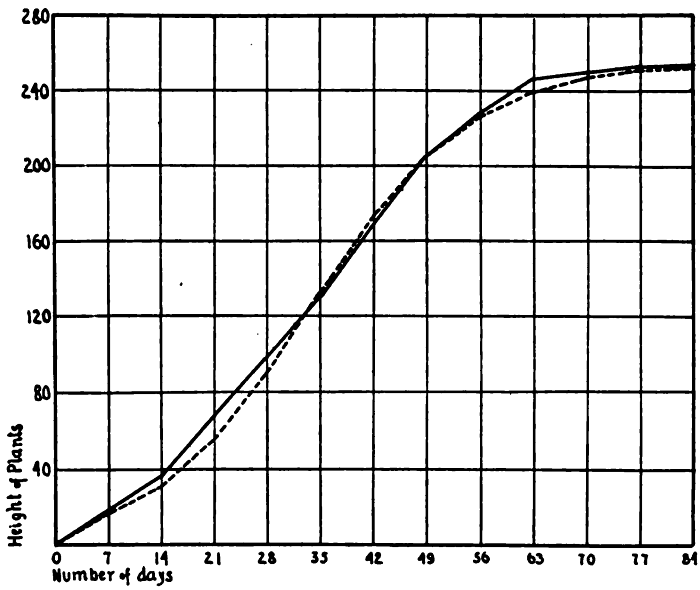


FIG. 3. COMPARISON OF OBSERVED AND CALCULATED VALUES FOR THE MEAN HEIGHT OF HELIANTHUS

Observed ————————

Calculated — — — — — — — —

the thirty-fifth day, thus agreeing with the computed value of t_1 = 34.2 days, the time at which half the final height is attained and at which growth is most rapid.

The assumption having been made that the growth was more largely governed by internal than external factors, and positive evidence in favor of the assumption having been obtained, it is next in order to investigate the relationship between growth and some of the more prominent factors of the external environment.

Temperature is known to have a potent effect upon growth, especially if it departs widely from the optimum requirements of the organism. In a prob-

lem like the present, we are more concerned with the temperature summation than with the mean temperature unless the range is very large.

It is clear that we shall not arrive at a correct value if we take an arithmetical average of the maximum and minimum daily temperatures, because we do not in that way take any account of the time during which either prevailed, or of the range of temperatures. For example, the minimum temperature on a given day may be 50° and the maximum temperature 90°, an average of 70° but if the maximum temperature prevails for only two hours out of the twenty-four, while the temperature varies between 50° and 65° for most of the daily period, it is obvious that the mere arithmetical mean, 70°, is a false expression of the temperature. The values must be weighted in order to give an average which correctly represents the temperature condition.

A method of measuring temperature summations has been employed which is believed to be fairly satisfactory. It consisted in finding the product of hours multiplied by degree of temperature above 40°F. and is expressed in degree-hours. A degree-hour may be regarded as one degree of effective temperature acting for one hour. The point 40°F. was arbitrarily chosen as a basal point, at or near which plant growth will proceed. The method of obtaining the summation of effective temperature consisted in measuring with a planimeter the area between the pen tracing and the 40°F. line on thermograph records obtained from a self registering thermograph situated about 100 yards from the plantation of sunflowers. This method gives a direct index of temperatures above the 40° point, but does not take into account the efficiency of temperatures as assumed by the van't Hoff-Arrhenius principle.

The coefficient of correlation between the degree-hours and the increase in height of the sunflower plants for each seven-day interval was calculated. Its value turned out to be $r = 0.199 \pm 0.187$. There are some indications here of a positive correlation, but, since the probable error nearly equals the coefficient in magnitude, no reliance can be placed upon the existence of a correlation.

Reference to the graph showing temperature summations in figure 2, shows little correspondence with the curve representing growth increases, except in the first twenty-one days of the period.

In a somewhat similar way we have investigated the possibility of a correlation between growth rate and the coefficient of the evaporating power of the air, the latter value being obtained from the readings of a spherical porous-clay atmometer-bulb located about one hundred yards from the plants. The coefficient of correlation for these values was even less than that in the foregoing case, being 0.041 ± 0.202. The coefficient in itself is so small as to lack significance, and when compared with its probable error it fails entirely to indicate any correlation between these two factors.

These statements are not to be construed as arguments against the effect of temperature and transpiration upon the rate of growth of plants. Our argument is merely intended to emphasize the greater importance of the inter-

nal factors in determining the growth, reproduction and senescence of the plant, factors which are so potent that they overbalance external factors so long as the latter do not too closely approach minimum or maximum values.

That the oncoming of reproductive processes induces changes in the growth rate of the organism is a fact too well known to require comment, but is well illustrated by the behavior of the sunflower.

It appears that growth, its rate, its grand period, and, to some extent, its amount are so steadily controlled by factors inherent in the genetic constitution of the sunflower that these factors are prepotent unless the external conditions depart widely or repeatedly from the optimum. Plants in this respect are more sensitive to variations in their external environment than animals, yet these studies show that even plants are not entirely dependent upon environmental (external) conditions for determining their growth rate.

It may be of interest to inquire whether the formula of autocatalysis applies, as well to the smaller plants as to the medium and large plants of this group and how the mean values of K for different groups agree.

TABLE 3

HEIGHT AND GROWTH CONSTANTS OF PLANTS ENDING IN THEIR GROWTH IN DIFFERENT QUARTILES

	QUARTILE			
	I	II	III	IV
Final height of plants.....	198 cm.	238 cm.	272 cm.	312 cm.
Mean value of K.........	0.0440 ± 0.0011	0.0421 ± 0.0016	0.0429 ± 0.0017	0.0443 ± 0.0023
Standard deviation of mean value of K.......	0.0052 ± 0.0008	0.0079 ± 0.0011	0.0079 ± 0.0012	0.0111 ± 0.0016

As a basis of classification we divided the plants into quartiles, based upon their heights at maturity. Quartile I, contained the smallest plants, quartile II, the next larger and so on. Since 58 is not exactly divisible by 4, the quartiles were not exactly of equal size: quartiles I and III contained 15 plants each and quartiles II and IV contained 14 each. An average of the heights of each group of plants at each time interval, t, gave a corresponding value of x, from which the several values of K were computed. (See table 3).

The mean values of K are remarkably constant for the different quartiles, in fact all are within the range of their probable errors. This may be regarded as evidence that the growth constant has the same value for all classes of plants in this population without regard to their relative heights, since the relation between the final height and the height at any given time obeys the same principle.

A brief consideration of a parallel case will make it evident that such a relation must exist in this sort of reaction. The inversion of cane sugar is a familiar case of autocatalysis in which one of the products catalyzes the reaction.

It is plain that the amounts of invert sugar finally formed in four different flasks may vary, depending upon the amount of cane sugar originally present, yet the constant of autocatalysis remains the same and the time may be the same in each case. This relation exists because the quantity dx/dt is proportional to the quantity of cane sugar remaining, which may be represented by $(a - x)$ where a is the original quantity of cane sugar at the beginning, i.e., when $t = 0$.

The senior author has discussed the distribution of these plants in the several quartiles in a paper soon to be published (Reed, 1919), showing that said distribution is due to some agency operating to cause variability in height other than purely casual agencies which might be expected upon the basis of pure chance. The data presented in table 3, indicate that no difference in the growth constants exists which can account for the larger or smaller size of a part of the population. Whether the differences in the amount of material produced, i.e., the size of the plants, is to be referred to differences in amount or activity of the catalyst, or of the substrate, cannot be discussed upon the basis of the data now in hand.

An additional point may be discussed in this connection, viz., "Are the values of the growth constants more widely dispersed from their means in one quartile than in another?" We may take the standard deviations of the means as a measure of the dispersion of the individual values. Reference to table 3 shows that the standard deviations increase from the lower to the upper quartiles, which would seem to indicate that the growth rate of the larger plants fluctuates more from the mean than is the case in the smaller plants.

Summary.—1. Measurements of sunflower plants at intervals of seven days showed that their growth rate approximated closely the course of an autocatalytic reaction.

2. The close correspondence of the actual mean height of plants to that required by the equation of autocatalysis is taken to indicate that the growth rate is governed by constant internal factors rather than by external factors which would be expected to be more casual in their influence.

3. The growth rate showed no strong correlation either with temperature-summations, or with transpiration-summations.

4. The value of the growth constant was not perceptibly different for the larger or smaller plants in this population.

*Paper 56 from the University of California, Graduate School of Tropical Agriculture and Citrus Experiment Station, Riverside, California.

Church, A. H., *quoted* in Cockerell, T. D. A., *Amer. Nat., Lancaster*, **49**, 1915, (609).

Elderton, W. P., *Biometrika, London*, **1**, 1902, (153–163).

Miyake, K., *Soil Science*, **2**, 1916, (481–492).

Pearl, R., and Surface, F. M., *Zs. indukt. Abstam.- Vererbungslehre*, **14**, 1915, (97–203).

Reed, H. S., (1919), *Amer. J. Bot.*, (in press).

Roberston, T. B., *Arch. Entwicklungsmechanik*, **26**, 1908, (108–118); *Ibid.*, **37**, 1913, (497); *Berkeley, Univ. Cal. Publ. in Physiology*, **4**, 1915, (211–288).

Shull, G. H., *Bot. Gaz., Chicago*, **45**, 1908, (103–116).

ON SOME METALLIC DERIVATIVES OF ETHYL THIOGLYCOLLATE[1]

By Charles A. Rouiller

Pharmacological Laboratory, Johns Hopkins University

Communicated by J. J. Abel, March 10, 1919

In 1910 Abel[2] discovered that thioglycollic ester dissolves antimony trioxide with the greatest ease, forming an antimony derivative, $Sb(SCH_2CO_2C_2H_5)_3$, according to the equation: $2Sb_2O_3+6HSCH_2CO_2C_2H_5=2Sb(SCH_2CO_2C_2H_5)_3+3H_2O$. The antimony compound separates as a heavy oil which, when treated in absolute alcohol with ammonia, yields the corresponding amide, $Sb(SCH_2CONH_2)_3$, obtained by precipitation from alcohol with ether as a colorless or slightly reddish, semiresinous mass soluble in water in all proportions with neutral reaction. The experiments of Rowntree, carried out in collaboration with Abel, showed that the new amide is a very powerful trypanosomicidal substance.

Professor Abel found that the thioglycollic ester reacts energetically with mercuric oxide also, and in order to determine whether the reaction discovered by him is of general applicability, he suggested to the writer that he try the action of various other metallic oxides on the ester. It was hoped that the resulting products might be so slightly soluble as not to be toxic when applied on open wound surfaces, but yet soluble enough to be antiseptic and bactericidal. Abel's expectation that his reaction would prove to be general has been confirmed; whether the products formed are of pharmacological and therapeutic value we have not yet had an opportunity to determine.

Following Abel's general method (for details see the forthcoming paper in the Journal of the American Chemical Society), the compounds listed below have been prepared and analyzed.

Triethyl bismuthtrithioglycollate, $Bi(SCH_2CO_2C_2H_5)_3$

Diethyl mercurydithioglycollate, $Hg(SCH_2CO_2C_2H_5)_2$

Ethyl silverthioglycollate, $AgSCH_2CO_2C_2H_5$.

A copper compound with 9.3 % of copper for which no simple formula can be derived; the normal compound, $Cu(SCH_2CO_2C_2H_5)_2$, would contain 21.06% of copper.

Zinc, arsenic, and tin compounds were also prepared but have not yet been analyzed.

[1] A more detailed report of this investigation will appear in the May issue of the Journal of the American Chemical Society.

[2] Rowntree, L. G., and Abel, J. J., *J. Pharmacology Exper. Therapeutics*, 2, 1910, (108).

AN ANALYSIS OF THE RELATION BETWEEN THE TEMPERATURE AND THE DURATION OF A BIOLOGICAL PROCESS

By Selig Hecht

Department of Biochemistry and Physiology, College of Medicine, Creighton University, Omaha

Communicated by J. Loeb, February 17, 1919

1. Biological processes exhibit variations in rate at different temperatures. The speed of a given activity increases with an increase in the temperature, usually in the form of an exponential function, occasionally in a linear manner. At higher temperatures, however, the rate of increase falls off. The velocity of the process reaches a maximum at a critical temperature, above which the activity declines rapidly or ceases entirely.

Such vital processes are undoubtedly conditioned by chemical and physical reactions. It should therefore be possible to analyze these variations in terms of the changes usually associated with the relation between chemical and physical reactions and the temperatures at which they occur. This has been difficult to accomplish except in the most general terms. The variations at higher temperatures are attributed to the destructive effect of heat on protoplasm, etc., but no really quantitative analysis of such variations is attempted. It has, however, been possible to accomplish this in a study of the photic sensitivity of *Mya arenaria.*

2. The mollusc *Mya* responds to illumination by a rapid retraction of its siphons. Its reaction time is composed of two parts. The first is a sensitization period during which the animal must be exposed to light in order to respond at the end of the usual reaction time. The second is a latent period during which *Mya* may remain in the dark. At the end of this period the organism responds as if it had been exposed to light for the entire reaction time. The composition of the reaction time is strikingly clear, because the sensitization period is very short,—less than 0.10 second at moderate intensities. The latent period, however, is well above 1.0 second at room temperature.

The sensitization period is conditioned by the velocity of a photochemical reaction. The more intense the stimulating light, the shorter is the time necessary to expose Mya in order to produce a response.

The latent period, however, depends on a reaction, the velocity of which is determined by the amount of substance formed during the exposure to light. The reciprocal of the latent period measures the velocity of its determining reaction. The relation between the velocity and the exposure time is strictly linear for exposures shorter than the sensitization period. Since it is generally true that the velocity of a catalyzed reaction is a linear function of the concentration of catalyst, the latent period is very probably the result of a

chemical reaction which is catalyzed by the substance formed during the exposure to light.

This intimate connection between the two phases of the reaction time makes it possible to investigate the relation between the temperature and the latent period, free from the usual 'rate of heating' and 'time factor' difficulties. By bringing the animal to the desired temperature in the dark, errors resulting from the change of the system during the preliminary and subsequent heatings are largely eliminated. This is because the products of the reaction concerned with the latent period occur only as an immediate result of the exposure to light. They can, therefore, be influenced only after such exposure. The changes which then occur are exactly the ones which it is desired to measure.

The results obtained by determining the latent period at different temperatures, at a constant intensity, will be found in detail in a future number of the Journal of General Physiology. They are, in general, similar to the many that have already been published, with the exception that they are amenable to a clear analysis. With this we shall now be concerned.

3. The relation between the velocity constants (K_1 and K_0) of a chemical reaction at different temperatures (T_1 and T_0), is given by Arrhenius in the following equation:

$$\frac{K_1}{K_0} = e^{\frac{\mu}{2}\left(\frac{T_1-T_0}{T_1T_0}\right)}.$$

The temperatures are absolute, e is the Naperian base, and μ is a constant characteristic of a given chemical reaction. Since only the ratio between the velocity constants is required, there may be substituted in their places the reciprocals of the time required to accomplish a given amount of work.

Using the reciprocals of the latent period at different temperatures in this manner, μ shows a constant value of 19,680 from 13° to 21°C. Between these temperatures the latent period, therefore, varies as if it were conditioned by a single chemical reaction. Above 21°, however, the value of μ decreases steadily until at 31° it is 11,210. This regular decline in the value of μ is equivalent to saying that above 21° the latent period becomes longer than it would be if the process as a whole showed a constant value of μ. At higher temperatures, therefore, the operation of a secondary factor becomes evident. The effect of this is to increase the duration of the reaction concerned fundamentally with the latent period.

4. The steady decrease in the value of μ with rising temperature indicates an increasing effect of the second factor. The results of its operation below 21° are probably not measurable. It should be pointed out that 21°C. is the mean water temperature of the hottest month at Woods Hole.

If we conceive the substance formed during the latent period to be thermolabile, and also that a definite quantity of it is required to cause a response of the animal, the presence of this secondary, modifying factor may be quantitatively determined. At higher temperatures the thermolabile substance is

rendered inactive in appreciable amounts. Consequently the reaction producing the thermolabile substance must proceed longer in order to make up the amount of it required for a response. The interval of time in which all this occurs is very small at temperatures above 21°,—less than one second. The portion of the reaction isotherm which is involved may therefore be considered a straight line. The amount of thermolabile substance formed will thus be a linear function of the time.

By using the original, constant value of μ (=19,680), it is possible to calculate the duration of the latent period at higher temperatures, free from the influence of the additional variable. The differences between the calculated time and the observed time serve as a measure of the effect of the second factor. The difference in time divided by the calculated time gives the per cent of the theoretically necessary amount of thermolabile substance which is inactivated at any temperature.

It has been repeatedly shown that the course of inactivation reactions and of spontaneous decompositions agrees well with that of a reaction of the first order. Therefore, from the above results, isotherms may be constructed showing the partial course of the inactivating reaction here concerned, at different temperatures. It is thus possible to find the time required to inactivate a given amount of thermolabile substance at the various temperatures. The reciprocals of the time may now be substituted in the above equation of Arrhenius, and the value of μ determined.

When these steps are followed, the inactivating reaction shows a constant value of μ = 66,800. High values for μ, of the same order of magnitude as here given, obtain generally for spontaneous destructions, inactivations, and coagulations. The fact that the value of μ corresponds to those found for similar processes strengthens the force of the reasoning which underlies the analysis here presented.

It is, therefore, permissible to conclude that the duration of the latent period in the photic response of *Mya*, at the temperatures studied, is conditioned by two chemical reactions. The first is the fundamental reaction which determines the nature of the latent period. The second is a reaction which inactivates the principal product of the fundamental reaction, thereby prolonging the latent period at higher temperatures.

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 MAY 15, 1919 Number 5

SUPERPOSED OR DUPLICATED SPECTRUM FRINGES*

By Carl Barus

Department of Physics, Brown University

Communicated, February 17, 1919

1. *Introductory.*—To obtain sharp spectrum fringes it is necessary, as a rule, to use a slit narrow enough to show the Fraunhofer lines. Hence there is sometimes a deficiency of light from this reason alone. It occurred to me on producing identical fringes of inclination (achromatics or monochromatics) and of color (dispersion), that by their superposition a slit of any width (or an entire absence of slit) would be admissible, without destroying the fringes in the impure spectrum resulting.

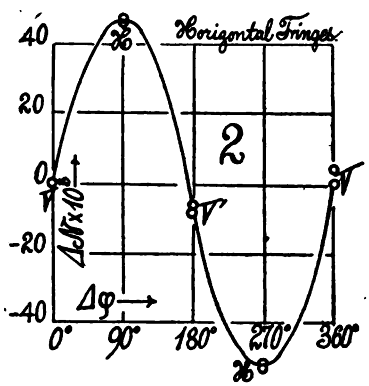
Furthermore if the edge of the prism is rotated around the axis of the spectro-telescope 180°, the inclination of all spectrum fringes must be symmetrically reversed; i.e., inclination up toward the right (positive) will become inclination down on the right (negative) to the same amount. The identical result may also be reached independently by displacing one of the mirrors of the interferometer parallel to itself (path difference) until the fringes passing through their maximum size reach the opposed inclination and size. Hence there must be a relation of a periodic kind between the displacement of mirror (ΔN) and rotation of the spectro-telescope ($\Delta\varphi$), by which sharpness of fringes in the absence of a slit is conditioned.

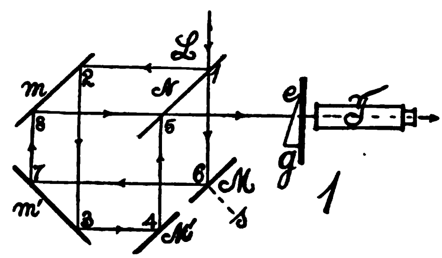
This device of locating an angle of rotation of the telescope by sharpness of fringes, may possibly be used for other purposes something after the manner of the halfshade or the sensitive tint; for if small, they jump suddenly out of an intensely brilliant unbroken spectrum band, when a definite $\Delta\varphi$ is reached.

Finally, as the fringes are examples of interference of intense non-reversed spectra, they should be available in such experiments as described in my last paper, for instance.

*Advance account, from a Report to the Carnegie Institution of Washington.

2. *Apparatus.*—To fix the ideas it will be necessary to give a diagram of the apparatus (fig. 1) employed. It is the selfadjusting interferometer, very serviceable here because of the large number of separate adjustments to be made, each of which might otherwise require long searching for fringes. White light L from a collimator takes the paths $12345T$ and $16785T$, N being a halfsilver. The telescope T is provided with the direct vision grating g, capable of rotating around the axis $5T$ (angle $\Delta\varphi$). T and g are preferably rotated together, as a rigid system. The mirror MM' consists of two independent, nearly coplanar parts, as shown, one of which, M for instance, may be displaced parallel to itself by the micrometer screw along the normal s (displacement ΔN). Path difference to the amount $2 \Delta N \cos 45°$ is thus introduced more than sufficient to pass the spectrum fringes through their maximum sizes between extremes of hair lines. By rotating m on a horizontal axis and m and M' on vertical axes, fringes of all sizes and inclinations when at their maximum may be obtained. The character of the





fringes due to inclination is shown by the achromatics and hence the adjustment is made with reference to them. They depart but little, relatively speaking, from their slope throughout the experiment.

3. *Observations.*—For the present purposes, the case of achromatic fringes, horizontal, vertical, and at about 45°, respectively, will suffice. Moreover, relatively small fringes, requiring much larger displacements (ΔN) than very large fringes, will generally be preferable.

Figure 2 gives an example of the results for horizontal achromatic or monochromatic fringes, the ordinates showing the displacement of micrometer ΔN (at M fig. 1) in 10^{-3} cm., and the abscissas the corresponding rotation of spectro-telescope (gT, fig. 1), needed to produce sharp fringes in the spectrum of an indefinitely wide slit. When the fringes are small, a few degrees of excessive rotation $\Delta\varphi$, either way, will cause them to vanish completely, so that the orientation for sharp fringes is quite sensitive. The symbols H (horizontal) and V (vertical) refer to the orientation of the edge of the prism, or the lines of the grating. The plane of dispersion is thus normal to H and V.

Hence it appears that horizontal fringes are left unchanged when the plane of dispersion is horizontal (edge of prism vertical), which is to be expected; for in such a case the light is permanently absent at the absorption bands due to the inclination fringes. On the other hand when the plane of dispersion is vertical, the nearly horizontal fringes have to pass from the positive to the negative inclination through their maximum size, when the telescope is rotated over 180°, and hence ΔN is very large, particularly so when the fringes are relatively small. In this large displacement of mirror ($\Delta N = .10$ cm., nearly) small monochromatic fringes will not change their inclination much; but their size will change considerably, and thus at $\Delta\varphi = 90°$ they are large and at $\Delta\varphi = 270°$, small.

Exactly the opposite conditions are met with when the fringes are nearly vertical, and the data may be omitted here. In figure 2, V' lies somewhat below V, as I could not (for incidental reasons) obtain adequately horizontal fringes without extreme difficulty. But this amounts merely to a slight shift of phase in the diagram. The vertical fringes were larger and hence a smaller double amplitude of displacement ($\Delta N = .07$ cm.) was here recorded.

Finally in the results for achromatic fringes at about 45° (estimated by the eye) the maxima were somewhere near $\Delta\varphi = 45°$ and 135°. Though the results were less smooth here, from deficiencies in the orientation (45°) of the achromatics, there was no fault to be found with the clearness of fringes, or with their abrupt evanescence.

If the spectrotelescope Tg (fig. 1) with a very fine slit is rotated, the fringes remain parallel to the length of the spectrum passing through a symmetrical case where the spectrum is reduced to a single fine colored line parallel to the slit. The fringes remain nearly parallel to the edge of the prism. Hence if any form coincides with the achromatic or monochromatic fringes, it will be retained on opening the slit wide, whereas the other forms, inasmuch as they require a fine slit, will vanish with the Fraunhofer lines. In the absence of a slit, the whole colored field bursts into sharp fringes, whenever the proper angle $\Delta\varphi$ of the telescope is reached. If the slit is a little too broad to show the solar lines distinctly, the monochromatic fringes may often be detected crosshatching the vague Fraunhofer lines, even when the spectrum fringes are still strong.

If the fringes of a fine slit are at say 45° to the axis, their inclination will change to 135° on passing the symmetrical stage; but there is apt to be both a change of size and angle in such cases.

Summary.—It has been shown in the experiments that the fringes (monochromatic) due to differences of inclination of rays, and the fringes (dispersion) resulting from differences in wave length of rays may be made of nearly equal size by displacing any mirror of the rectangular interferometer normal to itself (ΔN). The fringes will not, however, generally have the same inclination. This may be imparted to the spectrum fringes by rotating the spec-

tro-telescope (prism edge) on its axis until the inclinations also coincide. In reality the phenomenon is more complicated as the spectrum fringes change both size and inclination on rotation of the spectrum. In case of the completion of this twofold adjustment the slit of the collimator may be made indefinitely wide or removed altogether (undesirable light is to be screened off). The spectrum fringes may thus be given any intensity of illumination at pleasure, while the wave length corresponding to any fringe may be found by narrowing the slit until the Fraunhofer lines reappear. When the fringes are small the orientation of the spectro-telescope revolving around its axis may be determined by the appearance and evanescence of fringes. On the other hand the spectro-fringes, particularly if large, remain clearly enough in the field for the observation of the motion of a large number (i.e., for interferometry), before they vanish.

Similar results were obtained in broadening the vertical string of interference beads of reversed spectra. An account of these experiments will have to be omitted here, as they are much more complicated.

AN ELECTROMAGNETIC HYPOTHESIS OF THE KINETICS OF HETEROGENEOUS EQUILIBRIUM, AND OF THE STRUCTURE OF LIQUIDS

By William D. Harkins

Kent Chemical Laboratory, University of Chicago

Communicated by J. Stieglitz, March 24, 1919

While Gibbs,[1] in his remarkable treatise "On the Equilibrium of Heterogeneous Substances" has given a very broad treatment of his subject from the thermodynamic standpoint, nothing is included which would give any idea of the probable distribution of a component between a set of phases from a knowledge of the properties of only the pure component and of those of the phases before any of this component has been added to them. It is the purpose of this paper to indicate that the general nature of such a distribution can be predicted in most cases from the standpoint of the hypothesis that it is determined mainly by the intensity and nature of the electromagnetic field surrounding the molecules, and by the motion of the molecules and atoms. There is considerable evidence that the atom consists of a positively charged nucleus surrounded by a system of negative electrons. On such a basis it is to be expected that the atom, and therefore the molecule, would be surrounded by an electrostatic field. Inasmuch as there is much evidence from the magnetic properties of substances that the electrons are in motion, this is also to be considered as a magnetic field. Such a composite field is usually said to be electromagnetic.[2]

That the application of this hypothesis is not a single problem, is indicated by the electrical duality of such a field, and by the fact that while a part of the combinations between atoms or molecules in such a system may be of the nature of primary valence unions,—presumably a fitting of one or more of the outer electrons of one atom into the electronic system of another atom,—other molecules may be grouped together, though very much less firmly, by forces which still remain after all of the primary valence combinations have been made. In this preliminary paper only the more general features of the hypothesis will be considered,—that is only those which can be treated on the basis of a general knowledge of the intensity of the electromagnetic field around the molecule. The greatest obstacle in this connection is the meagerness of our knowledge of the characteristics of this field, which in this paper will be designated as the *stray field* of the molecule, since it gets out beyond the electronic constituents of the molecule.

The first problem which will be considered is: given two components (A) and (B), each in a phase by itself and both phases in the liquid state at the common temperature (T), when will these two phases be miscible and when will they be practically insoluble in each other? The relation is not difficult to find, for we know that (A) mixes with itself; so perfect miscibility should result when the stray fields around the molecules of (B) are sufficiently like those around the molecules of (A). Likeness of the fields in this sense means likeness in intensity, and presumably in the rate at which this intensity falls off with the distance from the molecule. A sufficient likeness of stray fields is also the condition which must hold if Raoult's law

$$p_A = P_A \frac{N_A}{N_A + N_B} = P_A x_A, \qquad p_B = P_B \frac{N_B}{N_A + N_B} = P_B x_B$$

is to be valid. Here p_A and p_B are the vapor pressures of (A) and (B) in the mixture, P_A and P_B are the vapor pressures of the pure liquids, and x_A and x_B are the mol fractions in the mixture. If (A) is a liquid, but the state of (B) is unknown, then (B) is apt to be a liquid if the pressures and temperatures of both are the same, though the size of the molecule is a factor which also has an effect. If the stray fields around the molecules of (A) and (B) are sufficiently different, then the two substances will be practically insoluble in each other—if the difference is extreme, one of the substances will be a gas and the other a solid, if they are at ordinary room temperatures.

Since the intensity of the stray field falls off more rapidly with the distance in the case of some molecules than with others (it probably decreases more rapidly around small *atoms* than around large atoms), it is not possible to give a list arranged in the order of increasing intensity of the stray field which is correct in all respects. Thus, while the intensity of the field close to the atoms of the heavy metals is very high, it undoubtedly decreases rapidly with the distance. On the other hand there are facts which seem to indicate

that the forces around the oxygen or nitrogen atoms of organic compounds or of water, or such atoms as are commonly called 'polar,' extend to a greater distance, although their intensity is less at the ordinary atomic distances. However, the following list may be considered to give something of this order of increasing intensity for a limited number of substances. This list has been obtained for the most part from a consideration of the *surface tension relations* of substances, and is as follows, beginning with those substances around whose molecules the stray field is weakest: helium, neon, hydrogen (molecular, not atômic), argon, krypton, xenon, nitrogen, oxygen, methane, carbon monoxide, and the following organic compounds—saturated aliphatic hydrocarbons, aromatic hydrocarbons, sulphides, mercaptans, halogen derivatives (methyl chloride, carbon tetrachloride, chloroform, and ethylene chloride, with rapidly increasing fields), unsaturated hydrocarbons, ethers, esters, nitro compounds, nitriles, aldehydes, ketones, alcohols, amines, acids, and unsaturated acids. Following these are water, molten salts, heavy metals, boron, and carbon. The list of organic substances is arranged for derivatives with short hydrocarbon chains. A lengthening of the chain causes a displacement in the direction of lower intensity for polar derivatives, but probably toward higher intensity in the case of the hydrocarbons themselves. It will be seen that in general the greater the distance between the substances in this list, the less their solubility in each other, the closer together, the more soluble. For organic substances, though the present list is much more extensive, it is in agreement with that found by Rothmund from solubility data.[3] It is well known that metals in general give concentrated solutions only with metals, carbon, and other similar substances; molten salts dissolve salts or water; pairs of organic liquids are miscible unless the members of the pair lie at the very opposite extreme of the list of organic substances; water dissolves salts or organic substances which are close to it in the list. An interesting illustration of this relation is given by data on the organic halogen derivatives listed above. The solubility of carbon tetrachloride per 100 grams of water is 0.80 grams, while that of chloroform, which lies closer to water, is 0.822 grams; and methylene chloride, approaching water still more closely, has a solubility of 2.00 grams. This is also the order of increasing hydrogen content of the molecule, but that this is not the determining factor is indicated by the fact that methyl chloride and methane, similar compounds containing still more hydrogen, are much less soluble in water. In organic compounds the intensity of the stray field is much higher adjacent to what are commonly called double bonds, than it is near single bonds, and this intensity grows much larger still if triple bonds are introduced. Corresponding to this the solubility of ethane, with its single bond between two carbon atoms, is 0.0507 volumes of gas per volume of liquid; that of ethylene with its double bond is 0.1311, or more than twice as great; while acetylene with its triple bond has a solubility of 1.105, or about 22 times more than that of the single bonded compound.

The above list, giving the order of intensity of the stray electromagnetic fields, will be seen to give the substances in increasing order of cohesion. The atoms or molecules giving the lowest cohesion, such as He, Ne, A, Kr, Xe, etc., are just those which, according to the valence theory of G. N. Lewis (*J. Amer. Chem. Soc.*, **38**, 1916, (762–85)) have *complete* outer shells of eletrons (2 for He, 8 for Ne and A). The substances with the highest cohesion, such as carbon, silicon, iron and cobalt, ruthenium, and tungsten and osmium, are elements which lie in the periodic system exactly half way between the elements of lowest cohesion, so high cohesion may be said to result when the outer shell of electrons is half filled or half complete. In other words a low stray field is found when according to the theory of Lewis the atomic or molecular outer shell of electrons is complete, the highest stray field, when the shell is half filled, that is when there is the possibility of the greatest number of electronic linkages directly between the atoms, without the intermediate formation of molecules of a simple type containing only a few atoms. Accompanying a high field the substance is found to have a low atomic volume, a high cohesion, melting point, a low coefficient of expansion, a low compressibility, etc. When the field is low in intensity, the properties are just the opposite.

In analyzing a solubility problem it is well to consider the attraction between the molecules of (A), between those of (B), and also that between (A) and (B). Consider octane and water which are mutually insoluble. It has sometimes been considered that this insolubility is due to the fact that water molecules attract each other more than they do molecules of octane, and that octane molecules attract each other more than they do molecules of water. Now the work of this laboratory shows that while the molecules of water do attract each other much more than those of octane, on the other hand the molecules of octane attract those of water very slightly more than they do those of octane. The much greater attraction of the water molecules for each other is a sufficient cause to produce immiscibility, since it is only necessary that when a group of water molecules is once formed, the mutual attraction shall be great enough to cause the molecules of water to leave the group less often than they enter it, so long as there is an appreciable quantity of water in the octane. The octane molecules are thus left in a phase by themselves.

Since an increase in the intensity of the stray field around the molecule is accompanied by an increase in molecular attraction, it causes an increase in the cohesion of the liquid or of the solid, so in its application to pure liquids the above theory gives predictions which are similar to those obtained by that of Hildebrand,[4] that liquids of like cohesions are miscible, while those whose cohesions are very unlike are practically insoluble in each other. However, solubility is a molecular scale phenomenon, so there are certain advantages of a molecular theory of the phenomenon. Moreover, the theory presented

here has a much wider range of application, since it may be used to indicate the internal structure of a liquid, to predict the distribution (partition and adsorption) of components between different phases, interfaces, and surfaces (these will be designated by the term *regions*), and is of great importance in theories of ideal and non-ideal solutions:—in other words it is a theory of what is called by Washburn[5] the '*thermodynamic environment*.' This in the sense of our theory would be designated as the *electromagnetic environment*.

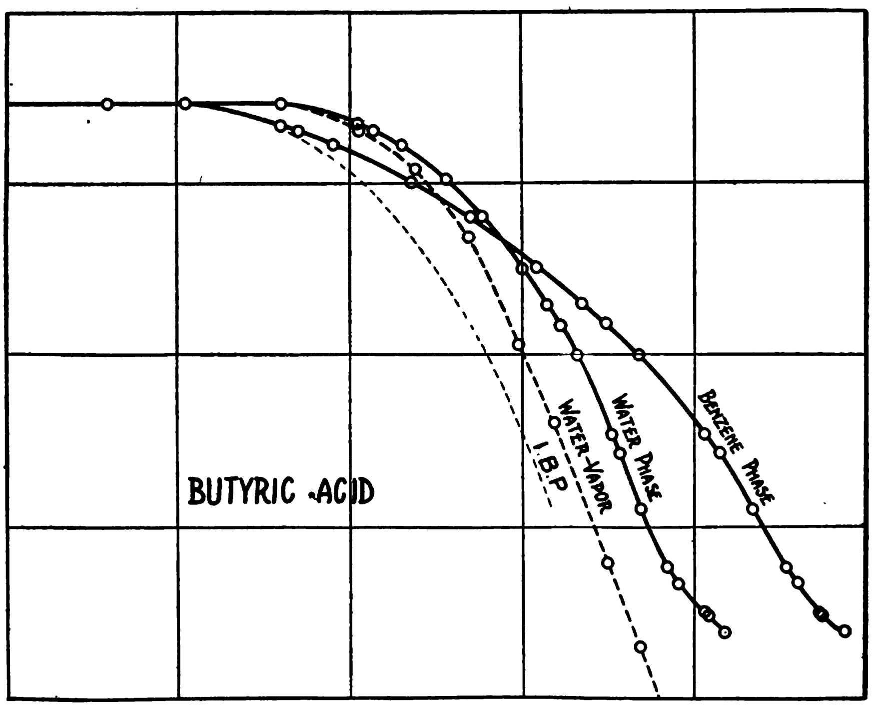
Application to Interfaces and to Distribution between Regions.—While in applying the hypothesis, the intensity of the stray fields around the molecules is of primary importance, at least one additional principle must be used if the direction which any change will take by itself is to be predicted. As might be expected the second law of thermodynamics is of fundamental importance in this connection, and for this purpose it may be stated in the form: Any change which takes place by itself in a system will proceed in the direction which will result in a decrease in the free energy of the system. Thus a surface will decrease in area by itself, but will not increase. Since a rapid variation of the intensity of the stray field with the distance in any direction, is accompanied by a high concentration of free energy, the second law indicates that in any change which takes place by itself, the variation in the stray field becomes less abrupt. If we imagine the surface of a liquid up to a bounding, surface plane, to have just the same structure as the interior of the liquid, then the actual surface always has a smaller free energy[6] than would be given by calculation for this imaginary surface, and therefore the drop in intensity of the stray electromagnetic field at the actual surface is always less than it would be at a surface of the structure of the imaginary surface. Since a molecule is often made up of several species of atoms, the stray field around it is often unsymmetrical. Thus many organic molecules, such as the primary normal alcohols, acids, amines, nitro compounds, nitriles, ethylene and acetylene derivatives, etc., consist of a paraffin chain around which the stray field has a relatively low intensity (a so-called non-polar group), while at the other end of the molecule there is a group containing oxygen or nitrogen, sometimes with metals in addition, around which the intensity of the stray field is relatively high (a polar group). Such molecules may be designated as *polar-nonpolar*, and designated by the symbol o——, where o represents the polar, and —— the nonpolar end of the molecule. If molecules of this type, such as butyric acid (C_3H_7COOH), are put in a two phase system consisting of a polar liquid such as water, and a nonpolar liquid such as octane, then the free energy of the interface will be less when the transition from one liquid to the other is made by molecules of butyric acid, with their polar ends turned toward the water, and their nonpolar ends turned toward the octane, since in this way the abruptness of the transition is decreased.

The problem here arises as to the distribution of molecules of the polar-nonpolar type between the two liquid phases, their surfaces, and the interface

between them. It may be considered that each region (phase, surface, or interface) exerts a certain restraining force upon (has affinity for) the solute molecules. Since at equilibrium the thermodynamic potential of the solute is the same in all of the regions, it may be considered that the concentration of the solute (at equilibrium) in each phase, interface or surface, gives an index of the restraining force exerted by that region upon the solute molecules. Let us now assume that we have a number of exactly similar two phase systems, each of which consists of equal volumes of a polar liquid, such as water, and a nonpolar liquid such as octane, with an interface of a definite area between them, and into each of these systems we put N molecules of the polar-nonpolar type o——. The hypothesis indicates that with a given polar group the distribution of the N molecules will vary in such a way that with an increase in the length of the nonpolar part of the molecule, the number of molecules, and therefore the restraining force in the octane will increase, while in the water both of these will decrease. The reverse of this occurs when with a given nonpolar chain, there is an increase in the number of polar groups. The greatest restraining force would be exerted on such molecules when they are in the interface, where the nonpolar end of the molecules could turn toward the nonpolar liquid, and the polar end toward the polar liquid. Since the restraining force is greatest at the interface, the concentration in this region should also be the greatest, which agrees with the facts as found by experiment.

In the preceding paragraph it was assumed that the water is completely covered by the octane. Let us now assume that the water has a surface, or a water-vapor interface as well, and that benzene is the nonpolar (slightly polar) liquid. At the water-vapor interface at ordinary temperatures, the drop in average intensity between the stray field in the water and that in the dilute vapor (in which the intensity is practically zero), is much greater than that between water and benzene, so the restraining force on molecules of the polar-nonpolar type should be much greater at the former interface. In complete agreement with this idea experimental tests indicate that *the restraining force is about three times greater at the water-vapor interface than at the water-benzene interface*, when butyric acid is the solute, and somewhat similar results are obtained when the solute is acetic acid. The experimental work was carried out in 1913 by E. C. Humphrey, and was afterward repeated and made more precise by H. H. King and H. McLaughlin. The above results may be expressed in other words as follows: the thermodynamic potential of butyric acid is about three times as great in the water-benzene interface as it is in a water surface, when the concentration of the butyric acid in both of these interfaces is the same, or the drop in mechanical potential which a molecule of butyric acid undergoes in passing from water into the interface is much greater when the second phase is a dilute vapor than it is when the second phase is a nonpolar liquid. This is an illustration of a general case,

and similar relations hold for any molecule of the polar-nonpolar type with reference to the pair of interfaces, polar liquid-vapor, and polar liquid-less polar liquid. Similar work is now in progress, a metal being used as one of the phases, and vapor, water, or an organic liquid as the other. The results of this work are very interesting, but space is wanting for their discussion. They indicate that, corresponding to the greater drop in intensity of the stray field between metal and vapor, in comparison with that between water and vapor, the adsorption at the metallic surface is much greater than that at a water surface for the same concentration of the adsorbed component in the nonmetallic phase.



A striking result obtained was that the number of molecules per square centimeter in the 'constant concentration' film between water and benzene was found to be 2.79×10^{14}, while that between water and air was found to contain 2.77×10^{14} molecules, showing that the number of molecules in such a film is independent of the presence of the second liquid phase, and therefore depends only on the dimensions of the molecules.

These results were obtained by the drop weight and capillary height methods for determining surface tension, as they have been developed in this laboratory.[7] For such work a high degree of precision is essential. The results for butyric acid are presented in a graphic form in Figure 1, where

the ordinates give the surface tension; the abscissae, the logarithm of the concentration, and the adsorption is obtained from the slope of the respective curve by means of the Gibbs equation, $\mu = \frac{1}{iRT}\frac{d\gamma}{d\ln c}$ where μ is the adsorption, γ is the surface tension, and c is the concentration of the solute. The curve for the water-vapor interface lies to the left of that for the water phase of the two phase system, which indicates that the restraining force of the former interface is the higher. The curve marked I. B. P. represents that which would be obtained for the benzene of the two phase system if the butyric acid in this phase were not associated. The horizontal distance between this curve and that for the actual benzene phase is a measure of the degree of association.

When applied to the structure of a liquid our hypothesis indicates that when the molecules are not wholly symmetrical, there will be more or less grouping of the parts of the molecules around which the stray field is the most intense, the extent of the grouping being a function of the temperature also. It is probable that liquids of this class have more, but a less definite, structure than is commonly supposed. If molecules of the polar-nonpolar type are dissolved in a nonpolar liquid this grouping is commonly spoken of as association of the solute; in a polar solvent it is considered as the solvation or hydration of the solute, in pure liquids it is designated as the association of the liquid. These groupings are constantly being modified by the kinetic agitation, the other factors being the intensity of the stray field, and the space available for the grouping.

A discussion of electrical theories of surface phenomena will be found in papers by Hardy;[8] Langmuir;[9] Harkins, Brown, and Davies;[10] Harkins, Davies and Clark,[11] and by Frenkel.[12] The present paper will be published in a much more complete form in the Journal of the American Chemical Society, in which there will be taken up the question of what is known as specific adsorption.

[1] Gibbs, *Trans. Conn. Acad., New Haven*, Oct. 1875–May 1876, (108–248), and May 1877–July 1878, (343–524).

[2] Lewis, Wm. Mc., *Phil. Mag., London*, 28, 1914, (104–16).

[3] Rothmund, *Loslichkeit und Loslichkeitsbeeinflussigung*, Leipzig, 1907, p. 118.

[4] Hildebrand, *J. Amer. Chem. Soc., Easton, Pa.*, **38**, 1916, (1452–73).

[5] Washburn, *Physical Chemistry*, Chapter XIII, pp. 134–42, also pp. 143, 224, 273.

[6] Harkins, Davies, and Clark, *J. Am. Chem. Soc.*, **39**, 1917, (553–70, 594–5).

[7] Harkins and Humphrey, *Ibid.*, **38**, 1916, (228–46); Harkins and Brown, *Ibid.*, **38**, 1916, (246–52), and April 1919.

[8] Hardy, *London, Proc. Roy. Soc., B*, **86**, 1911–12, (634).

[9] Langmuir, *J. Amer. Chem. Soc.*, 39, 1917, (1848–1906); these Proceedings, **3**, 1917, (251–7). Abstract in *Met Chem. Eng.* **15**, 1916, (468).

[10] Harkins, Brown, and Davies, *Ibid.*, 39, 1917, (354–64).

[11] Harkins, Davies, and Clark, loc. cit., 1917, (541–96).

[12] Frenkel, *Phil. Mag., London*, 33, 1917, (297–322).

HYDROGEN OVERVOLTAGE

By Duncan A. MacInnes and Leon Adler

Chemistry Department, University of Illinois

Communicated by W. A. Noyes, March 29, 1919

Hydrogen overvoltage may be defined as the difference of potential that exists between a reversible hydrogen electrode, and an electrode, in the same solution, at which hydrogen, H_2, is being formed from hydrogen ions. A reversible hydrogen electrode is one at which the reaction

$$2H^+ + 2\epsilon = H_2 \qquad (1)$$

(ϵ = electron) has reached equilibrium. This equilibrium is attained, and maintained during the passage of very small currents, only on electrodes covered with a dispersed 'noble' metal, such as platinum. If stronger currents are passed across such a metal-electrolyte boundary, or if other electrodes are used, the reaction does not take place under equilibrium conditions and an overvoltage appears. For instance, it requires an overvoltage of about 0.2 volt to liberate hydrogen gas from a polished platinum surface, and about 0.7 volt from a lead surface.

On attempting to determine the overvoltage of small electrodes of 'platinized' platinum the authors observed some interesting fluctuations in the measured voltage, a typical series of measurements being plotted in figure 1. Here ordinates represent overvoltage in millivolts and abscissae time in seconds. Bubbles were evolved at the points marked by small circles, there being one bubble to each fluctuation. At the low current densities used, the bubbles came off at a single point on the electrode, making it appear probable that the nucleus of the next bubble remained on the electrode after each bubble had separated.

When Reaction 1 takes place it is probable that the greater part of the liberated hydrogen goes directly into solution. Unless carried away by diffusion, stirring, or other means, the concentration of dissolved hydrogen, in immediate contact with the electrode, will tend to rise and produce a supersaturated solution. It is this layer of dissolved hydrogen that is responsible for the overvoltage. However, if there is a nucleus of *gaseous* hydrogen on the electrode a portion of the liberated hydrogen will enter this gaseous phase. A gaseous nucleus will thus play a similar rôle to that of a small crystal in a supersaturated salt solution. It is evident that hydrogen gas in the form of small bubbles must have a larger energy content, per mol of gas, than the same volume of undispersed gas, as energy must be expended in overcoming the surface tension in the formation of the bubbles. Such bubbles will be more soluble (i.e., remain at equilibrium with more concentrated dissolved hydrogen) than the undispersed gas. This is analogous to

the increase of solubility produced by the fine grinding of solids. An electrode in equilibrium with small bubbles will thus reach a higher potential than one in contact with undispersed gas.

With the foregoing in mind we can proceed to an explanation of the voltage fluctuations. At *a* (fig. 1) a bubble has separated from the electrode, leaving a nucleus behind. As the electrolysis proceeds this nucleus will grow, obtaining hydrogen from the supersaturated solution or from the electrode. However, as the nucleus increases in size, the energy necessary to produce further increases in volume must decrease, as the ratio

$$\frac{\text{increase in surface}}{\text{increase in volume}}$$

is continuously decreasing. If the bubble is growing slowly the hydrogen bearing solution surrounding the electrode will tend to get into equilibrium with the bubble. This accounts for the decrease in overvoltage from *a* to *b*.

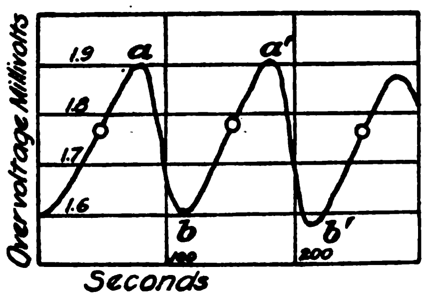


FIG. 1

At *b* the buoyant effect of the electrolyte is sufficient to overcome the attraction of the electrode for the bubble, which breaks away, leaving a nucleus behind. Bubbles are, however, seldom observed to come off at points corresponding to *b*; they probably remain in the pores of the electrode, and are pushed out by the growth of the following bubble. From *b* to *a'* the concentration of dissolved hydrogen is increasing to a value such that the nucleus can again grow, when the processes described above are repeated.

It is evident that it makes no difference, in so far as the absorption of energy is concerned, whether small bubbles are evolved, or a solution of hydrogen in equilibrium with the bubbles is formed. In the following paragraph it is assumed that all the hydrogen is liberated as bubbles.

The number of bubbles of radius r that can be formed from a mol of hydrogen of volume V is $3V/4\pi r^3$. To obtain the surface energy of such a system this must be multiplied by $4\pi r^2$ and the surface tension γ. Substituting RT/p for V (R, T and p are the gas constant, the absolute temperature and the pressure respectively) and equating the surface energy with the electrical

energy, $2FE$, in which F is the Faraday equivalent and E the overvoltage, we obtain:

$$2FE = \frac{4\pi r^2 RT}{4/3\pi r^3 p}\gamma = \frac{3RT}{pr}\gamma \qquad (2)$$

Substituting 75.6 dynes per sq. cm. for γ and appropriate values for the other terms, equation 4 becomes, for 25°C. and one atmosphere

$$E = \frac{2.87 \times 10^{-2}}{4}. \qquad (3)$$

For platinized platinum electrodes this relation was found to hold quantitively. For instance, in one experiment the bubbles were found to have

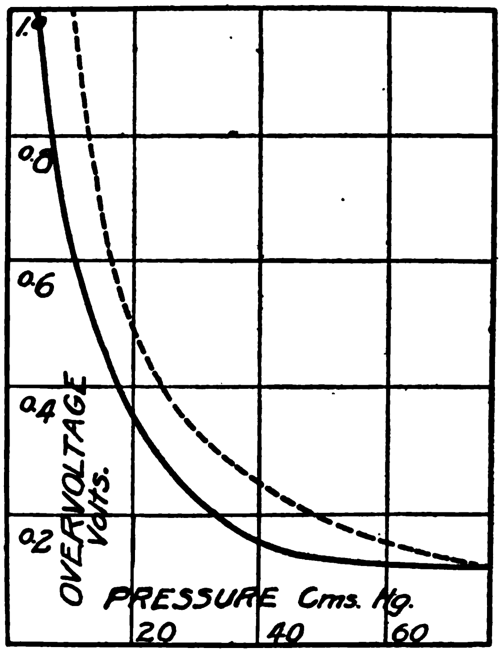


FIG. 2

radii of 0.017 mm., corresponding to an overvoltage of 1.6 mv. The observed values fluctuated between 1.9 and 1.6 mv. the latter potential, according to the theory, being that corresponding to the fully formed bubble. For electrodes of other metals difficulty was encountered in measuring the bubbles, but they were found to be smaller than those for platinized platinum in all cases observed.

If the bubble radius does not change with the pressure (as was found to be the case, up to 16 atm. pressure, with platinized platinum electrodes) the overvoltage (according to equation 2) should increase as the pressure decreases, and *vice versa*. This prediction was found to be verified in some

unpublished work by Goodwin and Wilson of the Massachusetts Institute of Technology. In figure 2 the solid line shows the experimentally determined variation with the pressure of the overvoltage of nickel. Similar curves were found for mercury and lead. The dotted line shows the variation as calculated by Equation 2, using the overvoltage at one atmosphere as a basis for computing the values for the other pressures. The difference between these two curves may be explained by an increase of stirring at the lower pressures, since many more bubbles are produced per mol of gas.

It appears quite probable, then, that the factor that determines the overvoltage of an electrode at any one pressure is the size of the gaseous nuclei that can cling to it. A number of observers have called attention to the fact that electrodes with low overvoltages are those that have large adsorptive powers. This adsorptive power is undoubtedly related to the attraction of an electrode for a gaseous nucleus.

AN APPROXIMATE LAW OF ENERGY DISTRIBUTION IN THE GENERAL X-RAY SPECTRUM

By David L. Webster

Department of Physics, Massachusetts Institute of Technology

Communicated by E. H. Hall, April 9, 1919

In the spectra of X-rays as ordinarily determined there are factors of absorption in the anticathode, the glass of the tube, the reflecting crystal and the ionized gas, and of efficiency of reflection that are all functions of the frequency. Fortunately, except at the discontinuities of any of these absorptions, the unknown factors vary continuously with frequency, so that the measured intensities in the spectrum represent the energy distribution qualitatively, but by no means quantitatively. The problem of the present paper is to combine other available data in such a way as to find an approximate law of energy distribution, not involving unknown absorption factors, and avoiding also any *a priori* assumptions about the emitting mechanism. The data are incomplete and this work is merely a first approximation.

For data we have (a) some graphs of intensity against potential at constant frequency (where the unknown factors are all constant in each graph), and (b) the total energy measurements by Beatty,[1] who made the absorption negligible by using a thin window and no crystal. Some of the intensity-potential graphs were obtained in the course of experiments for another purpose with a rhodium target by the author,[2] and with platinum by the author and Dr. H. Clark,[3] and others were obtained by taking points at the same wave length from intensity-wave length graphs drawn for tungsten by A. W. Hull,[4] Hull and Rice[5] and Ulrey,[6] and for molybdenum by Hull.

In the experiments on rhodium and platinum, the spectrometer was kept at a fixed wave length and the potential was changed between readings.

This gave directly a series of intensity-potential graphs that are represented with fairly good accuracy by the law

$$I(V,\nu) = k(\nu)\left\{(V - H\nu) + H\nu p(\nu)\left(1 - e^{-q(\nu)\left(\frac{V}{H\nu}-1\right)}\right)\right\},$$

where $I(V,\nu)d\nu$ is the intensity in the frequency range $d\nu$ per electron striking the target from potential V, and H is the ratio of Planck's h to the charge of the electron, and $k(\nu)$, $p(\nu)$ and $q(\nu)$ are functions of ν. Since p and q are pure numbers they are independent of the arbitrary intensity unit, and can be determined wherever the data are available, though not very accurately because the term containing them is rather small. In the few data available for rhodium, p is of the order of 0.06 to 0.08 and q about 12 to 16, making pq about 1. The work on tungsten and molybdenum gives only a few points on each intensity potential graph, and because of the smallness of the exponential term and its disappearance at potentials large enough for really accurate intensity measurements, it is impossible to get an accurate test of this law except with more points than one can get from these graphs. But the data obtainable show that the relation between I and V is not far from linear, and the only definite curvature seems to be something of the type indicated by the above equation. In platinum, we have data scattered over the range from 1.33 to 0.43 Å, but most of them rather rough. But to an accuracy of 20 or 30%, they seem to indicate constant values for both p and q, with $p = 1/5$ and $q = 13$, so that $pq = 5/2$. Fortunately the smallness of the p and q terms makes their influence on the determination of k also small, although the existence of the terms themselves may be of considerable theoretical importance. For the present, therefore, we shall include these terms in the calculation, but neglect any changes of p and q with ν.

An important point to be deduced from the graphs of 'intensity' against ν is the fact that they are smooth and regular, so that k must have no discontinuities or sharp curvature in its graph against ν. As a trial value we shall therefore assume first that $k(\nu) = k\nu^n$, with k and n both constant. The total energy is then

$$E \doteq \int_0^{\frac{V}{H}} I(V,\nu)d\nu = k\left\{\frac{1}{n+1} - \frac{1-p}{n+2} - p\int_0^1 x^{n+1}e^{-q\left(\frac{1}{x}-1\right)}dx\right\}\frac{V^{n+2}}{H^{n+1}},$$

If $k(\nu)$ is not expressible in this way, but as a power series in ν, then this equation gives a power series for E in terms of V, and if the coefficients for such a series are found experimentally those of the series for k can be computed from them. If E depends on a single power of V, then k must depend on a single power of ν.

Now, Beatty's work indicates that $E = \text{constant} \times V^2$. Hence, we infer that $n = 0$ and $k(\nu) = k = \text{constant}$, and

$$E = k\left\{\frac{1}{2} + \frac{p}{2} - p\int_0^1 xe^{-q\left(\frac{1}{x}-1\right)}dx\right\}\frac{V^2}{H},$$

or approximately

$$E = k\left\{\frac{1}{2} + \frac{p}{2} - \frac{p}{q}\right\}\frac{V^2}{H}.$$

In a previous paper by the author[7] it was shown that, neglecting absorption in the target, the intensity per electron and per atom and per unit interval of frequency from an extremely thin target would be

$$i\,(V, \nu) = \frac{bD_V I(V,\nu)}{2\,NV},$$

where b is the coefficient of the Thomson-Whiddington law in the form $V_0^2 - V_x^2 = bx$, and N is the number of atoms struck per unit length of the electrons' path. This equation is of course subject to some error due to absorption in the target, and as one may readily prove, could be corrected by adding to i a term

$$\frac{a\mu}{N}\, I\,(V, \nu),$$

where a is the ratio of the distance travelled in the target by the emerging X-rays to the distance travelled by the cathode ray before it emits X-rays, and μ is the linear absorption coefficient. This correction is important only at low frequencies. Neglecting it, we have

$$i\,(V, \nu) = \frac{bk}{2\,NV}\left\{1 + pqe^{-q\left(\frac{V}{H\nu}-1\right)}\right\},$$

with k, p and q all constant. It must be remembered that the data are very incomplete and this result is unreliable and is presented only for lack of anything better. But the true law must have something of the same general characteristics as this, and certain conclusions about the emitting mechanism can be drawn from that fact.

First, let us assume that some form of quantum law governs the radiation of frequencies different from V/H as well as at that one. We are then practically though not rigorously led to one of two alternatives. If the quantum law merely regulates the frequency being emitted at any instant in terms of the energy still available at that instant for radiation, so that the radiation by one electron is not monochromatic, then every electron radiating at all must radiate some energy at the highest possible frequency, V/H, and presumably give up all its energy to radiation. But this would make $i(V, \nu)$ independent of ν, and is therefore improbable. The more probable alternative is that the radiation by one electron is monochromatic, and the quantum law gives its

frequency in terms of the total energy radiated. In this case only a few of the electrons radiating will do so at frequencies very near the limit, V/H, though more will radiate there than in an equal frequency interval at a frequency not too far below that one. Adopting this conclusion, the chance that an electron radiates a fraction of its energy within a range ϵ to $\epsilon + d\epsilon$ is $f(V, \epsilon)d\epsilon$, where

$$f(V, \epsilon) = \frac{bk}{2Nh} \frac{1 + pqe^{-q\left(\frac{1}{\epsilon}-1\right)}}{V\epsilon}.$$

Let R be the radius of the atom and r the distance from the particle causing radiation to the path the electron would have taken if not deviated. Then

$$\frac{d(r^2)}{R^2} = -f(V, \epsilon)\, d\epsilon, \text{ with } \epsilon = 1 \text{ at } r = 0,$$

or

$$\frac{r^2}{R^2} = \int_{\epsilon}^{1} f(V, \epsilon)\, d\epsilon.$$

If we assumed the frequency to be determined by the energy transferred in a collision of two equal repelling particles, one of which was initially at rest, a definite value of ν could be predicted for any assumed values of r and V, and the forms of $f(V, \epsilon)$ and $i(V, \nu)$ could be found. Several such assumptions have been tried, but none give the forms of f and i required above. A detailed discussion would be out of place with such rough preliminary data, and any such assumption would be improbable, but it seems likely that more exact data of this type ought to throw valuable light on the mechanism of radiation. It is hoped that with apparatus now being constructed it will be possible to get such data.

[1] Beatty, R. T., *Proc. Roy. Soc., London*, A **89**, 1913, (314–27).

[2] Webster, D. L., these PROCEEDINGS, **2**, 1916, (90–4); *Physic. Rev., Ithaca*, **9**, 1916, (599–613).

[3] Webster, D. L., and Clark, H., these PROCEEDINGS, **3**, 1917, (181–5).

[4] Hull, A. W., *Amer. J. Roentgenology* **2**, 1915, (893–9).

[5] Hull, A. W., and Rice, M., these PROCEEDINGS, **2**, 1916, (265–70).

[6] Ulrey, C. T., *Physic. Rev., Ithaca*, **11**, 1918, (401–10).

[7] Webster, D. L., *Ibid.*, **9**, 1917, (220–5).

LINEAR ARRANGEMENT OF GENES AND DOUBLE CROSSING OVER

By Harold H. Plough

Department of Biology, Amherst College

Communicated by T. H. Morgan, April 11, 1919

In his recent criticism of the theory of linear arrangement of the genes in the chromosome, Prof. W. E. Castle states that that theory calls for a number of unproved secondary hypotheses, among which is the assumption of

TABLE 1

NUMBER	TEMPERATURE	TOTAL NUMBER	PER CENT CROSSOVERS	PER CENT INCREASE OVER CONTROL	PER CENT CROSSOVERS	PER CENT INCREASE OVER CONTROL
b pr c			b–pr		pr–c	
	deg. C.					
1	9.0	995	13.5	125	25.8	32
2	13.0	2,972	13.5	125	27.2	39
3	17.5	2,870	8.3	38	23.0	17
4 (Control)	22.0	15,000	6.0		19.6	
5	29.0	4,269	8.8	47	22.5	14
6	31.0	3,547	14.0	133	26.7	36
7	32.0	4,376	15.7	162	26.5	35
S^1 b c			S^1–b		b–c	
8 (Control	22.0	6,009	38.4		21.1	
9	31.5	3,769	37.8	Decrease	33.2	58
b pr vg			b–pr		pr–vg	
10 (Control)	22.0	2,139	10.0		15.1	
11	31.5	1,099	15.1	50	20.2	34

Data from H. H. Plough, *J. Exp. Zool.*, **24**, No. 2.

double crossing over. There are, however, a number of facts, each independent of the original hypothesis, that require double crossing over in their explanation. Some time ago[1] I demonstrated that treatment of female flies with temperatures both above and below the 'normal' (20°–27°C.) resulted in an increase in the percentage of crossing over in the second chromosome. This increase for a short chromosome 'distance' was roughly proportional to the number of degrees above or below the 'normal.' Bridges[2] had already shown that age increased crossing over and more recently it has appeared probable that other environmental agents may cause a similar increase. In all cases, as pointed out in the original paper, the increase is much greater for short regions than for longer ones, and finally, for genes which normally

give a crossing over percentage of 35% or more no increase at all results. A summary of these facts extracted from my earlier paper is given below. The percentage of increase over the control is given. Only those data which were collected at the same time are entirely comparable with each other.

As a whole this table shows conclusively that small percentages of crossing over are increased markedly by high or low temperatures, while larger percentages show a much less significant increase. For instance, as a result of exposure of the F_1 females to 32°C. (no. 7 in the table) the crossing over value between black and purple was increased from 6 in the control to 15.7—an increase of 162%—while purple-curved in the same experiment showed an increase from 19.6 to 26.5 or only 35%. In no. 9 in the table it is shown that while the black-curved value was increased by 58% as a result of a temperature of 31.5°C., the value of the star to black region (38.4 units) not only did not increase, but actually showed a slight decrease. The amount of the increase is in all cases related to the crossover value involved. The last named case was explained in my former paper as follows (p. 157):

> The first brood data show very clearly that while the blackcurved region of the chromosome shows an unquestionable increase of more than 50%, no increase at all is registered in the test between star and black. This can mean only that with such long distances any increase in the actual amount of single crossing over is compensated by a similar increase in double crossing over, and thus no increase at all appears in the percentage registered by the count.

The data show that the percentage of increase caused by high or low temperature is roughly in inverse proportion to the size of the crossover value involved. On Castle's three dimensional scheme these facts necessitate the view that long chromosomal 'distances' are less affected by temperature than are short ones. On the hypothesis of linear arrangement this relation is consistently explained by the assumption that the amount of double crossing over is increased by high and by low temperatures.

[1] Plough, H. H., *J. Exp. Zool.*, **24**, 1917, (147).
[2] Bridges, C. B., *Ibid.*, 19, 1915, (1).

THE SPATIAL RELATIONS OF GENES

By A. H. Sturtevant, C. B. Bridges, and T. H. Morgan
Columbia University and Carnegie Institution of Washington

Communicated April 11, 1919

Castle[1] has proposed an arrangement of linked genes in three dimensions, based on the assumption that the distance between any two loci is exactly proportional to the observed crossover value. He believes that this system gives a better agreement between observation and expectation than does the hypothesis of a strictly linear arrangement that we have developed.

According to Castle, "that the arrangement of the genes within a linkage system is strictly linear seems for a variety of reasons doubtful." The only one of these reasons specified is that "it is doubtful, for example, whether an elaborate organic molecule ever has a simple string-like form." The suggestion that chromosomes are organic molecules is probably not intended to be taken seriously. The further argument, that organic molecules probably never have a simple string-like form, is scarcely to the point, for *chromosomes* do have a thread-like form. The comparison between crossing over and organic substitution reactions seems forced, for these reasons, as well as on strictly chemical grounds.

Castle presents two cases as "critical" ones showing that the arrangement cannot be linear; viz., the relations of bifid and of abnormal to yellow and white. The case of abnormal presents some complications,[2] but bifid may be accepted as a crucial case. On the basis of the data summarized by Morgan and Bridges[3] and used by Castle, the crossover values for yellow, white and bifid are:

Yellow white	1.1
White bifid	5.3
Yellow bifid	5.5

Each of these values is based on several thousand flies, so that the probable errors arising from random sampling are small. Castle points out that no one of the three values is either the sum or the difference of the other two, and therefore concludes that the placing of the three loci in a line cannot represent their relations to each other. But these data come from several sources; and it has frequently been pointed out that crossover values are subject to variation, due to genetic factors,[4] to environmental causes,[5] or to differential viability.[6] In such a case as the present one, then, strictly comparable data can be obtained only from experiments in which all three loci are followed at the same time.[7] Such crucial data for yellow, white, and bifid had been given by Morgan and Bridges[3] and by Muller,[4] in papers which were known to Castle. These experiments may be summarized as follows:

	NON-CROSS-OVERS	YELLOW SEPARATES FROM OTHER TWO	WHITE SEPARATES	BIFID SEPARATES	TOTAL
Morgan and Bridges	487	3	0	16	506
Muller	673	12	0	27	712
Total	1160	15	0	43	1218
Percentage		1.2	0.0	3.5	

Here we have crossover values that are strictly comparable with each other, and they show:

Yellow white	1.2
White bifid	3.5
Yellow bifid	4.7

These relations, in which the sum of the smaller values is exactly equal to the larger one, can be represented[8] only by placing the three loci in a straight line, in the order yellow, white, bifid.[9]

It will be seen that this linear arrangement is necessitated by the fact that white never separated from yellow and bifid; i.e., no 'double crossover' occurred. By considering data involving only short distances (e.g., white bifid club, bifid club vermilion, club vermilion miniature, vermilion miniature sable, sable rudimentary forked, rudimentary forked bar, and forked bar fused; or still better, certain unpublished data including the loci ruby, cut, tan, garnet, etc.), it is possible to handle the whole X-chromosome of Drosophila as made up of successive overlapping sections in which double crossing over either does not occur or is so rare as to be negligible. Since these sections must be represented as straight lines, and since they overlap, the whole X-chromosome must be represented as a straight line.

For longer sections of this same chromosome, however, double crossovers do occur, so that the distances apart on the straight line are no longer proportional to the crossover values. The 'map-distance' for white forked, for example, is 55.4, though the crossover value observed[10] is 43.9.

Castle is disturbed by such map distances greater than 50. He says: "A crossover value greater than fifty cannot exist. For there must be either linkage or no-linkage. But no-linkage means 50% crossovers, and linkage means less than 50% crossovers. Hence a value greater than 50% cannot occur." It is evident that the conclusion of this curious syllogism depends solely on the definition of linkage contained in the second half of the second premise. It is true that crossover values significantly greater than fifty have never been found, but this is due to the frequency of double crossovers—to the fact that 'coincidence' is high for long distances. But coincidence is known to be a variable quantity, so that there would seem to be no *a priori* reason to suppose that crossover values greater than fifty are impossible. Furthermore, as has been explained above, map distances greater than 50 have never been intended to represent observed crossover values greater than fifty.

It is to be observed that the three dimensional figure given by Castle might seem capable of reduction to a curved line lying in one plane. An examination of his figure 2 shows that the only loci far from the plane that includes most of the group are bifid, depressed, shifted, lethal 3, furrowed, and lethal sc. Let us then examine these individually.

Bifid has already been shown to lie in the same straight line with yellow and white, both of which are in Castle's thickly populated plane.

Morgan and Bridges give three crossover values for depressed, two of which (white depressed and depressed vermilion) are stated to be based on 59

flies each. Such meager data are clearly insufficient to establish accurate crossover values. But the values given by Morgan and Bridges are themselves incorrect, as is shown by their account of the actual experiment (p. 68). The total number of flies is 69, not 59, and the crossover values accordingly become 17.4 for white depressed (instead of 20.3) and 14.5 for depressed vermilion (instead of 17.0). The sum of these values, 31.9, is not significantly different from 30.5, the accepted value for white vermilion. This relation, and the fact that the 69 flies included no double crossover, give no warrant for supposing that the locus of depressed is outside of the plane that includes white, vermilion, and bar.

Only two crossover values are given for shifted—shifted vermilion and shifted bar. It is evidently impossible to place a point in space by means of its distances from only two other points. Three are necessary, as Castle himself points out in connection with abnormal and bifid. There is thus no reason whatever for placing shifted out of the common plane.

The aberrant position of lethal 3 in Castle's model is evidently due in large part to the fact, shown by Morgan and Bridges (p. 75), that the experiments involving this locus gave a value of 2.3 for vermilion miniature, which is less than the mean value, 3.1, used by Castle. The values obtained in the lethal 3 experiments were:

Lethal 3 vermilion	6.8
Vermilion miniature	2.3
Lethal 3 miniature	9.3

The third value is substantially the same as the sum of the first two—hence the three loci must be represented as lying very nearly in a straight line.

In the case of furrowed, the values are all based on a few flies (240 or less in every case), and the three most important values (vermilion furrowed, miniature furrowed, and furrowed sable) were all obtained in different experiments, so that comparisons are hazardous. Furrowed is a character that is often difficult and sometimes impossible to classify, so that there is a large probability of error in the counts that have been reported.

In the case of lethal sc there is only one available crossover value (bar lethal sc, 8.3) small enough so that double crossing over would be negligible; and there are no data involving lethal sc and more than one other locus at the same time. The data are therefore not of the type to give a decisive answer as to the relations of the locus.

A careful examination of the data thus shows that a single plane suffices for the representation of the loci dealt with by Castle. Within this plane the positions of the remaining loci fall, in Castle's figure, roughly into a single curved line. The only noticeable exception is the locus of lemon. This locus is based on only 241 flies, and these gave rather too small a crossover value for the well established white vermilion distance, so that this discrepancy is not significant. The arrangement of the genes is thus approximately linear, but

the line is curved instead of straight. This curvature is due to the phenomenon of double crossing over, and the degree of curvature is dependent on the frequency of double crossovers; i.e., upon 'coincidence.' This method of representing coincidence, however, leads to inconsistencies. As was pointed out above, it is possible to build up the whole X-chromosome from successive overlapping sections, each of which is so short as to include few or no double crossovers. Yet double crossovers are not rare in longer sections. If ABC and BCD are sections so short that no double crossing over occurs within each, section ABCD may be long enough so that double crossing over in which one crossover is between A and B and the other between C and D may still occur; and in that case it becomes necessary to represent D in two positions at the same time. Such situations are actually known in Drosophila. Hence the curved line cannot give a consistent scheme; nor can any other scheme based on the assumption that long distances, as well as short ones, are to be represented as proportional to observed percentages of crossing over.

According to Castle the supposition that double crossovers do occur is "an unproved secondary hypothesis." Of course, if three genes are imagined as not lying in a straight line, a single plane may separate any one from the other two. Under these conditions double crossing over has no meaning. But the fact remains that when three linked genes are studied simultaneously, one pair of contrary classes is *always* small, and this class is always the one that is the double crossover class on the linear arrangement scheme. Some method of accounting for the smallness of this class is evidently demanded. Castle recognizes this, and suggests that the explanation may be either that "only transverse breaks occur, of which two taking place simultaneously are required to produce the difficult regrouping" (i.e., the familiar double crossover explanation), or that "transverse breaks are more frequent than oblique longitudinal ones, of which a single one would suffice to accomplish the regrouping, if the genes are not strictly linear in arrangement." Muller[4] has published data bearing on this point. He followed simultaneously eleven loci in the X-chromosome of Drosophila, and obtained some double crossover classes that according to Castle's model are impossible with a single plane. However, if the loci all reduce to the curved line discussed above, than any so-called double crossover is possible with a single separation plane. But those classes now termed triple and quadruple crossovers are all impossible with a single plane of crossing over. In the case of the X-chromosome over a dozen triple crossovers have been observed, and in the case of the second chromosome they are so frequent that 131 were observed in a single experiment. It follows that, even if the genes are not arranged in a straight line, the occurrence of double crossing over is an established fact, not an unproved hypothesis.

If crossover planes do not occur longitudinally, or occur thus less often than transversely, it is difficult to see how the distances apart of the loci in Castle's model can be proportional to the crossover values, except in the case where

the line joining the loci is perpendicular to the average direction of crossover planes. The crossover value for lines in any other directions would seem to be proportional to their *projections* on such a perpendicular. In short, both of Castle's alternative subsidiary hypotheses to account for the rarity of double crossovers are inconsistent with his primary assumption that distances are proportional to crossover values.

[1] These PROCEEDINGS, 5, 1919, (25).

[2] Abnormal is a character that is separable from wild-type only under certain environmental conditions; and is also simulated by accidental abnormalities. It is, therefore, not a reliable character to use in linkage experiments.

[3] *Carnegie Inst., Washington, Pub.*, No. 237, 1916.

[4] Sturtevant, these PROCEEDINGS, 3, 1917, (555), and Muller, *Amer. Naturalist*, 50, 1916, (193, 284, 350, 421).

[5] Bridges, *J. Exp. Zool.*, 19, 1915, (1), and Plough, *Ibid.*, 24, 1917, (147).

[6] Bridges and Sturtevant, *Biol. Bull.*, 26, 1914, (205), and Morgan and Bridges, loc. cit., note 3.

[7] That Castle tacitly recognized this is shown by the fact that he omitted lethal 2 from his model, without even mentioning the fact. It is obvious from an examination of the data in this case that the crossover values are inconsistent with those obtained in other experiments not involving lethal 2.

[8] Provided the relations of the loci are to be represented by straight lines joining them and proportional in length to the corresponding crossover values.

[9] It does not follow that 4.7 is to be taken as the locus of bifid, for the mean value indicated by all bifid experiments is the one most likely to coincide with future experiments; i.e., to give the best predictions.

[10] Castle concluded that the white forked value of 45.7, used by him, is somewhat too high, which is true; but there were available to him more than 40,000 flies (Bridges, *Genetics*, 1, 1916, (1), and Weinstein, *Ibid.*, 3, 1918, (135)) giving the lower value, in contrast to the less than 4000 flies on which the high value was based.

A COMPLETE APPARATUS FOR ABSOLUTE ACOUSTICAL MEASUREMENTS

By Arthur Gordon Webster

Clark University, Worcester, Mass.

Read before the Academy, November 17, 1915

The apparatus here briefly described is the result of researches begun many years ago in the attempt to solve the problem of measuring the intensity of sound at any given point of space in terms of absolute units, by means of instruments that may be reproduced from specifications, and that shall be convenient and portable. For this purpose three things are requisite; first, a source of sound that shall continuously produce a simple tone of known intensity—this will be denoted by the term 'phone;' second, an instrument for measuring in absolute units a constantly maintained simple tone, here called

a 'phonometer;' third, in order to check the theory of the two instruments a series of experiments on the propagation of sound from one to the other, free from the effect of disturbing bodies. Such experiments must be made either in a room so padded as to absorb all sound, which is at present impossible, or else out of doors at a distance from all objects except the flat surface of the earth, of which the coefficient of reflection is measured. This has been the method here pursued. While many observers have attacked one or both of the instrumental problems, it is not known that any has previously attacked all three parts above mentioned.

1. Phonometer.—In order to attain the required sensitiveness for the small amount of energy concerned, it is necessary to use the principle of resonance, and a coupled system of two degrees of freedom is used, the first consisting of a tunable air resonator, the second, of a tuned diaphragm which is to make the motion visible. Inasmuch as the sensitiveness will ultimately depend on the damping intrinsically residing in the diaphragm, it is desirable to reduce this as much as possible. For many years the best thing found was a glass diaphragm, tuned by adding weights. Finally it was found that mica diaphragms could be had having a smaller damping than glass. Finally, however, after an attempt to measure the loudness of some of the fog-signals on the Maine coast, all of which have different pitches, it was found necessary to have a diaphragm susceptible of gradual tuning, which could notbe obtained by variation of mass. Accordingly the instrument was entirely redesigned, and the tuning was done by varying the potential, rather than the kinetic energy of the diaphragm, by stiffening it with a string tuned by tension. And now the chief improvement was made of abolishing the diaphragm, and since the resonator has a hole for the sound to enter, it was determined to replace the diaphragm by a rigid piston placed centrally in the hole, leaving an annular aperture for the sound to enter, and at the same time freeing us from the necessity of using the calculated equivalent area of the diaphragm. These improvements are embodied in the instrument shown in Figs. 1 and 2.

The phonometer is shown in elevation in Fig. 1, where 1 is the cylindrical resonator, sliding in the cylindrical piece screwed to the main casting A, resting on two feet and the cylinder 8. The end of the resonator is closed by a glass window 15. For years all measurements were made by the Michelson interferometer viewed stroboscopically, as it was desired to have the diaphragm absolutely free to move. For purposes of portability and convenience this has now been given up, and replaced by observing in a telescope the displacement of a mirror, as in Max Wien's phonometer. The interferometer, 14, (not shown) is attached temporarily at the front, or receiving end, to check the indications of the mirror. In the micrometer eyepiece, 2, adjustable with five degrees of freedom, is observed the image of the filament of the small lamp 4, fed through a key 5 by three dry cells, 10, placed in the cylindrical holder upon which the tube 7 is carried by a bayonet joint. The

vertical image of the filament is drawn out into a horizontal band. Details are shown in Fig. 2. The aluminium disk 12 is hung in the center of the hole in the front plate by means of the three steel wires clamped at one end by the spider 13, and at the other wound on the pins 8, turned by a key from the

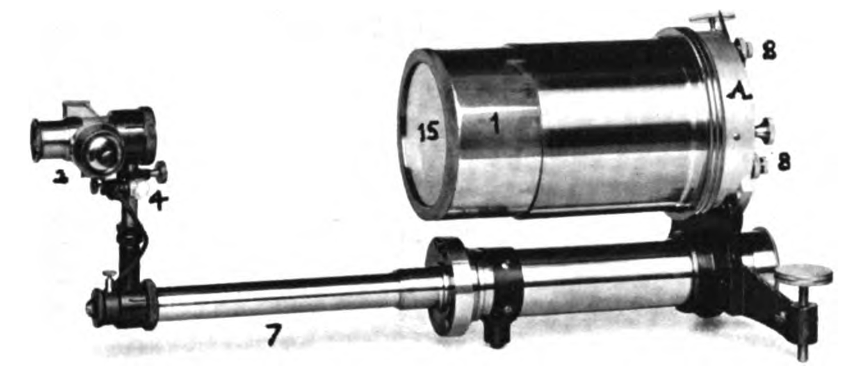
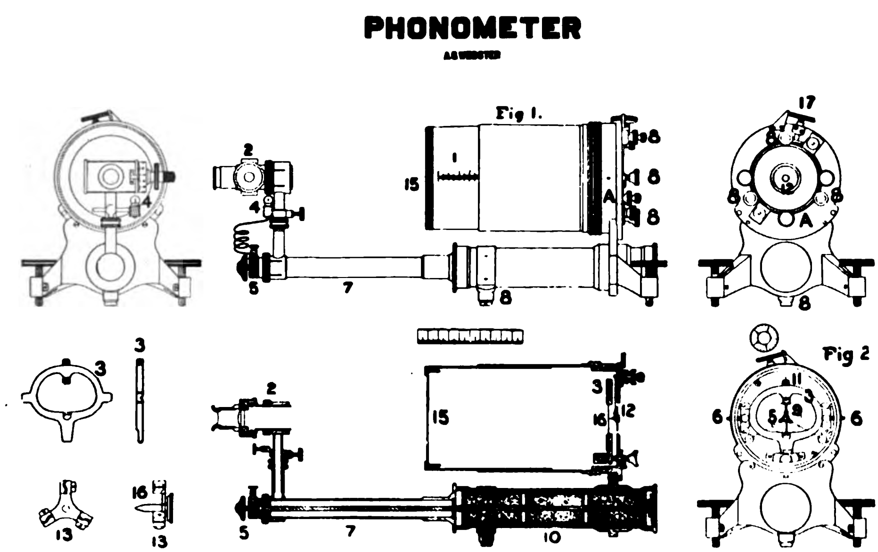


FIG. 1 A



FIGS. 1 and 2

outside, one controlled micrometrically by a lever and screw 17, by which the fine tuning is done. The wires pass over either one of two sets of three bridges cast on the front plate. The objective of the telescopic observing device is

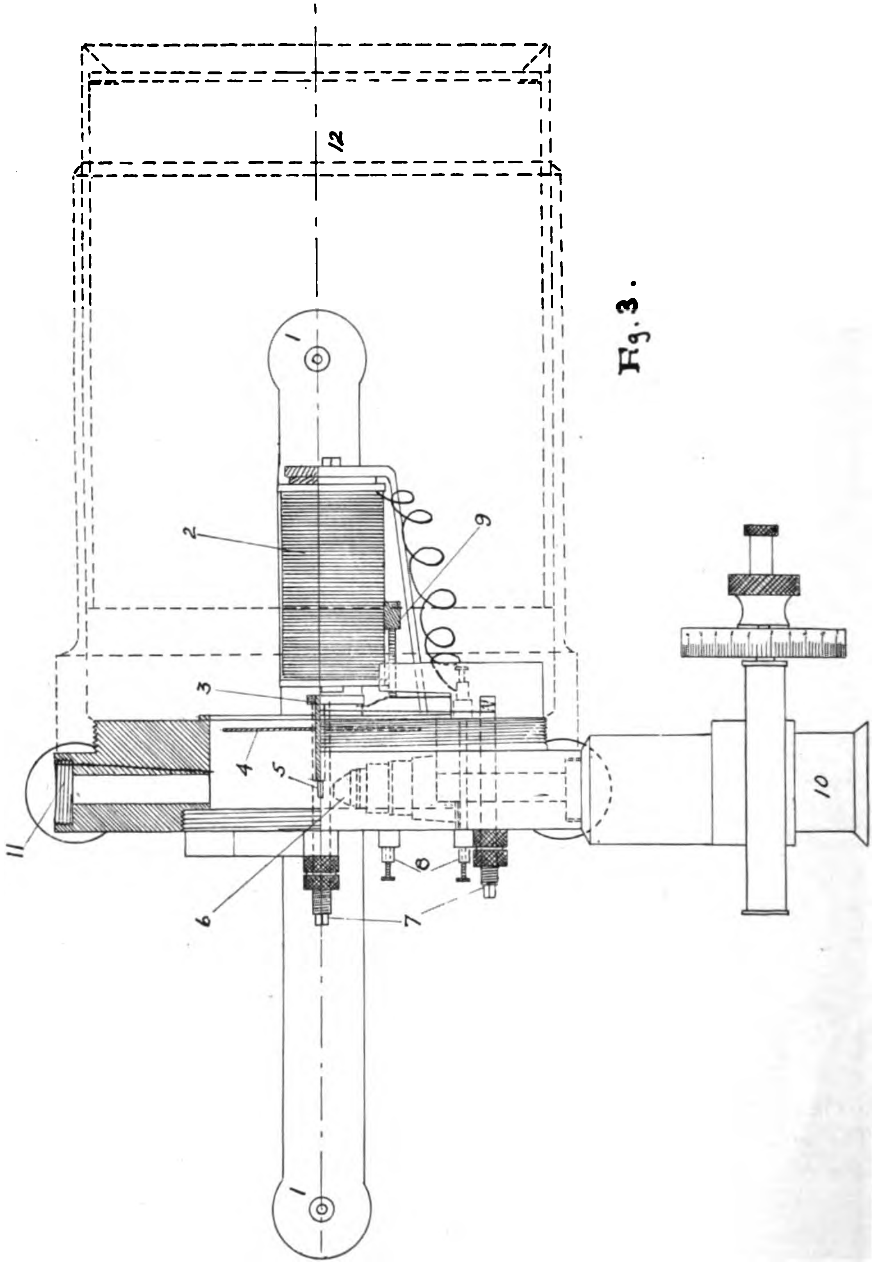


Fig. 3.

the small concave mirror 5, the back of which constitutes its own short lever, being directly pushed by a point 16 carried by the spider 13, which carries the disk 4. Instead of being pivoted in jewels, the mirror is carried by a torsion strip 9, cut from thin sheet steel with a square side projection upon which the mirror 5 is cemented. The strip is held by end clamps, and its tension may be adjusted by a screw 11, bearing on the spring 10, (inside). Two micrometric adjustments are secured by the strip being carried on a rocker 3, pivoted on two screws 6, allowing of sidewise displacement in order

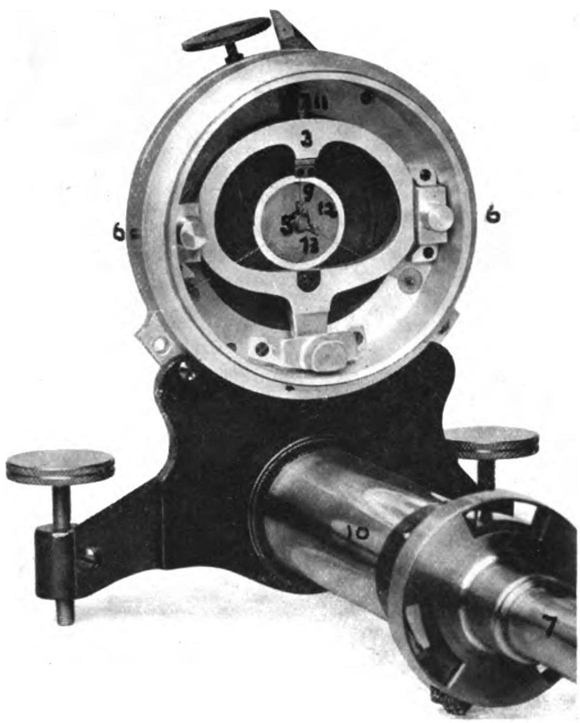


FIG. 2A

to change the lever arm of mirror 5, and thus the magnification of the motion. The motion of the image sidewise in the field of the ocular is obtained by a slow motion of the rocker, controlled by a screw at its lower end, accessible from outside. All other adjustments of the image are made at the ocular end. With the usual leverage of one-fourth to one-third of a millimeter between the point and the axis of the mirror, and a distance of forty centimeters to the reticule in the ocular, a magnification of about 2400 is obtained, and as one can read to one-tenth of a millimeter on the reticule, we may detect a dis-

placement of 1/24,000 millimeter of the vibrating disk, which is of the same order as could be used with the interferometer, and far better than could be done with a microscope. This phonometer is as sensitive as the normal ear (but for a very limited range).

2. *Phone*.–The phone, after having gone through many different forms, has now assumed a form closely connected with the phonometer, particularly as to the possibility of tuning. Fig. 3 shows the form now chiefly used. The disk 3, placed centrally in the hole of the resonator, supported in its wires and tuning pins 7, is actuated by an electromagnet 9, the current of which is inter-

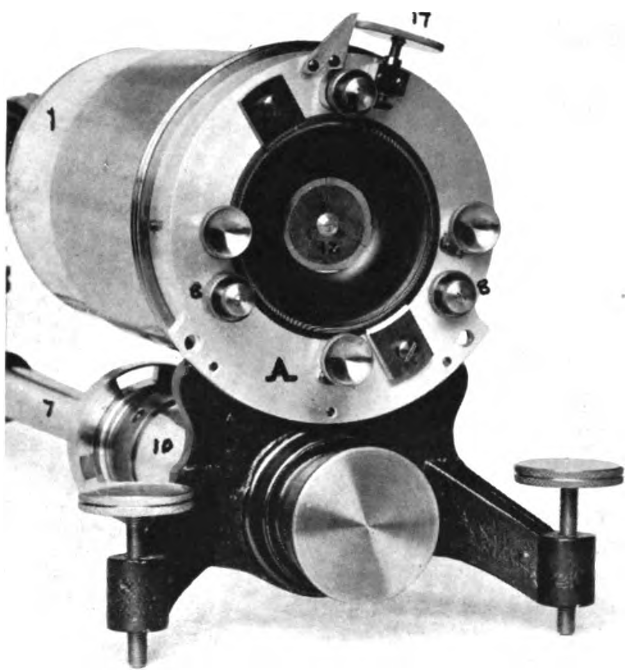


FIG. 2B

rupted by a separate electrically-maintained tuning-fork with a platinum point dipping into mercury. If exact purity of sound is not important, and the slight fizzing always associated with a dry contact is unobjectionable, a spring interrupter may be used, and the phone is self-contained, but generally it is better to have a set of forks to be used in the interchangeable stand, to control the phone at various pitches.

During the last year I have made the acquaintance of the pliotron and found it to be a wonderfully convenient instrument for the production of

harmonically varying currents. I accordingly adopted it instead of the interrupting tuning fork, inasmuch as it could be instantaneously tuned to any pitch whatever, and we are therefore freed from the necessity of being confined to four or five pitches, but have perfectly continuous range. The intensity of the sound emitted is measured by the amplitude of vibration of the disk, read off by the micrometer 10, focused on a fine slit ruled in the silvering of a piece of cover-glass 5 carried by the disk, and illuminated by an incandescent lamp and condenser not shown. It may be remarked that this phone as it stands may be used as a phonometer for tolerably loud sounds, such as those of singing near by, which gives a considerable vibration in the microscope. It should be likewise stated that this phone produces sound more efficiently than any other known instrument.

The *theory* of the phonometer and the phone is given in another paper.

NATIONAL RESEARCH COUNCIL

EXTRACT OF MINUTES OF JOINT MEETINGS OF THE EXECUTIVE BOARD OF THE NATIONAL RESEARCH COUNCIL WITH THE COUNCIL OF THE NATIONAL ACADEMY OF SCIENCES

AT THE NATIONAL RESEARCH COUNCIL BUILDING

MEETING OF JANUARY 14, 1919, AT 10 A.M.

Present: Messrs. Abbot,* Bogert, Clevenger, Cross,* Hale,* Howe, Howell,* Hussey, Johnston, Mendenhall, Michelson,* Millikan, Noyes,* Russell, Walcott,* Welch, Woods, Yerkes, and by invitation, Leuschner and Nichols.

Mr. Hale presided during the first part of the meeting while the business of the National Research Council was under consideration.

The minutes of the meetings of the Executive Board, December 21, 1918, and of the Interim Committee, December 31, 1918, and January 6–7, 1919, were approved as circulated.

Mr. Hale presented a report of the Committee on the Patent Office, appointed December 4, 1917, L. H. Baekeland, Acting Chairman. The report was duly considered and approved and it was

Moved: That the necessary funds be set aside for printing the report of the Committee on the Patent Office in full. (*Adopted.*)

The Chairman presented the resignation of Mr. S. L. G. Knox as Scientific Attaché to the Embassy in Rome, in charge of the Research Information Service in Italy and it was

Moved: That the resignation of Mr. S. L. G. Knox be accepted and that the National Research Council express its appreciation of the important services which he has rendered to the Council and convey to him a vote of thanks. (*Adopted.*)

Moved: That the Committee on Relations with Educational Institutions be increased to include as members Mr. S. P. Capen of the Bureau of Education, Mr. R. M. Yerkes, Mr. A. O. Leuschner, and Mr. H. E. Gregory. (*Adopted.*)

The Chairman called upon Mr. Leuschner to state the situation as to the Smith-Howard Bill for the promotion of research in connection with educational institutions, which is now under consideration by Congress. After discussion it was

Moved: That a Committee of seven with full power to act be appointed by the Chairman to consider the whole question of a Bill in Congress to provide appropriations for research, and that the President of the National Academy of Sciences and the Chairman of the National Research Council be members of the Committee. (*Adopted.*)

*Members of the Council of the National Academy of Sciences.

At this point Mr. Walcott took the chair and Mr. Hale presented a draft of a proposed Constitution of the National Research Council. After discussion of this draft it was

Moved: That the draft o the Constitution be referred back to the Committee with the request that it report at the next meeting, and that copies of the draft of the Constitution as revised be circulated among the members of the Council of the National Academy of Sciences and of the Executive Board of the National Research Council, prior to the meeting when its adoption is to be considered. (*Adopted.*)

The meeting adjourned at 1.00 pm.

PAUL BROCKETT, *Assistant Secretary.*

MEETING OF FEBRUARY 11, 1919, AT 9.30 A.M.

Present: Messrs. Clevenger, Cross,* Hale,* Howe, Hussey, Johnston, Manning, Millikan, Noyes,* Pupin,* Stratton, Walcott,* Welch, Woods, Woodward, and by invitation, Leuschner. Mr. Hale in the chair.

The minutes of the meeting of the Council of the National Academy of Sciences, on December 30, 1918, and of the joint meeting of the Executive Board of the National Research Council and the Council of the National Academy of Sciences, on January 14, 1919, were approved, as were the minutes of the several meetings of the Interim Committee, on January 21, 30, and February 5, 1919, except that two motions of February 5 relative to publication were amended to read as follows:

That the publication of a series of bulletins on the general activities of the National Research Council be authorized, and that the matter be left to the Chairman and Secretary with power.

That accounts of scientific work done under the auspices of the National Research Council shall in general be published in scientific periodicals, provided that the connection of the Council with the work is acknowledged in a footnote or otherwise; and that the Council procure, and pay the cost of, a sufficient number of reprints of such articles.

The Chairman presented a revised plan of Organization of the National Research Council which was discussed in detail.

Moved: That the Executive Board of the National Research Council approve and recommend to the Council of the National Academy of Sciences the adoption of the Organization of the National Research Council as presented. (*Adopted.*)

The organization as finally adopted is given on page 184.

The Council of the National Academy of Sciences and the Executive Board of the National Research Council adjourned at 1.08 p. m.

* Members of the Council of the National Academy of Sciences.

The Council of the National Academy of Sciences resumed session at 1.09 p.m., Mr. Walcott in the chair.

The President presented the Organization of the National Research Council recommended by the Executive Board for adoption by the Council of the National Academy.

Moved: That the organization as submitted and approved by the Executive Board be approved by the Council of the National Academy of Sciences. (*Adopted.*)

The meeting then adjourned.

The Council of the National Academy of Sciences and the Executive Board of the National Research Council reassembled in joint session at 2.45 p.m.

Present: Messrs. Clevenger, Cross,* Hale,* Howe, Hussey, Johnston, Millikan, Noyes,* Pupin,* Walcott,* Woods, and by invitation, Leuschner. Mr. Noyes in the chair.

The Chairman of the National Research Council presented the following communication from Mr. Theodore N. Vail, President of the American Telephone and Telegraph Company:

The Bell Telephone System, composed of the American Telephone and Telegraph Company, its associated and operating, and the Western Electric companies acting through the American Telephone and Telegraph Company, guarantees to the National Research Council $25,000 a year if not less than $250,000 a year for five years be raised for the use of the Council.

The Bell Telephone System to have the benefit of the advice, assistance and coöperation of the Council in all matters relating to its industry.

Moved: That all matters relating to publication and publicity be referred to, and carried on under, the Research Information Service. (*Adopted.*)

Moved: That the scale of annual salaries of the National Research Council be as follows: Chairman of the National Research Council, $10,000; Chairmen of Divisions of Science and Technology, $6000, with an additional allowance of $1000 for traveling expenses if they are sent abroad by the Council; Scientific Attachés, $6000, until the Council is in a position to pay more. (*Adopted.*)

Moved: That Mr. J. C. Merriam be elected Acting Chairman of the National Research Council from March 15 until a new Chairman takes office. (*Adopted.*)

Moved: That Major R. M. Yerkes be elected Chairman of the Research Information Service. (*Adopted.*)

Moved: That the resignation of Mr. Graham Edgar as Secretary of the Washington office of the Research Information Service be accepted with regret; and that the Council express to him its appreciation of his work. (*Adopted.*)

Moved: That Mr. G. S. Fulcher be elected as Secretary of the Washington office of the Research Information Service, his appointment to date from February 1, 1919. (*Adopted.*)

The resignation of Mr. Yerkes as Chairman of the Psychology Committee was received and accepted with the understanding that it is to go into effect at the time of the organization of the Division of Anthropology and Psychology.

* Members of the Council of the National Academy of Sciences.

Moved: That the Council express its appreciation of the very valuable work which Mr. Yerkes has done as Chairman of the Psychology Committee and in connection with the organization of the Division of Psychology of the Surgeon General's Office, and convey its thanks to him. (*Adopted.*)

Moved: That the resignation of Mr. H. A. Bumstead as Scientific Attaché to the American Embassy in London, in charge of the Research Information Service in Britain, be accepted with regret; and that the National Research Council express appreciation of the admirable work which he has done and convey its thanks to him. (*Adopted*).

Moved: That Mr. C. E. Mendenhall be elected to take the place of Mr. Bumstead as Scientific Attaché to the American Embassy in London. (*Adopted.*)

Moved: That the resignation of Rear-Admiral Roger Welles from the Research Information Service be accepted and that an expression of thanks be extended to him for the able service which he has rendered the National Research Council. (*Adopted.*)

Moved: That the Executive Board request the President of the National Academy to ask the President of the United States to designate Captain George W. Williams to take the place of Admiral Welles on the Research Information Service. (*Adopted.*)

Moved: That a vote of thanks be extended to Major-General W. C. Gorgas, Surgeon General U. S. A., for the valuable service which he has rendered to the National Research Council. (*Adopted*).

Moved: That the President of the National Academy of Science be requested to ask the President of the United States to designate Surgeon General M. W. Ireland to take the place of General Gorgas on the Research Council. (*Adopted.*)

On recommendation of Secretary Johnston of the National Research Council it was

Moved: That the following allotments be authorized from the appropriation from the President's emergency fund of $61,000 available for expenses of the Council from January 1 to June 30, 1919: General and Office Administration, $33,000; Research Information Service, $20,000. (*Adopted.*)

The Chairman of the Engineering Division *moved:* That Mr. Clevenger be elected Vice Chairman of the present Division. (*Adopted.*)

The Chairman of the Engineering Division *moved:* That the organization of a Manganese Saving Committee be authorized, with Mr. J. R. Cain as Chairman; and that an allotment of $150 from the funds available for the Engineering Division be placed at the disposal of this Committee. (*Adopted.*)

The Vice-chairman of the Division of Medical Sciences made the following statements concerning grants recommended by the Executive Committee of the Division:

Dr. C. A. Kofoid, representing the section on Protozoology of the Medical Zoology Committee of this Division has been engaged in examining troops at Hoboken returning from France, studying the flora of human intestinal protozoa. This work has been interfered with on account of the changes in the status of personnel due to demobilization. It is important that this work should be continued for two essential reasons: such an opportunity for so complete a study will hardly present itself again in the near future, and we have no treatise at present which deals with any comprehensive, extensive study of the flora of parasites in the human intestines; also the results of this work will determine whether our troops acquired foreign parasites. A sufficient number of men who did not go to France are being examined for control. To continue this work it is necessary to have a permanent assistant. Dr. Kofoid has requested that we appropriate funds sufficient to pay the salary of such an

assistant, and further requests that Dr. Olive Swazey of the Department of Zoology at the University of California be appointed at a salary of $2100 per annum and traveling expenses from the University of California to Hoboken. The Executive Committee of the Medical Division recommends this appropriation for one year.

A grant of $500 has been approved to support investigations on measles now in progress at Camp Devens conducted by Dr. Sellards. This amount is to be paid to Colonel F. F. Russell to be expended at his discretion.

Moved: That the above recommendations of the Division of Medical Sciences be approved. (*Adopted.*)

Mr. Hale *moved:* That the President of the American Association for the Advancement of Science, or a representative of the Association, be made ex-officio a member of the Executive Board. (*Adopted.*)

Moved: That the Executive Board of the National Research Council recommend to the Council of the National Academy of Sciences that authority be given for the inclusion of the following as a Section in the Plan of Organization of the National Research Council.

Article VIII—Amendment of the Organization

Section 1. This organization may be amended by a two-thirds vote of the Council of the National Academy of Sciences at any stated meeting, provided that notice of the proposed amendment has been given to each member of the Council of the National Academy at least thirty days in advance of the stated meeting. (*Adopted.*)

The meeting adjourned at 4 p.m.

Paul Brackett, *Assistant Secretary.*

ORGANIZATION OF THE NATIONAL RESEARCH COUNCIL

PREAMBLE

The National Academy of Sciences, under the authority conferred upon it by its charter enacted by Congress, and approved by President Lincoln on March 3, 1863, and pursuant to the request expressed in an Executive Order made by President Wilson on May 11, 1918, hereto appended, adopts the following permanent organization for the National Research Council, to replace the temporary organization under which it has operated heretofore.

ARTICLE I. PURPOSE

It shall be the purpose of the National Research Council to promote research in the mathematical, physical, and biological sciences, and in the application of these sciences to engineering, agriculture, medicine, and other useful arts, with the object of increasing knowledge, of strengthening the national defense, and of contributing in other ways to the public welfare, as expressed in the Executive Order of May 11, 1918.

ARTICLE II. MEMBERSHIP

Section 1. The membership of the National Research Council shall be chosen with the view of rendering the Council an effective federation of the principal research agencies in the United States concerned with the fields of science and technology named in Article I.

Section 2. The Council shall consist of

1. Representatives of national scientific and technical societies;
2. Representatives of the Government, as provided in the Executive Order;
3. Representatives of other research organizations and other persons whose aid may advance the objects of the Council.

ARTICLE III. DIVISIONS

Section 1. The Council shall be organized in Divisions of two classes:

A. Divisions dealing with the more general relations and activities of the Council;

B. Divisions dealing with related branches of science ond technology.

Section 2. The initial constitution of the Divisions of the Council shall be as follows:

A. Divisions of General Relations

I. Government Division,
II. Foreign Relations Division,
III. States Relations Division,
IV. Educational Relations Division,
V. Industrial Relations Division,
VI. Research Information Service,

B. Divisions of Science and Technology

VII. Division of Physical Sciences,
VIII. Division of Engineering,
IX. Division of Chemistry and Chemical Technology,
X. Division of Geology and Geography,
XI. Division of Medical Sciences,
XII. Division of Biology and Agriculture,
XIII. Division of Anthropology and Psychology.

Section 3. The number of divisions and the grouping of subjects in Article III, Section 2, may be modified by the Executive Board of the National Research Council.

Section 4. The Divisions of General Relations shall be organized by the Executive Board of the National Research Council (Article IV, Section 2).

Section 5. To secure the effective federation of the principal research agencies in the United States, provided for in Article II, a majority of the members of each Division in the group of Science and Technology shall consist of representatives of scientific and technical societies, chosen as provided for in Article V, Section 2. The other members of the Division shall be nominated by the Executive Committee of the Division, approved by the Executive Board of the National Research Council, and appointed in accordance with Article V, Section 4.

Section 6. The Divisions of the Council, with the approval of the Executive Board, may establish sections and committees, any of which may include members chosen outside the membership of the Council.

ARTICLE IV. ADMINISTRATION

Section 1. The affairs of each Division shall be administered by a Chairman, a Vice-Chairman, and an Executive Committee, of which the Chairman and the Vice-Chairman shall be ex-officio members, all of whom shall be elected annually by the Division and confirmed by the Executive Board.

Section 2. The affairs of the National Research Council shall be administered by an Executive Board, of which the President and Home Secretary of the National Academy of Sciences, the President of the American Association for the Advancement of Science, the chairmen and vice-chairmen of the Divisions of Science and Technology, and the chairmen of the Divisions of General Relations shall be ex-officio members. The Executive Board may elect additional members, not to exceed ten in number, who, if not already members of the National Research Council, shall be appointed thereto, in accordance with Article V, Section 4.

Section 3. The officers of the National Research Council shall consist of a Chairman, one or more Vice-Chairmen, a Secretary, and a Treasurer, who shall also serve as members and officers of the Executive Board of the Council.

Section 4. The officers of the National Research Council, excepting the Treasurer, shall be elected annually by the Executive Board. The Treasurer of the National Academy of Sciences shall be ex-officio Treasurer of the National Research Council.

Section 5. The duties of the officers of the Council and of the Divisions shall be fixed by the Executive Board.

ARTICLE V. NOMINATIONS AND APPOINTMENTS

Section 1. The government bureaus, civil and military, to be represented in the Government Division, and the scientific and technical societies, to be represented in the Divisions of Science and Technology of the National Research Council, shall be determined by joint action of the Council of the National Academy of Sciences and the Executive Board of the National Research Council.

Section 2. Representatives of scientific and technical societies shall be nominated by the societies, at the request of the Executive Board, and appointed by the President of the National Academy of Sciences to membership in the Council and assigned to one of its divisions.

Section 3. The representatives of the government shall be nominated by the President of the National Academy of Sciences after conference with the Secretaries of the Departments concerned, and the names of those nominated shall be presented to the President of the United States for designation by him for service with the National Research Council.

Section 4. Other members of the Council shall be nominated by the Executive Committees of the Divisions, approved by the Executive Board, and appointed by the President of the National Academy of Sciences to membership and assigned to one of the Divisions.

Section 5. Prior to the first annual meeting of the Council following January 1, 1919, all Divisions shall be organized by appointment of their members in accordance with Article II and Article V, Sections 1 to 4.

Section 6. As far as practicable one-third of the original representatives of each scientific and technical society and approximately one-third of the other

original members of each of the Divisions of Science and Technology shall serve for a term of three years, one-third for a term of two years, and one-third for a term of one year, their respective terms to be determined by lot. Each year thereafter, as the terms of members expire, their successors shall be appointed for a period of three years.

Section 7. The government representatives shall serve for periods of three years, unless they previously retire from the government office which they represent, in which case their successors shall be appointed for the unexpired term.

Section 8. As far as practicable a similar rotation shall be observed in the appointment of the members of the Divisions of General Relation.

ARTICLE VI. MEETINGS

Section 1. The Council shall hold one stated meeting, called the annual meeting, in April of each year, in the city of Washington, on a date to be fixed by the Executive Board. Other meetings of the Council shall be held on call of the Executive Board.

Section 2. The Executive Board and each of the Divisions shall hold an annual meeting, at which officers shall be elected, at the time and place of the annual meeting of the Council, unless otherwise determined by the Executive Board, and such other meetings as may be required for the transaction of business.

Section 3. Joint meetings of the Executive Board of the National Research Council and the Council of the National Academy of Sciences shall be held from time to time, to consider special requests from the government, the selection of organizations to be represented in the National Research Council, and other matters which, in the judgment of the President of the National Academy, requires the attention of both bodies.

ARTICLE VII. PUBLICATIONS AND REPORTS

Section 1. An annual report on the work of the National Research Council shall be presented by the chairman to the National Academy of Sciences, for submission to Congress in connection with the annual report of the President of the Academy.

Section 2. Other publications of the National Research Council may include papers, bulletins, reports, and memoirs, which may appear in the Proceedings or Memoirs of the National Academy of Sciences, in the publications of other societies, in scientific and technical journals, or in a separate series of the Research Council.

EXECUTIVE ORDER ISSUED BY THE PRESIDENT OF THE UNITED STATES, MAY 11, 1918

The National Research Council was organized in 1916 at the request of the President by the National Academy of Sciences, under its Congressional charter, as a measure of national preparedness. The work accomplished by the Council in organizing research and in securing co-operation of military and civilian agencies in the solution of military problems demonstrates its capacity for larger service. The National Academy of Sciences is therefore requested to perpetuate the National Research Council, the duties of which shall be as follows:

1. In general, to stimulate research in the mathematical, physical and biological sciences, and in the application of these sciences to engineering, agriculture, medicine, and other useful arts, with the object of increasing knowledge, of strengthening the national defense, and of contributing in other ways to the public welfare.
2. To survey the larger possibilities of science, to formulate comprehensive projects of research, and to develop effective means of utilizing the scientific and technical resources of the country for dealing with these projects.
3. To promote co-operation in research, at home and abroad, in order to secure concentration of effort, minimize duplication, and stimulate progress; but in all co-operative undertakings to give encouragement to individual initiative, as fundamentally important to the advancement of science.
4. To serve as a means of bringing American and foreign investigators into active co-operation with the scientific and technical services of the War and Navy Departments and with those of the civil branches of the Government.
5. To direct the attention of scientific and technical investigators to the present importance of military and industrial problems in connection with the war, and to aid in the solution of these problems by organizing specific researches.
6. To gather and collate scientific and technical information at home and abroad, in co-operation with Governmental and other agencies and to render such information available to duly accredited persons.

Effective prosecution of the Council's work requires the cordial collaboration of the scientific and technical branches of the Government, both military and civil. To this end representatives of the Government, upon the nomination of the President of the National Academy of Sciences, will be designated by the President as members of the Council, as heretofore, and the heads of the departments immediately concerned will continue to co-operate in every way that may be required.

WOODROW WILSON.

The White House,
May 11, 1918.

MINUTES OF JOINT MEETING OF THE EXECUTIVE BOARD AND THE COUNCIL OF THE NATIONAL ACADEMY OF SCIENCES

AT THE NATIONAL RESEARCH COUNCIL BUILDING

MARCH 11, 1919, AT 9.45 A.M.

Present: Messrs. Abbot,* Bogert, Bumstead, Clevenger, Cross,* Hale,* Howe, Hussey, Johnston, Leuschner, Manning, Michelson,* Millikan, Noyes,* Pupin,* Walcott,* Washburn, Woods, Yerkes.

Minutes of the joint meeting of February 11, as circulated by mail, were approved.

Mr. Hale presented a cablegram from Dr. Schuster, Secretary of the Royal Society, in regard to the admission of neutral nations to the International Research Council.

Moved: That the Council of the National Academy of Sciences and the Executive Board of the National Research Council recommend to the International Research Council that, on the conclusion of peace, nations with sufficient scientific development, which have been neutral during the war, be invited to join the International Research Council. (*Adopted.*)

Moved: That the foregoing resolution be submitted to the National Academy of Sciences for approval. (*Adopted.*)

It was the sense of the joint meeting that the International Research Council should be requested to take action at an early date in the matter of inviting such neutrals to join the Council.

Upon motion of Mr. Hale, acting upon the suggestion of Mr. Elihu Root, Resolved: The Council of the National Academy of Sciences recommends to the Academy the addition of the following paragraph to Section 2, Article II, of the Constitution of the Academy: "It shall be competent for the Council to call in experts or men of special attainments not members of the Academy, to aid, under special permanent organizations, in promoting the objects of the Academy."

Mr. Hale, member for the United States of the Executive Committee of the International Research Council, reported that in accordance with a resolution passed at the November meeting of the International Research Council in Paris, a tentative plan for the formation of an American Section of the proposed International Astronomical Union had been adopted with the approval of the President of the National Academy of Sciences and that the Section had organized under the tentative plan at a meeting held in Washington. The tentative organization provides that the initial membership shall include representatives of organizations as follows:

National Academy of Sciences	5
American Astronomical Society	10
American Physical Society	3

* Members of the Council of the National Academy of Sciences.

American Mathematical Society	3
U. S. Naval Observatory	1
U. S. Nautical Almanac Office	1
U. S. Coast and Geodetic Survey	1

Messrs. Hale and Leuschner then presented the minutes of the organization meeting of the Section, as printed on page 193.

Moved: That the Council of the National Academy of Sciences and the Executive Board of the National Research Council approve the provisional organization of the American Section of the International Astronomical Union. (*Adopted.*)

Mr. Leuschner then presented the following recommendation of the Division of Physical Sciences:

Moved: That in accordance with its offer to serve as the agency in Astronomy of the Council, the American Section of the International Astronomical Union be constituted a Section on Astronomy of the Division of Physical Sciences. (*Adopted.*)

Mr. Leuschner reported that in harmony with a further resolution of the International Research Council, a committee consisting of Messrs. R. S. Woodward, L. A. Bauer, William Bowie, A. O. Leuschner, Whitman Cross, and C. F. Marvin, had been appointed to consider the question of the tentative organization of an American Section of the proposed International Geophysical Union, and that a report submitted by this committee to the Chairman of the Council had been carefully considered by the Executive Committee of the Division of Physical Sciences which had voted to submit the following recommendations:

That the tentative formation of an American Section of the proposed International Geophysical Union be approved, with the provision that the subjects to be included and their grouping be determined by the Section after its organization.

That the question of the formation of an American Geophysical Society be referred to the Section.

Mr. Leuschner then presented the following provisional nominations on behalf of the Executive Committee of the Division of Physical Sciences for membership in the Section:

Messrs. S. J. Barnett, Joseph Barrell, L. A. Bauer, H. B. Bigelow, W. R. Blair, E. H. Bowie, William Bowie, J. C. Branner, E. W. Brown, Whitman Cross, R. A. Daly, A. L. Day, J. F. Hayford, W. J. Humphreys, A. O. Leuschner, G. W. Littlehales, G. F. MacEwen, C. F. Marvin, Max Mason, A. G. McAdie, A. A. Michelson, R. A. Millikan, F. R. Moulton, H. F. Reid, Frank Schlesinger, R. B. Sosman, J. T. Watkins, H. O. Wood, R. S. Woodward.

Moved: That these recommendations and provisional nominations of the Division of Physical Sciences be approved. (*Adopted.*)

Mr. Hale brought to the attention of the meeting the question of a popular journal to be published jointly by the National Academy of Sciences and the American Association for the Advancement of Science, and stated that the matter would be brought up later for further consideration.

Mr. Yerkes brought up for consideration the question of securing funds for the preparation of a suitable scale for the mental rating of school children, and stated that the General Education Board was prepared to make an appropriation of $25,000 to meet the expenses of the work if requested to do so by the National Research Council. On motion, the Chairman of the Council was authorized to make such application to the General Education Board.

Mr. Noyes *moved:* That the Executive Board approve the undertaking by the Council of a project of which the broad purpose is to promote fundamental research in physics and chemistry in educational institutions—the project to consist primarily in the organization and maintenance of a system of National Research Fellowships to be awarded to persons who have already demonstrated a high order of ability in research, for the purpose of enabling them to carry on investigations at educational institutions making adequate provision for the prosecution of research in physics and chemistry. (*Adopted.*)

The constitution of a Board to formulate and administer this project was approved, with the proviso that if any member cannot give active service to the work his name shall be withdrawn and another substituted. It was also agreed that Mr. Hale be asked to act temporarily as Chairman.

Mr. Hale proposed that the Division of Foreign Relations include a representative each of the State Department, the National Academy of Sciences, the American Association for the Advancement of Science, the American Philosophical Society, the American Academy of Arts and Sciences; Honorable Elihu Root, former Secretary of State; the Chairmen of the National Research Council and of the Research Information Service; together with one representative each of leading international scientific and technical organizations in which the United States takes part, such as the International Surgical Association, American Section of the International Astronomical Union, the American Section of the International Geophysical Union, International Bureau of Weights and Measures, etc.

Moved: That the foregoing plan of organization be approved, that the Chairman of the National Research Council be authorized to proceed with its completion; and that the Foreign Secretary of the National Academy be the Chairman of the Division of Foreign Relations. (*Adopted.*)

Mr. Howe *moved:* That all committees of the various Divisions serve until the annual meeting and that the list of committees and their membership be revised each year at the annual meeting of the Executive Board, in accordance with recommendations to be submitted by the Divisions. (*Adopted.*)

Moved: That the questions of this year's annual meeting, of the National Research Council, of the beginning of the financial year, and of the date of appointment of members, be referred to the Interim Committee, with power to act. (*Adopted.*)

Moved: That consideration of the following amendment to the Organization of the National Research Council be postponed until the next stated meeting of the Council of the National Academy of Sciences:

ARTICLE VIII—AMENDMENT OF THE ORGANIZATION

Section 1. This organization may be amended by a vote of the Council of the National Academy of Sciences at any stated meeting, provided that notice of the proposed amendment has been given to each member of the Council of the National Academy at least thirty days in advance of the stated meeting. (*Adopted.*)

Moved: That the Budget Committee be directed to confer with the Finance Committee of the National Academy of Sciences and with Mr. Gano Dunn concerning a plan for the administration of the financial affairs of the National Research Council under the provision of the new organization by which "the Treasurer of the National Academy of Sciences shall be ex-officio Treasurer of the National Research Council" and to submit such a plan with recommendations. (*Adopted.*)

In accordance with the resolution, passed at a previous meeting, that the Budget Committee make recommendations for allotments to the several Divisions on the basis of their minimum requirements for the period ending June 30, 1919, the Budget Committee presented the following resolutions:

That the balances of allotments to the several Divisions, amounting in all to $45,378.99 be and are hereby reverted to the unappropriated fund.

That appropriations to the several Divisions amounting in all to $39,200, be made for the period February 1 to June 30, 1919.

On this basis the unappropriated funds of the Council derived from the Carnegie Foundation are $50,810.01; the unappropriated fund available from the Government is $8,000.

Moved: That the foregoing recommendations of the Budget Committee be approved. (*Approved.*)

The Chairman presented a check from Dr. W. W. Keen of Philadelphia, amounting to $106.22, being the proceeds from the recent sales of the book which he prepared for the National Research Council entitled "Treatment of War Wounds."

Moved: That a vote of thanks and appreciation be extended to Dr. Keen for this further gift to the funds available for research in connection with the Division of Medicine and Related Sciences. (*Adopted.*)

Moved: That the new Divisions be considered to be effective when the divisional nominations have been approved by the Executive Board, and that present officers of Divisions shall continue to serve until their successors shall have been elected. (*Adopted.*)

Mr. Hale reported on the present status of the organization of the Divisions and presented the nominations received to date for membership on the Divisions of Physical Sciences, Chemistry and Chemical Technology, and Engineering.

Moved: That the nominations as presented be approved, and that the approval of the nominations required to complete the foregoing Divisions be referred to the Interim Committee with power. (*Adopted.*)

Moved: That the question of the publication of the following bulletins be referred to the Interim Committee, with power:

1. National Importance of Scientific and Industrial Research (Hale and others)
2. Report of Committee on Patent Office (Baekeland)
3. Report of the Psychology Committee of the National Research Council (Yerkes)

4. Some Problems of Sidereal Astronomy (Russell)
5. Refractories (Washburn)
6. Industrial Research (Jewett)
7. Scientific Abstracts (Fulcher)

Moved: That Mr. C. P. Townsend be elected a member of the Patent Office Committee. (*Adopted.*)

Moved: That Mr. James R. MacColl be elected a member of the Advisory Committee of the Industrial Research Section. (*Adopted.*)

Moved: That the resignation of Mr. W. F. Durand as Scientific Attaché to the American Embassy at Paris, in charge of the Research Information Service in France, be accepted with regret and that the National Research Council express appreciation of the admirable work which he has done and convey its thanks to him. (*Adopted.*)

Moved: That the resignation of Mr. Edgar Buckingham as Associate Scientific Attaché at Rome, and technical assistant with the Research Information Service in Italy, be accepted with regret and that a vote of thanks be extended to him for the work which he has rendered in this service. (*Adopted.*)

Moved: That the resignation of Mr. Charles E. Mendenhall as Vice-Chairman of the Division of Physics, Mathematics, Astronomy and Geophysics be accepted with regret; and that the Council express to him its appreciation of the valuable services which he has rendered. (*Adopted.*)

Moved: That Mr. A. O. Leuschner be elected Acting Chairman of the Division of Physics, Mathematics, Astronomy and Geophysics. (*Adopted.*)

Moved: That the resignation of Mr. John Johnston be accepted with regret and that the Chairman be requested to express to him the great appreciation of the work which he has done for the National Research Council, and that the acceptance of this resignation be effective when his successor is appointed. (*Adopted.*)

Moved: That the transportation expenses of the members of the Division of Chemistry and Chemical Technology attending the first meeting of the Division, to be held in Washington March 21 and 22, be met by the Council, as a means of assuring a full attendance at this initial meeting; and that the Interim Committee be authorized to make arrangements for the payment of expenses of members attending the initial meetings of other Divisions. (*Adopted.*)

Upon the request of Mr. Merriam the Chairman presented the following resolution:

Moved: That the sum of $500 be set aside for general maintenance of the Division of Educational Relations from the present date to June 30, 1919. (*Adopted.*)

The Treasurer, Mr. Cross, presented the financial statement for the month ending February 28, 1919, which was accepted.

The meeting adjourned at 1 p.m.

PAUL BROCKETT, *Assistant Secretary.*

ORGANIZATION MEETING OF THE AMERICAN SECTION OF THE PROPOSED INTERNATIONAL ASTRONOMICAL UNION

At the organizing meeting of the International Research Council held in Paris in November 1918, it was decided to establish an International Astronomical Union, to continue and extend the work formerly conducted by such international astronomical organizations as the committee of the Carte du Ciel, the International Union for Co-operation in Solar Research, and similar bodies

less formally constituted which dealt with various questions relating to astronomy and its applications. The International Research Council adopted a resolution requesting the National Academy of Sciences, or the corresponding organization in each of the countries represented, to take the initiative in organizing the section to represent that country in the International Astronomical Union. The tentative plan of organization of the American Section of the Astronomical Union, as approved by the President of the National Academy of Sciences, involved the representation of the various interests concerned as given below.

Upon the call of Dr. George E. Hale, acting for the National Academy of Sciences, the organization meeting for the American Section of the proposed Astronomical Union was held in the office of the National Research Council, Washington, D. C., March 8, 1919. The delegates who had been appointed by the presidents of the respective societies, or by the government, were as follows:

National Academy of Sciences, 5.—H. D. Curtis acting for W. W. Campbell, G. E. Hale, A. A. Michelson, F. R. Moulton, Frank Schlesinger.

American Astronomical Society, 10.—C. G. Abbot, S. I. Bailey, E. W. Brown, E. B. Frost, A O. Leuschner, S. A. Mitchell, W. J. Humphreys, H. N. Russell, Joel Stebbins, (absent, J. F. Hayford).

American Mathematical Society, 3.—Frank Morley, (others to be appointed).*

American Physical Society, 3.—Henry Crew, (absent, J. S. Ames, Theodore Lyman).

U. S. Naval Observatory, 1.—J. A. Hoogewerff, accompanied by W. S. Eichelberger, Asaph Hall, F. B. Littell.

U. S. Coast Survey, 1.—William Bowie.

The meeting organized by appointing Mr. Hale as chairman and Mr. Stebbins secretary. There followed a general discussion of the present international situation of science, and it was agreed that the Union should take the place of previous international bodies in astronomy.

It was voted that the organization of the Section should be considered temporary until after the proposed conference in Paris in July 1919.

The Section voted that the chair appoint a Committee on Committees, to act temporarily as an Executive Committee, which should consider the general matter of business, appoint all committees, and add additional members to the Section. Appointed: W. W. Campbell, chairman; C. G. Abbot, E. W. Brown, Frank Schlesinger, Joel Stebbins, secretary.

The Committee added the following to the membership of the Section: W. S. Adams, R. G. Aitken, E. E. Barnard, L. A. Bauer, Benjamin Boss, W. S. Eichelberger, W. J. Hussey, V. M. Slipher.

In regard to membership of enemy nations in the Union, the Section voted to adopt as representing the sentiments of the meeting the Declaration of the Interallied Conference on International Scientific Relations, held at the Royal Society in London on October 9 to 11, 1918. (See appendix to this report).

* The following representatives have since been nominated: George David Birkhoff, W. D. MacMillan, R. S. Woodward.

In regard to the admission of neutral nations to the Union, the Section voted that it be the sense of the meeting that nations which had been neutral in the war should be admitted into the International Astronomical Union on the conclusion of peace.

Mr. Schlesinger outlined the kind of astronomical work that requires international coöperation:

First: Work too extensive to be undertaken except by international coöperation; the Carte du Ciel, for example; or the plan of Selected Areas.

Second: Undertakings in which there is a geographical necessity for international coöperation. Variation of latitude; longitudes; variable stars; continuous observation of solar phenomena, etc.

Third: Matters of convention. Uniformity of nomenclature, notation and units. Examples, unit for stellar distances (four now in use); classification of spectra; use of probable or mean error or of average deviation; notation for celestial mechanics; notation for the reduction of photographic plates, etc.

Fourth: The avoidance of duplication. Calculations for the national Almanacs and for special ephemerides, such as comets, asteroids, and variable stars; astronomical abstracts; and news of new comets, variable stars, novae, asteroids and the like.

The Section discussed the various fields in astronomy in which committees should be formed to make report at another meeting of the Section, which would give instructions to the delegates to the proposed Paris conference. The following committees were authorized by the Section. The Executive Committee later made the appointments:

Committee on the Variation of Latitude: F. B. Littell, chairman; A. O. Leuschner, Frank Schlesinger. It was voted to ask the American Section of the International Geophysical Union to appoint a similar committee to confer and make a joint recommendation on the organization and method of handling the work on the variation of latitude.

Committee on Standards of Wave-Length: Henry Crew, chairman; W. S. Adams, Keivin Burns, W. W. Campbell, C. E. St. John.

Committee on Solar Rotation: C. E. St. John, chairman; W. S. Adams, Frank Schlesinger.

Committee on Eclipses: S. A. Mitchell, chairman; E. E. Barnard, H. D. Curtis.

Committee on Stellar Classification: H. N. Russell, chairman; Miss Annie J. Cannon, R. H. Curtiss.

Committee on Asteroids and Comets: A. O. Leuschner, chairman; E. W. Brown, G. H. Peters.

Committee on Almanacs: W. S. Eichelberger, chairman; E. W. Brown, R. H. Tucker.

Committee on Radial velocities: W. W. Campbell, chairman; W. S. Adams, J. S Plaskett.

Committee on Double Stars: R. G. Aitken, chairman; Eric Doolittle, W. J. Hussey.

Committee on Notation, Units, and Economy of Publication: W. J. Humphreys, chairman; E. B. Frost, A. O. Leuschner.

Committee on Meridian Astronomy: Benjamin Boss, chairman; F. B. Littell, Frank Schlesinger.

Committee on Abstracts and Bibliographies: F. E. Fowle, chairman; H. D. Curtis, G. S. Fulcher.

Committee on Research Surveys: G. E. Hale, chairman; F. R. Moulton, Harlow Shapley.

Committee on Stellar Photometry: F. H. Seares, chairman; S. I. Bailey, F. C. Jordan, J. A. Parkhurst, Joel Stebbins.

Committee on Wireless Determination of Longitude: J. A. Hoogewerff, chairman; W. W. Campbell, J. J. Carty. This committee was requested to study the feasibility of deter-

minations of longitude by wireless at widely distributed stations, and report on what seems to be the proper time and method for such undertakings.

Committee on Solar Radiation: Mr. C. G. Abbott was asked to prepare a report on Solar Radiation.

Committee on the Spectroheliograph: The Mount Wilson Solar Observatory was asked to prepare a report on work with the Spectroheliograph.

Committee on Reform of the Calendar: R. T. Crawford, chairman; W. W. Campbell, Harold Jacoby.

The question of delegates to the Paris meeting was left to the Executive Committee with power.

It was voted that the Section offer to act in astronomical matters as the agent of the Division of Physical Sciences of the National Research Council.

Various other items of organization and scientific interest were discussed by the Section at the morning and afternoon sessions, and in the evening, without formal action.

JOEL STEBBINS, *Secretary.*

Declaration in regard to Enemy Nations

When more than four years ago the outbreak of war divided Europe into hostile camps men of science were still able to hope that the conclusion of peace would join at once the broken threads, and that the present enemies might then once more be able to meet in friendly conference, uniting their efforts to advance the interests of science; for ever since the revival of learning in the Middle Ages the prosecution of knowledge has formed a bond strong enough to resist the strain of national antagonism. And this bond was strengthened during the latter part of last century, when branches of science developed requiring for their study the co-operation of all the civilized nations of the world. International Associations and Conferences rapidly multiplied, and the friendly intercourse between the learned representatives of different countries grew more intimate, in spite of their political differences, which were admitted, but not insisted upon.

In former times war frequently interrupted the co-operation of individuals without destroying the mutual esteem based on the recognition of intellectual achievements; peace then soon effaced the scars of a strife that was ended. If to-day the representatives of the Scientific Academies of the Allied Nations are forced to declare that they will not be able to resume personal relations in scientific matters with their enemies until the Central Powers can be re-admitted into the concert of civilized nations, they do so with a full sense of responsibility, and they feel bound to record the reasons which have led them to this decision.

Civilization has imposed restrictions on the conduct of nations which are intended to serve the interests of humanity, and to maintain a high standard, of honour, such as the recognition of the sanctity of treaties—especially those designed to apply to a state of war—and the avoidance of unnecessary cruelties inflicted on civilians. In both these respects, the Central Powers have broken the ordinances of civilization, disregarding all conventions, and unbridling the worst passions which the ferocity of war engenders. War is necessarily full of cruelties; individual acts of barbarity cannot be avoided and have to be borne. It is not of these we speak, but of the organized horrors encouraged and initiated from above with the sole object of terrorizing unoffending communities. The wanton destruction of property, the murders and outrages on land and sea, the sinking of hospital ships, the insults and tortures inflicted on prisoners of war, have left a stain on the history of the guilty nations, which cannot be removed by mere compensation of the material damage inflicted. In order to restore the confidence without which no scientific intercourse can be fruitful, the Central Powers must renounce the political methods which have led to the atrocities that have shocked the civilized world.

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 JUNE 15, 1919 Number 6

THE THERMO-ELECTRIC EQUATION $P = TdV/dT$ ONCE MORE

By Edwin H. Hall

Jefferson Physical Laboratory, Harvard University

Communicated, March 27, 1919

At the Philadelphia meeting of the Academy in November, 1917, I questioned the validity of this equation as commonly understood, P being taken as the ordinary Peltier effect and V the Volta effect between any two metals. But I did not make my point of objection entirely clear and wish now to try again; for the matter is important.

It appears that Kelvin, who derived this equation from a course of theoretical reasoning, did not regard P as necessarily the Peltier effect only, believing that it might include some other, hitherto unknown, reversible heat effect accompanying movement of electric charge from one metal to another. O. W. Richardson, arriving at the same equation by a different course of reasoning, held P to be simply the Peltier effect. My colleague Professor Bridgman, repeating with some modifications of his own the argument of Kelvin and also that of Richardson, came at first to a conclusion sustaining that of Richardson, and submitted his unpublished paper to me for criticism.

I maintained that Kelvin's broader interpretation of the P was probably the correct one and ventured the suggestion that ionization and re-association within the metals according to the mass-law of equilibrium between free electrons and metal ions, furnished the reversible heat effect, additional to the ordinary Peltier effect, that Kelvin saw the need of. Bridgman felt, however, that I had not disposed of Richardson's argument, and accordingly I wish now to revise, but not to withdraw, my criticism.

The passage which I quoted from p. 28 of his *Emission of Electricity from Hot Bodies* ended with the equation

$$dS = \frac{1}{T}[d\,(nv\phi) + p\,dv],$$

in which dS is change of entropy of the system and "φ is the change in the energy of the system which accompanies the transference of each electron

from the hot body to the surrounding enclosure." Now I was wrong in objecting to this particular passage, for just here Richardson is speaking of an insulated piece of metal which is imagined to give off electrons that do work as a gas on a moving piston; and for such a 'virtual' operation his statement is doubtless true. But in his *experiments* he is really taking off a stream of electrons in the gas state from the metal, and an equal stream is constantly entering the metal by conduction, yet he assumes that the same Φ which occurs in the equation above quoted, for his case of virtual emission from an insulated body, holds as the heat of emission for his actual case.

It was this actual case, of electric flow, that I had in mind when I offered my criticism that his equation for dS fails when not only heat but substance also, that of the incoming electrons, is added to the system during the operation under consideration. I believe that this objection holds.

Bridgman, missing my point through my lack of precision in stating it, studied Richardson's argument again and presently made for himself the same discovery that I had made, so that he and I are now in agreement regarding the inaccuracy of Richardson's reasoning. He finds, moreover, that, when this inaccuracy is corrected, Richardson's line of argument leads to precisely the same result as Kelvin's.

Accordingly Bridgman now writes P' instead of P in the equation in question, meaning by P' the total reversible heat effect that accompanies a virtual movement of charge from one plate to another of a condenser made of different metals, though he may not subscribe entirely to my theory as to the action of ionization and re-association within the metals.

I hope that Professor Bridgman's paper, dealing with thermo-electricity in a broad way, will be published before long.

ON THE X-RAY ABSORPTION WAVE-LENGTHS OF LEAD ISOTOPES

By William Duane and Takeo Shimizu

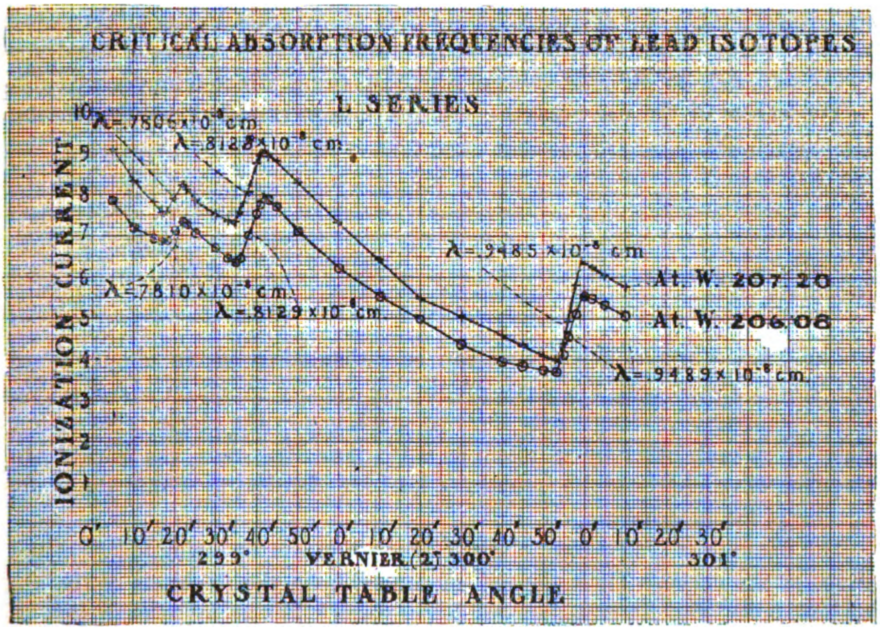
Jefferson Physical Laboratory, Harvard University

Communicated by T. Lyman, April 5, 1919

Researches on the transformations of radioactive substances have lead to the conclusion that chemical elements exist which have the same atomic number but different atomic weights. Such chemical elements have been called isotopes. In the reactions of ordinary chemical analysis isotopes behave in identically the same manner. It has not been found possible to separate isotopes from each other by means of purely chemical processes; although it seems probable that, since the atomic weights of isotopes differ from each other, they will act somewhat differently in those phenomena in which the mass of the atom enters as a factor. The ordinary line spectra of

chemical isotopes, also appear to be identical to a very high degree of precision, and recently Siegbahn and Stenström found no difference between their emission spectra in what is called the L series of X-rays.

The object of the research reported in this note has been to investigate the X-ray absorption spectra of chemical isotopes. In general each chemical element, except perhaps those of low atomic numbers, has several critical absorption frequencies, one connected with its K series and three connected with its L series of X-rays. These critical absorption frequencies mark points in the X-ray spectrum where sharp changes in the absorption of X-rays by the chemical element occur. The chemical element absorbs X-rays of higher frequency than the critical frequency to a much greater extent than it does X-rays of lower frequency.



In measuring the critical absorption frequencies of lead isotopes we have used the X-ray spectrometer described in the *Physical Review* for December 1917, page 624. A calcite crystal reflected the X-rays whose wave-length is given by the equation

$$\lambda = 2\,a \sin\theta = 6.056 \sin\theta \times 10^{-8} \text{ cm.}$$

(where θ is the grazing angle of incidence) into an ionization chamber. The ionization method of detecting the reflected beam of X-rays is far superior to the photographic method, both because, if properly used, it requires no correction for the penetration of the X-rays into the crystal, and also because it gives an estimate of the magnitude of the absorption.

The absorbing screen of lead was placed between the X-ray tube and the spectrometer. Professor Richards' laboratory kindly furnished us with the specimens of lead salts. Professor Richards has made accurate measurements of the atomic weights of lead isotopes, and the values he obtained for the two specimens we used were 207.20 for the ordinary lead and 206.08 for the radioactive lead. These differ from each other by more than $\frac{1}{2}$%.

In our experiments the X-rays came from a molybdenum target tube of the Coolidge type, and a constant difference of potential amounting to about 36,000 volts drove a current of 2 milliamperes through it.

The curves in the figure represent the ionization currents as functions of the readings of one of the verniers attached to the crystal table.

The three sharp drops in each curve correspond to the three critical absorption wave-lengths belonging to the L series of X-rays of each specimen of lead respectively. To get the grazing angles of incidence to substitute in the above formula for the wave-length we measured from the centres of the drops to the zero, 291° 55′ 40″, the value of which has been corrected for eccentricity.

The values of the wave-lengths, etc., have been collected together in the following table.

ABSORBING SCREEN	ATOMIC WEIGHT	$\lambda \times 10^8$ CM.	$\lambda \times 10^8$ CM.	$\lambda \times 10^8$ CM.
Ordinary lead	207.20	0.9485	0.8128	07.806
Radioactive lead	206.08	0.9489	0.8129	0.7810

The grazing angles of incidence can be estimated to within about 30″ of arc, which means that the wave-lengths are correct to within about 0.1%. Corresponding wave-lengths for the two specimens in the above table do not differ from each other by as much as 0.1%, and, therefore, the critical absorption wave lengths of the isotopes of lead are identical to within the limits of error of the experiments.

The magnitude of the characteristic absorption can be estimated from the drops in the curve. These depend, of course, upon the thickness of the absorbing layer, and its measurement is very much less accurate than that of the wave-length. It appears, however, that the relative change of absorption at the three critical wave-lengths is about the same for each isotope of lead.

SOME ORIENTING EFFECTS OF MONOCHROMATIC LIGHTS OF EQUAL INTENSITIES ON FUCUS SPORES AND RHIZOIDS

By Annie May Hurd

Botany Department, University of California

Communicated by J. M. Coulter, April 11, 1919

One of the most striking of the biological phenomena resulting from the action of light on organisms is the orientation of the first cleavage plane of germinating spores by unilateral illumination. Whenever such illumination is sufficiently intense the first cross wall forms perpendicular to the direction of the incident light. This phenomenon has been demonstrated in Equisetum, Puccinia, Fucus and some other algae[4,8,9,10,11] together with the fact that the cell on the shaded side of the spore becomes the rhizoidal cell irrespective of gravity. Thus in these and related forms the polarity of the plant is established by the direction of light stimuli.

The power of light waves to so orient the plant is, without doubt, the power to orient the spindle of the first dividing nucleus. The mechanics of such reactions may long remain unknown; but we have a suggestive and possibly the ultimate explanation in Child's[2] metabolic gradient theory. He has demonstrated in many marine plants and in some of the lower animals the existence of the so-called 'axial gradients.' By an axial gradient is meant the decreasing rate of metabolic processes from the apical to the basal end. We may suppose that such a gradient is produced within a germinating spore whenever there is sufficient difference in the amount of light energy received on two opposite sides to produce the requisite difference in the rate of the oxidation processes along the line of direction of illumination. If Child's supposition is correct, the cell on the shaded side of the spore becomes the rhizoidal cell by virtue of the fact that the least rapid rate of the oxidation reactions along the gradient determines the basal end, the most rapid the apical end, of the organism.

The purpose of the present investigation was to study the power of pure monochromatic lights to establish the polarity of the germinating spores of *Fucus inflatus*, and also to answer several questions concerning the negative phototropism of the young rhizoids; viz., the determination of the exact wave lengths responsible for the phenomenon; the relative importance of the quality and quantity factors in the illumination or the role of intensity of illumination apart from the kind of light; and whether all effective monochromatic lights produce the same result as white light.

To obtain the monochromatic light, seven Wratten filter screens were used, each transmitting a narrow range of wave lengths but altogether embracing the whole of the visible spectrum. The wave lengths to which each screen was transparent were determined by testing the light transmitted by each with

a direct-vision spectroscope with a wave length scale attached. These screens were then fitted as windows in the ends of boxes painted black on the inside. Culture dishes were made by cementing together microscope slides, so that the light entering the boxes through the screens fell on a flat side of the dish and thus entered the water normally with a minimum loss from reflection and refraction.

The electric arc was used as the source of light wherever possible because it gives all the desired wave lengths, with the result that the whole set of screens could be used in the same exposure, insuring for all the boxes identical conditions of temperature and duration of illumination.

The spectroscopic analysis of the light transmitted by the screens gave the exact wave lengths which would act on the cultures behind them. The next step was to devise a means of making the intensities of the lights acting in each box equal so as to eliminate that most important, and hitherto largely overlooked, factor in light reactions. This done, variations in results obtained behind different screens might safely be attributed to differences in the quality of the stimulus. There have been several methods devised by which the relative intensities of monochromatic lights can be measured and made equal.[1,3,5,6,7] But all of these involve special apparatus not available for use in this investigation; so a simpler method was devised whereby the relative intensities of the lights transmitted by the Wratten filter-screens were measured by means of a thermopile and galvanometer and made equal by varying the distances of the dark boxes from the electric arc such that at these distances the deflections of the galvanometer, when the thermopile was exposed in turn to the light behind each filter screen, were equal. This distance was also measured with a piece of white glass as a filter screen which represented the removal of the control culture from the arc. The instruments used in this energy calibration were a Hilger thermopile and a d'Arsonval galvanometer.

It seems necessary on account of the questions which have been raised during the course of this work, to state here that the thermopile is equally sensitive to the energy of the red and of the violet ends of the spectrum, and is, therefore, an accurate measure of the total amount of light acting behind each color screen. The difference between heat and light is only a matter of wave length. The thermopile measures light in terms of the electric current produced by the difference in temperature of the exposed and unexposed junctions; but it does so by virtue of the fact that the energy of whatever vibrations fall upon it, be they long and therefore heating in their physiological effect, or short and therefore perceived as light, is converted into heat energy upon being absorbed by the exposed junction of the thermopile. In other words, the light of the blue end of the spectrum produces an electromotive force much less than that of the infra-red but no less measurable.

Once these distances from the arc, at which the intensities of the light in each box are equal, are determined, the quantity of light energy can be varied

by multiplying or dividing all the distances by the same multiple and the intensities in all will still remain equal to each other. By means of a photometer the actual amount of light acting in each box can then be determined by measuring the intensity in candle meters behind the white glass control at the proper distance from the arc. Then from the law of inverse squares, viz., that the intensity of light per unit surface varies inversely as the square of the distance from the source, the absolute intensity at any distance from the arc can be computed. So a Sharpe-Millar photometer was used to measure the intensity of the naked arc at the distance of the white light control. But it was then necessary to correct the measurements so obtained for the absorption of light by the glass of the filters. This so-called absorption coefficient was obtained by measuring with a Lummer Brodhum photometer the intensity of a light both with and without a screen of clear glass equal in thickness to that of the filters. It was found that glass 1.5 mm. thick absorbed 12% of the light falling upon it. Therefore to obtain the intensity of the light actually entering each dark box, it was necessary to take 88% of the reading given by the photometric measurement of the unscreened arc at the previously determined distance of the white light control.

The following table lists the colored screens used with the wave lengths they transmitted and the distances from the arc at which they were placed to make the intensity of light behind each equal to 1800 meter candles. The lack of agreement between these values and the energy curve of the spectrum is due to the individual absorption of the filters and also to the fact that they do not all transmit the same number of wave lengths.

Table showing distances at which the intensities of light from an electric arc transmitted by Wratten light filters are equal

FILTER NUMBER	WAVE-LENGTHS IN ANGSTROMS	COLOR	DISTANCE FROM LIGHT	INTENSITY IN METER CANDLES
			cm.	
70	6600–7000	Red	320	1800
71	6200–6800	Red	275	1800
72	5900–6200	Orange	230	1800
73	5600–5900	Yellow	250	1800
74	5200–5600	Green	280	1800
75	4700–5200	Blue	250	1800
76	4000–4700	Violet	250	1800
Control	4000–7000	White	340	1800

To obtain the spores of *Fucus inflatus* for the experiments, the fruiting plants were collected at low tide, kept over night in damp newspapers, and the next morning were dried slightly by exposing them to the air for about half an hour. Then when the fruiting tips were submerged in sea water in the culture dishes, large numbers of eggs and sperms were extruded and settled to the bottom of the dish. After removing the piece of plant the culture dish was placed in one of the little racks made to fit in the dark boxes

behind the filter screens. The illumination of the cultures was continued six to eight hours, this time having been found more than enough to cause the first cleavage plane to be permanently oriented regardless of subsequent illumination.

In the experiments for which the naked arc was the source of light, the heating effect was so great that the spores were killed very quickly. The mercury vapor lamp was next used to obtain wave lengths of the blue end of the spectrum and a 1000 watt nitrogen filled Tungsten globe for the red. But as in the case of the electric arc, the Tungsten light killed the spores by the high temperature produced at the distances where it was necessary to place the cultures for a sufficiently intense illumination. With the mercury vapor lamp, however, positive results were secured. The wave lengths which were found to produce the orientation of the first cleavage plane such that all the first cross walls formed perpendicular to the direction of the incident rays, are those between 4000 and 5200 Ångstrom units. Behind the two other filter screens used with the mercury vapor lamp and transmitting wave lengths of 5200 to 5900 Ångstrom units, the spores germinated as if in darkness with the orientations of the first cleavage planes following no rule, and the rhizoids extending in all directions. However, the intensities of the lights behind these color screens were not equal when the mercury lamp was used because the shorter blue wave lengths predominated to so great a degree and hence produced greater intensities.

With regard to the phototropism of the young rhizoids, it was found that very weak white light, too weak to orient the cleavage planes, would cause the growing tips to turn sharply away from the source of light. With the intensity of illumination behind all the color screens equal to 1800 meter candles, only the blue and violet lights produced the phototropism. The other wave lengths at this intensity had no effect, the young rhizoids continuing in the direction in which they had started just as did those of the control in darkness. However when a more intense illumination was secured by placing the boxes in direct sunlight, the rhizoids behind the green filter, in addition to those behind the blue and violet ones, showed the same negative phototropism. This and subsequent experiments would lead us to believe that both quantity and quality, or intensity and wave length, are determining factors in the power of light stimuli to produce phototropisms.

In every culture of *Fucus inflatus* whether germinated in darkness or in strong unilateral light a most striking orientation of the first cross-wall with reference to adjacent spores appears. Wherever a group of spores are lying within about 0.2 mm. of each other, the first cleavage plane is perpendicular to the direction of the center of the group. The cell toward the interior invariably becomes the rhizoidal cell. This phenomenon was reported by Rosenvinge[9] in other species of *Fucus* and in *Ascophyllum*. For want of a better term I have called it *group orientation*. A study of the phenomenon was made to determine the strength of this stimulus, compared to that of

light, in its power to establish the orientation of the plant. It was at once very evident that for most spores the former stimulation is stronger when the spores are within a short distance of each other—0.2 mm. or often more—but beyond this distance, the chemical stimulus becomes too weak and only the light is able to determine the polarity of the plant. Only the comparatively isolated spores therefore show the orientation to light with the sources of illumination used here.

The phenomenon is very conspicuous in groups of 2, 3, or 4 eggs as well as in masses of 50 or 100. In these large groups it is made evident by the invariable rule that no rhizoid ever extends outward from a group. When two spores are within the distance through which the stimulus is effective, the first cleavage planes of the two are parallel and the rhizoids grow towards each other and often meet tip to tip. The groups of 5 or 6 often make symmetrical star-like designs when the rhizoids have grown and project beyond the group. The spores are more rarely affected in this way when the distance between them is over 0.3 mm. but the phenomenon is sometimes observed in spores as much as 0.5 mm. apart. Within a distance of 0.2 mm. there are practically no exceptions.

The relative sensitiveness of a spore towards light and towards this chemical (?) stimulus varies greatly for different spores. When cultures were placed in the window to get as strong a light stimulus as possible in order to determine at what distance from each other the eggs had to be not to show a greater sensitiveness towards the chemical stimulus than towards the light, it was found that this distance followed no rule, the spores showing the greatest individual differences. Of two spores lying within 0.3 mm. of each other one might be entirely oriented by the adjacent spore while the other, apparently like it, would show only the action of the light stimulus. In many cases two such spores would seem to show a resultant effect of the two stimuli so that both would be half turned towards each other with both rhizoidal cells showing a tendency to take a direction away from the light at the same resultant angle.

Rosenvinge ascribes this group orientation to a difference in the concentration of oxygen or of nutritive substances on the two sides of the spore. He thinks the rhizoid forms on the side toward the center of a group or towards another egg because the water on that side is less rich in the active substance than on the outer side as a result of their metabolism. Winkler[11] working with *Cystoseira barbata* found that a difference in oxygen concentration has no such effect. Apparently the phenomenon does not occur naturally in this species since his figure shows nothing but the effect of light. I have never seen a culture of *Fucus inflatus* with spores germinating so near each other, which showed only light orientation and not the group orientation. Almost invariably when the spores of this species germinate in such close proximity, light appears to have no power to establish the polarity of the plant.

The possibility that the group orientation is due to a polarity established by the position of the egg in the oogonium is suggested by finding many

groups of eight lying just as they escaped from the oogonial sac and conspicuously oriented with respect to each other. The fact that groups of ten or of two are as regularly oriented, would refute the suggestion; but in order to prove that the phenomenon is the result of a stimulus acting after the eggs leave the oogonium, a group of them were transferred to a watch crystal and mixed with the point of a needle until their relative positions were entirely changed. But when they germinated the characteristic orientation with respect to each other was found to be without an exception.

The only apparent explanation of the group orientation is that of a diffusion gradient of some substance emanating from a growing spore, or of some substance being used up by it. A continuation of the investigation of this problem will be an attempt to discover a substance which can so affect the dividing nucleus of the egg cell that its unequal distribution on the sides of the cell will orient the axis of the spindle. The effect of bubbling carbon dioxide and oxygen through cultures should be tried as being the most probable factors involved.

The substance or condition originating in the activity of adjacent spores which has so powerful an effect in orienting the first cleavage plane and in determining which cell shall become the rhizoidal cell has no power to cause any chemotropism of the rhizoids after they are started. No rhizoid has been found to have its direction modified by the presence of other spores adjacent to it. In the absence of any light stimulus the rhizoids continue in the direction that they take originally from the spore.

[1] Blaaw, A. H., *Rec. Travaux Bot. Néerlandais*, **5**, 1908, (209–372).
[2] Child, C. M., *Individuality of Organisms*, 1915.
[3] Day, E. C., *Bull. Mus. Comp. Zool.* **53**, 1911, (303–343).
[4] Farmer, J. B., and Williams, J. L., *Proc. Roy. Soc.*, **60**, 1896, (188–195).
[5] Kniep, G. and Minder, F., *Zs. Bot.*, **1**, 1909, (619).
[6] MacDougal, D. T. and Spoehr, H. A., *Science*, (N. S.), **45**, 1917, (616–618).
[7] Mast, S. O., *J. Comp. Neur. Psych.*, **17**, 1907, (99–179).
[8] Randolph, F. A., and Peirce, G. J., *Bot. Gaz.*, **40**, 1905, (321–350).
[9] Rosenvinge, M. L. K., *Revue Gen. Bot.*, **1**, 1889, (125–135).
[10] Stahl, E., *Ber. deuts. bot. Ges.*, **3**, 1885, (334–340).
[11] Winkler, H., *Ibid.*, **18**, 1900, (297–305).

ON THE MOST GENERAL CLASS L OF FRÉCHET IN WHICH THE HEINE-BOREL-LEBESGUE THEOREM HOLDS TRUE

BY ROBERT L. MOORE

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF PENNSYLVANIA

Communicated by E. H. Moore, April 15, 1919

§1 A class L of Fréchet[1] is a set of elements such that (1) if P is an element of L and P_1, P_2, P_3 . . . is a countable[2] sequence of elements belonging to L then the statement that P is the limit of the sequence P_1, P_2, P_3, . . .

has a definite meaning and the question whether this statement is true or false has a determinate answer as soon as the element P and the sequence in question are themselves determined, (2) if the element P is the limit of the sequence $P_1, P_2, P_3, \ldots$ and $n_1, n_2, n_3, \ldots$ is an infinite sequence of positive integers such that $n_1 < n_2 < n_3 \ldots$, then P is also the limit of the sequence $P_{n_1}, P_{n_2}, P_{n_3}, \ldots$, (3) if P is an element of L, P is the limit of the sequence $P, P, P, \ldots$, whose elements all coincide with the element P. An element P is said to be a *limiting element* of a sub-class M of the class L if P is the limit of some infinite sequence of distinct elements belonging to M. The set M is said to be *compact* if every infinite set of distinct elements belonging to M has at least one limiting element. The totality of all the limiting elements of a given set M is called the derived set of M. A class S is a class L in which the derived set of every set is closed. An element P belonging to L is said to be *interior to* the sub-class M of the class L if M contains P and at least one element of every sequence of distinct elements that converges to P. A family G of sub-classes of a class L is said to *cover* a subclass M of the class L if every element of M is interior to some member of the family G. A set of elements M is said to possess the Heine-Borel property if for every *countably* infinite family G of sub-classes of L that covers M there is a finite sub-family of G that also covers M. A sub-set M of L is said to possess the Heine-Borel-Lebesgue property if for *every* family G of sub-classes of L that covers M there is a finite sub-family of G that covers M.

In a recent paper[3] Fréchet has shown that in order that, in a given class S, a point-set[4] M should have the Heine-Borel property it is necessary and sufficient that the set M should be closed and compact. He also shows that the same conditions are necessary and sufficient in order that a point set M in a class V[5] should have the Heine-Borel-Lebesgue property. The Heine-Borel Theorem or the Heine-Borel-Lebesgue Theorem is said to hold true in a given space L if in that particular space every closed and compact point-set has the Heine-Borel property or the Heine-Borel-Lebesgue property respectively. Fréchet points out that in order that the Heine-Borel-Lebesgue Theorem should hold true in a given class L it is necessary, but not sufficient, that the class L should be a class S; and sufficient, but not necessary, that it should be a class V. He raises the question as to what property it is *necessary and sufficient* that a class L should possess in order that the Heine-Borel-Lebesgue Theorem should hold true in that class. In the present paper I will exhibit one such property.

I will call a family G of point-sets a *monotonic family* if, for every two point-sets of the family G, one of them is a subset of the other one. A sub-class M of a class L will be said to have the property K in case it is true that for every monotonic family G of closed sub-classes of M there is at least one point which is common to all the members of the family G. A class S in which every compact subset has the property K will be called a class S^*.

Lemma 1. *If in a class L every set of points which contains the point P in its interior contains at least one point of the set M distinct from P, then the point P is a limiting element of the set M.*

Proof. If P were not a limiting element of M then it would be interior to $L - M + P$. But this set of elements contains no point of M distinct from P.

Theorem 1. *If, in a class S^*, M is a closed and compact set of points and β is a well-ordered sequence of point-sets such that M is covered by the family composed of all the members of β, then there exists a member g of the sequence β such that M is covered by the family composed of g together with all those members of β that precede g.*

Proof. Suppose there exists no such member g. Then for each member x of the sequence β let M_x denote the set of all points P belonging to M such that P is not in the interior of x or of any member of β that precedes x. For every x, M_x contains at least one point. For every x, M_x is closed. For suppose P is a limiting element of M_x. If P were not in M_x then it would necessarily be in the interior either of the point-set x or of some point-set of the sequence β that precedes x and therefore, by a lemma of Hedrick's,[6] this particular point-set would contain, in its interior, at least one point of M_x, which is contrary to the definition of M_x. It follows that the family composed of all M_x's for all members x of the sequence β is a monotonic family of closed point-sets. Hence there exists a point P_0 which is in every M_x. But P_0 is in the interior of some point-set x_0 of the sequence β. Hence P_0 is not in M_{x_0}. Thus the supposition that Theorem 1 is not true leads to a contradiction.

Theorem 2. *In order that the Heine-Borel-Lebesgue Theorem should hold true in a given class S it is necessary and sufficient that that class S should be also a class S^*.*

Proof. Suppose that in a given class S^* the closed and compact point-set M is covered by the infinite family G of point-sets. The members of the family G can[7] be arranged in a well-ordered sequence β. By Theorem 1, there exists at least one member g of β such that M is covered by the family composed of g and all those members of β that precede g, together with not more than a finite number of those elements of β that follow g. Let g_1 denote the first[8] such g and let $g_2, g_3, g_4, \ldots g_n$ denote a finite set of members following g_1 such that M is covered by $g_1, g_2, g_3, \ldots g_n$ together with all those members of β that precede g_1. There clearly cannot exist a member of β immediately preceding g_1. I will show that g_1 is the first member of β. Suppose it is not. Then if g_1 and all the succeeding members of the sequence β are removed, there remains a well-ordered sequence β_1 whose members are the remaining members of β arranged in exactly the same order as in the original sequence β. Let us now construct a third well-ordered sequence β_2 having for its first n members the point-sets $g_1, g_2, g_3, \ldots g_n$ arranged in the order indicated and having as its remaining members the members of β_1 arranged in the same order as in β_1. The point-set β_2 has no

last member. The point-set M is covered by the family of point-sets composed of all the members of the sequence β_2 but there does not exist any member g of β_2 such that M is covered by the family composed of g together with all those members of β_2 that precede g. But this is contrary to Theorem 1. Thus the supposition that g_1 is not the first member of β has led to a contradiction. It follows that g_1 is the first member of β and that M is covered by the finite set of point-sets $g_1, g_2, g_3, \ldots g_n$. Thus the Heine-Borel-Lebesgue Theorem holds true in every class S^*.

Suppose now that the Heine-Borel-Lebesgue Theorem holds true in a given class S and that G is a monotonic family of closed, compact subsets of S. Let $\bar{g}$ denote any point-set of the family G. I will show that the members of G have at least one point in common. Suppose that this is not the case. Then, if P is a point of $\bar{g}$, there exists a closed point-set g_P of the family G that does not contain P. Hence, by Lemma 1, the point P is in the interior of some point-set R_P which contains no point of g_P. Let H denote the set of all R_P's for all points P of $\bar{g}$. By hypothesis, $\bar{g}$ is covered by a finite subfamily $R_{P_1}, R_{P_2}, R_{P_3}, \ldots R_{P_n}$ of the family H. But there exists an i $(1 \leq i \leq n)$ such that g_{P_i} is a subset of each of the point-sets $g_{P_1}, g_{P_2}, g_{P_3}, \ldots g_{P_n}$. It follows that no point of g_{P_i} is in any one of the sets $R_{P_1}, R_{P_2}, R_{P_3}, \ldots R_{P_n}$. Thus the supposition that there is no point common to all the members of G has led to a contradiction.

§2. I now raise the question whether it is not desirable to substitute, for Fréchet's definition of the word compact, a definition which is, for some spaces, (but not for spaces V) more restrictive than that of Fréchet. I will say that a monotonic family of point-sets is *proper* if there is no point that is common to all of its members. I will say that a set of points M is *compact in the new sense*[9] if for every proper monotonic family F of subsets of M there exists at least one point which is a limit point of every point-set of F.

Suppose that in a space L the infinite point-set N is compact in the new sense. The set N contains at least one countably infinite sequence of distinct points $P_1, P_2, P_3, \ldots$ For each n let t_n denote the point-set $P_n, P_{n+1}, P_{n+2}, \ldots$ The family of point-sets $t_1, t_2, t_3, \ldots$ is a proper monotonic family. Hence there exists a point P which is a limiting element of every one of these point-sets. It follows that if in a space L a point-set is compact in the new sense then it is also compact in the sense of Fréchet. That the converse is not true for every space S may be seen with the help of the following example.

Example. Let α be a well-ordered set of elements such that [10] (1) if K is any countable subset of the elements of α then there exists an element of α that follows all the elements of K, (2) if P is a given element of α the set of all those elements of α that precede P is countable. If a and b are two non-consecutive elements of α such that a precedes b then the set of all those elements of α which follow a and precede b will be called a segment. An element P of α will be said to be the *limit* of a countable sequence $P_1, P_2, P_3 \ldots$ of ele-

ments of α if, and only if, for every segment s that contans P there exists a positive integer δ_{PS} such that, for every n greater than δ_{PS}, P_n is in s. With respect to this conception of limit of a sequence, the elements of α evidently constitute a class S. The set M composed of all the elements of α is compact in the sense of Fréchet. It is not, however, compact in the new sense. For if for every element x of α, t_x denotes the set of all those elements of α which follow x then there exists no element which is a limiting element of every member of the proper monotonic family composed of all t_x's for all elements x of α. The set M, though closed and compact in the sense of Fréchet, does not possess the Heine-Borel-Lebesgue property.

It is easy to see that, in every class V of Fréchet, a point-set which is compact in the Fréchet sense is also compact in the new sense.[11]

By a proof in large part identical with the above proofs of Theorems 1 and 2, the truth of the following theorem may be established.

Theorem 3. *In a class S, in order that a point-set M should possess the Heine-Borel-Lebesgue property it is necessary and sufficient that M should be compact in the new sense and closed.*

[1] Fréchet, M., *Palermo, Rend. Circ. Mat.*, 22, 1906, (1–72).

[2] By a countable sequence is meant a sequence of the same order type as the sequence of positive integers arranged in the normal order.

[3] Fréchet, M., *Bul. sci. math.*, *Paris*, 45, 1917, (1–8). See also Chittenden, E. W., *Bull. Amer. Math. Soc.*, *New York*, 25, 1918, (60–65).

[4] In the present paper the elements of a class L will be called points.

[5] A Class V is a class L in which there exists a distance function. Cf. either of the above mentioned papers of Fréchet.

[6] Hedrick, E. R., *Trans. Amer. Math. Soc.*, *New York*, 12, 1911, (285–294). Cf. also Fréchet, loc. cit.

[7] For a proof that the elements of any set (whether countable or uncountable) can be arranged in a well-ordered sequence, see Zermelo, E., *Math. Ann.*, *Leipzig*, 59, 1904, (514) and 65, 1908, (107–128). Zermelo assumes the truth of the well-known Zermelo Postulate. With regard to this postulate, see a recent paper by Ph. E. B. Jourdain, *Paris*, *C. R. Acad. Sci.*, 166, 1918, (520 and 984).

[8] Every subset of a well-ordered sequence contains a first member, that is to say, a member that precedes every other member of that subset.

[9] It is clear that in every space S a point-set is compact in the new sense if and only if, it possesses the property K defined in §1 of the present paper.

[10] For a proof of the existence of a well-ordered sequence satisfying these two conditions cf. Hobson, E. W., *The Theory of Functions of a Real Variable*, Univ. Press, Cambridge, 1913, (177–181).

[11] Sometime after the manuscript of the present paper had left my hands I found that, in 1912, S. Janiszewski introduced an extended conception *of limit* and defined a point-set as "*parfaitement compact*, si de toute de ses éléments on peut extraire une suite du même type d'ordre et possédant un élémente limite." cf. J. éc. polytech., Paris, 16, 1912 (155). Compare also the example in §2 of the present paper with an example of Janiszewski's on page 167, loc. cit. It seems likely that in a class S a set is compact in my sense if and only if it is parfaitement compact in the sense of Janiszewski. I do not find however that Janiszewski has made any study of the Heine-Borel-Lebesgue Theorem in connection with his conception of compactness.

AN ELECTRODYNAMOMETER USING THE VIBRATION TELESCOPE

By Carl Barus

Department of Physics, Brown University

Communicated, April 21, 1919

1. *Introductory.*—The employment of a telescope with a vibrating objective did good service as an aid to the interferometry of vibrating systems. It seemed worth while therefore to see what could be got out of it, when used in connection with a telephone only, as a dynamometer. The experiments are of interest both because of the vibratory phenomena observable and in view of the peculiar method of optic observation developed. Its possible use for finding the magnetic field within a helix of unknown constants deserves mention.

2. *Apparatus.*—A front view (elevation) of the design is given in figure 1 and an enlarged detail (side view) in figure 2. The apparatus consists of a rigid rectangular frame-work of ½ inch gas pipe, *A*, *B*, *C*, *D*, *EE'*, *F* being the foot attached to a tripod. There may be a steadying foot at *C'*. *A* and *D* are attached to *EE'* by the stout clamps *c*, *c''*, so that *EE'* lying behind the plane of *ABCD*, admits of the attachment of a suitable clamp *c'*, by which the telephone *ih* may be held in the same plane. *B* and *C* may be forced apart slightly by the screw *n* controlled by the broad thumb nut *m*, the conical end of *n* rotating in a socket of the cap *p*.

The vibrating system consists of the bifilar wires of phosphor bronze *dd'* and the frame of the lens *f* which is the movable objective of the telescope *T*, the latter part containing the ocular and a plate micrometer (cm. divided in 100 parts). *T* may be at a considerable distance (50 cm. or more) from *f*, and supported by a convenient standard. The frame of the lens (which must hold it securely, cement being used if necessary) is of light sheet metal, the parts *gg'* being of sheet iron (about 0.05 cm. thick) so as to be attracted by the magnet, *i*, of the telephone. The stiff cross wires, *r*,*s*, of the frame are either soldered to the bifilar system *dd'* or otherwise attached to it (soft sealing wax does very well for temporary experimental purposes).

The attachment and tension-control of the bifilar suspension is finally to be described, as its period must be synchronized with the alternating current. Results are obtainable only when the two periods are strictly in unison. In figure 1 the wires *dd'* are looped around a groove in the pipe *D* below, and the upper ends of *dd'* after passing a similar groove in *A* are bent around the post *a*, *a'*, and wound respectively around the snugly fitting screws, *b*, *b'*, the ends being secured against sliding by a fine hack saw cut in the screws. To stretch a wire it is passed from the notch in *b* once or twice around it, thence around *a*, downward by the groove to *D* and then up in the corresponding way to *b'*.

The lens carriage gg' is then attached with cement (after the wires are evenly stretched) with the crosswires r, s on the opposite side of dd' to the pull of the magnet i. The magnet, in addition to the cement, thus guards against slipping. On turning the screws b and b' any degree of tension may be imparted to the wires bb', roughly. This simple device worked surprisingly well, and wires of different kinds may be easily inserted or replaced, the lens system being subsequently attached with cement; but it is better to loop the lower part of dd' around a special roller G, as indicated in figure 2, and used in my later tests, with the object of more easily reaching an equality of tension in the wires d and d'. The tensions are then roughly changed by the screw and nut u.

The approximate tension having thus been obtained by the screws u, b, b', the finer variations are imparted by the screw mn which flexes the elastic rectangle $ABCDE$ and thus gives to the bifilars d, d' exactly the tension required. It is at the thumb nut m that all adjustment is made during observation.

In my apparatus the rectangle was about 50 cm. long and 12 cm. wide. The wires d, d' about 1.5 cm. apart and each about 0.025 mm. in diameter. Wires as thick as this require sharp adjustment as to tension, but the method given proved quite satisfactory particularly as it is little disturbed by manipulation. The tension is sufficient to admit of an air gap of less than a millimeter between the plate g' and the magnet i of the telephone. Later the telephone was also put on a spring-micrometer screw for fine adjustment.

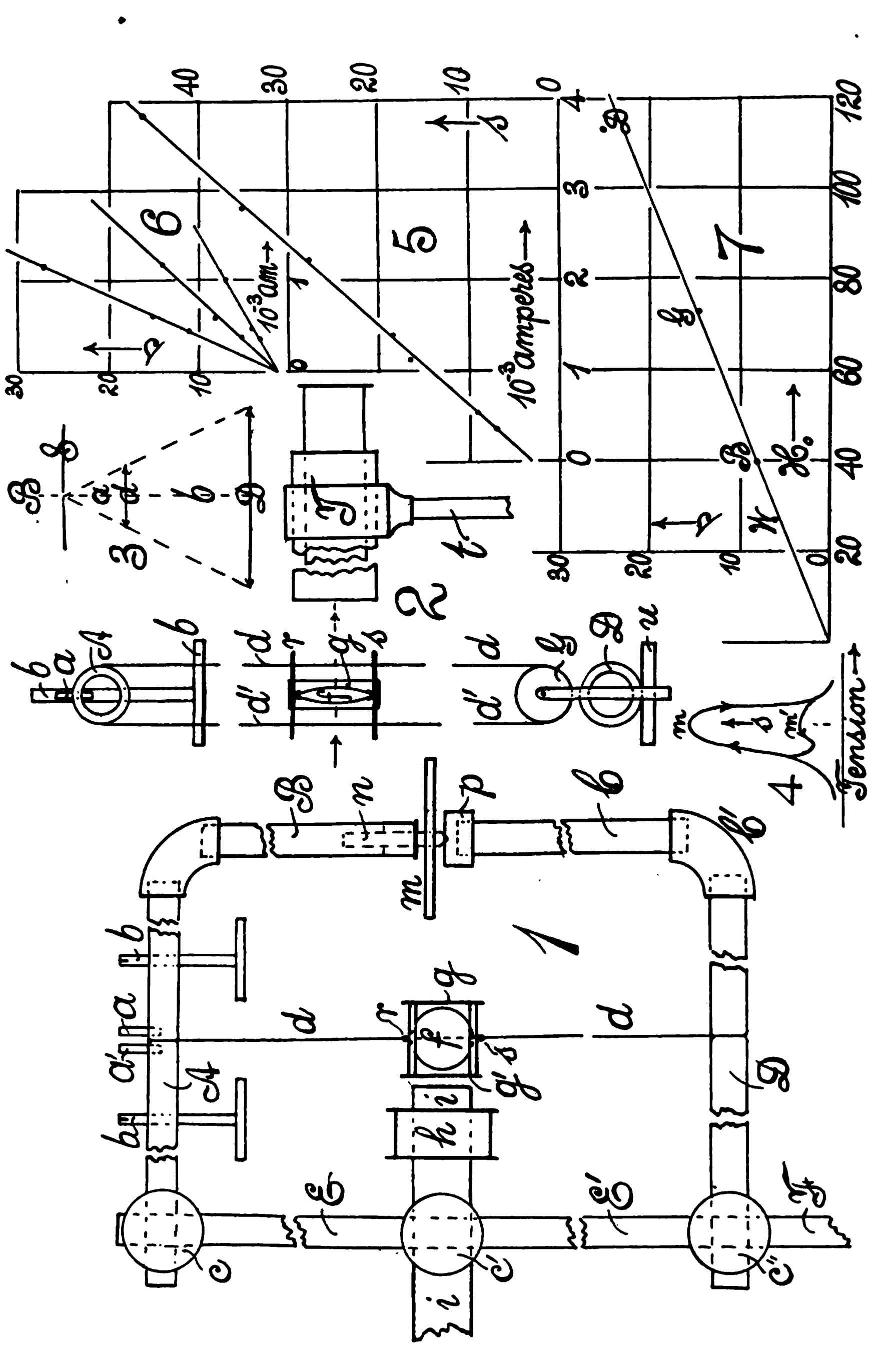
In the case of parallel rays, the displacement of the image in the ocular would be no larger than the displacement of the objective, f, figure 1. To obtain increased displacement, the method of figure 3 is available where S is the fine slit in front of a Welsbach burner at B. At d is the principal plane of the vibrating objective and at D the micrometer plate in the ocular. Again, if the length d represents the double amplitude of the objective and the sides of the triangle be drawn from S, the intercept D will represent the displacement in the ocular. If the distance Sd be a and dD, b, we may write

$$1/a + 1/b = 1/f \tag{1}$$

where f is the principal focal distance of the objective. Hence

$$D/d = b/f \tag{2}$$

Theoretically, therefore, any degree of magnification is possible by increasing b (the distance of T and f, fig. 2) and decreasing f. In the former case, some means of controlling the thumb nut m, figure 1, from a distance would have to be provided. In the latter the lens f would have to be achromatic. In the present experiments I first used an ordinary spectacle lens at f with a slotted screen between it and the slit to diminish chromatic aberration; but there is no objection to the use of an achromatic lens at f, as was done later, particularly since a breadth of a few millimeters of lens will suffice, for there is an abundance of light.


10⁻³ amperes→
10⁻³ am→
Tension→

To measure the width of the image band of light produced by the vibration, the ocular T should be on an axle t with slight friction. In my earlier improvised apparatus, single scale-parts (0.1 mm.) only could be guaranteed; but with a perfected optical system there is no reason why tenths of scale-parts should not be equally trustworthy. This makes a scale of 1000 parts in the ocular.

3. *Observations.*—As a source or alternating current I selected a small induction coil with a mercury break (Kohlrausch's design) to facilitate the initial tests. This was put in series with a rheostat (to 30,000 ohms), a Siemens's precision dynamometer reading to within milliamperes, an ordinary telephone to indicate the continuous action of the coil and the vibrator above described.

The Siemens's dynamometer was first standardized with a Clarke cell. Accepting the effective current i as being

$$i = C\sqrt{\varphi}$$

where φ is the deflection on a scale at about 1 meter of distance, the constant $C = 1.12 \times 10^{-3}$ relative to amperes was found and the mean resistance of the coils included about 250 ohms. Virtual currents of 10^{-4} amperes would escape detection.

The coil was now started and measurements made simultaneously both at the Siemens's dynamometer and at the vibrating telephone (slit distance a = 35 cm., ocular distance b = 75 cm.), with good results, the virtual currents of the dynamometer being compared with the width of the image band (in scale-parts) seen in the vibration telescope. To obtain different virtual currents, resistances from 10,000 to 2000 ohms were put into the circuit. The dynamometer showed deflections from 2 to 20 cm. on the scale. For larger deflections the coil current would have been too irregular for use.

The vibrator indicated about 10 scale-parts per milliampere, and readings even beyond 5 or 10 milliamperes would be possible. The deflections were fairly proportional to the current, indicating that with a good optical system virtual currents as small as 10^{-5} ampere should have been perceptible. The apparatus is thus very well worth further development and would be particularly useful where alternators with a definite period are in question.

The method of observation consists in gradually increasing the tension of the wire from a slightly low value, to beyond the maximum tension. In this case the band widens from a relatively small width to the maximum and then abruptly falls off to a small value. To repeat the observation the tension must often again be reduced to the low value and the whole operation repeated; but after some practice maximum may be reached in the reverse direction unless the band has become too narrow.

When the current is broken and thereafter closed, a low width of band is obtained which will not widen unless the operation described is carried out from low tension. In other words there are often two cases of equilibrium for each

current, corresponding to very different band widths. This is a curious result, for it means, virtually, that the magnetic forces and the stresses are in the relation of a doubly inflected curve to each other, so that there are three intersections, two for stable vibratory equilibrium; or else the two harmonic systems, the electrical and the mechanical, may vibrate in the same or in opposed phases. The cycle in figure 4 indicates the general relation of the band width s to the tensions, there being two maxima at m and m'.

Similarly each current requires its own particular maximum tension, which increases with the current. The difference is not large, but effective and for this reason the fine tension adjustment is essential.

4. Further observations.—The apparatus was then improved in a variety of ways, chiefly by the insertion of a small vibration objective, about 1 cm. in diameter, achromatic and with a focal distance of but 5.8 cm. In this case the distance a and b, figure 3, could be decreased to 7 cm. and 35 cm. and the observer was thus conveniently near the adjusting screw. The slit image was white and about a scale-part in width. There would have been no difficulty in using much greater magnification.

An example of the results (band-width s in scale parts) is given in figure 5. The constant of the dynamometer was now $C = 10^{-3} \times 0.87$ relative to amperes and the total resistance in circuit 710 ohms. The frequency, as before, was estimated at about 15 per second.

The results were a considerable improvement on the preceding and the discrepancies as a rule lie within 5×10^{-5} ampere. They are much more liable to be in the dynamometer than in the vibrator, as the former was not well adapted for these small currents. The deflections begin with 3 scale-parts (initial slit breadth) and not at zero; but as this appears merely as an initial constant, it is not of consequence.

If we compute the coefficient of induction as

$$L^2\omega^2 = \Delta(i^2(R + r)^2)/\Delta i^2$$

(Δ being a differential symbol) from the first and fifth, second and sixth, etc., observations, data for $L\omega$ and L follow.

5. Effect of frequency.—A special mercury interruptor was now made having as its distinctive feature contrivances by which the mercury surface was washed and thus could always be kept clean and bright. It was furthermore adapted to give different frequencies. The apparatus functioned admirably for days, frequent washing presupposed. Different frequencies, obtained by a sliding weight, were estimated from the moments of inertia as $n = 10$, 15, and 20. The latter could just be counted in groups of 4 vibrations with a stop-watch. Higher frequencies were obtainable by using stiffer springs.

An example of the results obtained with this apparatus is given in figure 6, for the phosphor bronze bifilar differently stretched. All gave evidence of the peculiar fact that the sensitiveness increases in marked degree with the frequency.

It is difficult to account for this effect of frequency, so peculiarly marked in the instance given, where the observations were good. If different harmonics are in action, the overtones would have to respond in case of the wires under less tension, and this seems unlikely. The behavior of very tense steel wires, moveover, was the reverse of this. It is probable that the phenomena of §6 contain the clue.

6. *Adjustable telephone.*—The question as to the most advantageous position of the telephone is thus variously important. Consequently the telephone *ih*, figure 1, was mounted on a stout flat steel spring controlled by a micrometer screw. By actuating this, the face near g' could be approached as near g' as permissible, or withdrawn to a remoter position, with precision. The experiments made at length showed that within a wide range of tensions and of telephone positions, a particular degree of approach of the telephone corresponded to each particular stress of wire. Unless these paired positions are selected, the bifilar system does not respond. Tense wires require a nearer telephone; less tense wires a more remote telephone (even 0.5 cm.), within wide limits. Therefore the condition of resonance may be reached either by adjusting the telephone on the micrometer screw for a given tension of wire; or by changing the tension of the wires for a fixed telephone. Between the admissible limits of tension, the sensitiveness passes through a flat maximum.

With a distance of not more than 50 cm. between slit and ocular, a judicious disposition of parts eventually gave me 40 ocular scale parts per virtual milliampere, viz.,

n	$i \times 10^3$	$s \times 10^2$
	amperes	*cm.*
15	0.67	27
20	0.67	27

so that here the above effect of frequency, n, is no longer apparent.

7. *Coil tester.*—An interesting application of the above apparatus, where a definite frequency is usually assigned, is its possible use for measuring the magnetic fields of different coils. For this purpose I wound a long thin primary of an induction coil, which when inserted into the coil to be tested, should, from the measurement of the current induced in the secondary in question and in the absence of other mutual inductions, give the constants of the secondary. As many of the coils to be tested were of small internal diameter, the primary was wound on a long iron tube, fine wire being necessary. The data are for the

Iron tube

Diameter outside.......0.635 cm.
Length ....55 cm.
Walls............0.08 cm. thick

Helix

Diameter outside..........0.7 cm.
Wire........0.034 cm. in diameter
n_1/l_1 ......21 turns per linear cm.

If L is the coefficient of induction per turn of primary, the total induction is

$$B = LN_1i \tag{1}$$

Hence the electromotive force induced in the secondary becomes

$$e = LN_1N_2(di_1/dt) \tag{2}$$

If the field of the secondary per unit current is put

$$H_2 = 4\pi N_2/l_2 \tag{3}$$

for N_2/l_2 turns per linear centimeter, and r the resistance of this coil and its circuit, we may compare any two coils by the equation

$$\frac{e_2}{e_2'} = \frac{N_2}{N_2'} = \frac{H_2l_2}{H_2'l_2'} = \frac{i_2r_2}{i_2'r_2'} \tag{4}$$

e_2 and e'_2 being the electromotive forces, and i_2 and i'_2 the currents induced in the two secondaries in question. Thus

$$\frac{H_2}{H_2'} = \frac{i_2r_2/l_2}{i_2'r_2'/l_2'} = \frac{s_2/l_2}{s_2'/l_2'} \tag{5}$$

it being assumed that the resistance r in the secondary circuit is made so large that the inductive resistance vanishes.

The coil tester was now thrust through a variety of helices, differing in shape and construction.

Figure 7 gives the results read off for these coils at the vibrator (s), when a high resistance was added to the circuit and Ri constant. The relation of H_0 and s as seen in figure 7, is linear for coils W, B, G, which were about of the same length and wound on non-conductors or split brass. The result for coil D, which was about 1.8 times longer is low, probably owing to mutual induction, as this coil was wound on a thick brass tube without fissure. Similarly it made very little difference whether the two helices wound side by side throughout G, were used in parallel or in series. Finally a number of single layer coils of lengths 10, 20, 30, 40 cm. were wound on stout glass tubing and compared with B. Provided care was taken to prevent the induced currents from heating the telephone, the band width s increased throughout proportionally to the number of turns of wire in the secondary and under good conditions at a rate of about $s = 0.06$ scale parts per turn of wire.

THE TEMPERATURE OF THE HUMAN SKIN

By F. G. Benedict, W. R. Miles, and Alice Johnson

Nutrition Laboratory, Carnegie Institution of Washington, Boston, Massachusetts

Read before the Academy, April 28, 1919

The clinical importance of records of body-temperature, as taken usually in the mouth and occasionally in the axilla or rectum, has quite obscured the physiological significance of the skin temperature. Extensive researches have shown that the temperature of the human body, deep in the body trunk or in any of the natural cavities, remains reasonably constant, although there is a diurnal ryhthm, with a minimum value at about 4 a.m. and a maximum at about 5 p.m. Simultaneous observations of temperature at different parts of the body show that there is almost always a parallelism in curves for temperature. Thus, the temperatures in the rectum or vagina, the well-closed groin or axilla, and the mouth, show similarly shaped curves, although at markedly different levels.

The technical difficulties in recording skin temperature have undoubtedly retarded extensive study in this phase of body-temperature research. The temperature of the human skin is the resultant of several factors, as heat is supplied from the subcutaneous tissues and lost from the surface of the body by radiation, conduction, and the vaporization of water. An attempt to secure accurate records of skin temperature by the application of an ordinary mercury thermometer is obviously useless, for but a small proportion of the bulb of such a thermometer comes in actual contact with the skin. Even thermometers constructed with a special bulb providing a large surface to apply to the skin have a like surface exposed to the environmental temperature. If, as is occasionally done, this outer surface is covered with non-conductive material, there is almost immediately a disturbance in the temperature of the skin due to the fact that there is a retardation of the normal loss of heat, with a consequent accumulation of heat from the subcutaneous tissue. The true temperature of the skin should therefore be recorded by an apparatus which is nearly instantaneous in action and sufficiently protected from the environment to insure a true record of the surface temperature, and not a resultant of skin and environmental temperature.

A method recently used at the Nutrition Laboratory consists of two copper-constantan junctions, one located in a constant-temperature bath—a Dewar flask—and the other applied to the skin. The resulting current which can be measured on any one of several types of galvanometers is directly proportional to the temperature difference between the two junctions. All thermometric lag is thereby eliminated and it is only necessary that the junction which is applied to the body should be properly protected from the environmental temperature. It was found that when this junction was backed with

a light tuft of cotton batting and installed rigidly in a piece of hard rubber, it was possible to apply it to the body and have it assume the surface temperature inside of a few seconds. Thus the period of application was so short that the protecting material did not sensibly affect radiation, conduction, or the vaporization of water.

In our ordinary skin temperature measurements, the junction in the constant temperature bath is kept at 31° or 32°C. When the other junction is exposed to usual room temperature, there is a large deflection of the galvanometer. On applying the junction to the surface of the skin, the amplitude of deflection decreases rapidly and with negative acceleration. The deflection level is reached in about six seconds after application of the junction. Thereafter the rate of movement of the galvanometer is extremely slow, being but a millimeter or two for each successive minute. The explanation of this is that the junction almost instantly assumes the true skin temperature. Thereafter the skin temperature gradually rises, as the protection of the junction with cotton batting and hard rubber stops the loss of heat by radiation, conduction, and vaporization of water, and there is then a 'building up' of the temperature due to the subcutaneous source of heat. As a matter of fact, observations taken for some time show that this rising temperature continues, the length of time depending in large part upon the amount of hard rubber and cotton batting used. For all of our work we have assumed that under these conditions the true skin temperature is obtained at the end of about six seconds after application of the junction.

In connection with an extended series of metabolism experiments on the influence of temperature environment upon the metabolism, it was found that the subject used (a professional artist's model) could withstand exposure to temperatures as low as 14°C. for several hours without shivering. These conditions presented unusual opportunities for studies of skin temperature which would give evidence first, as to the rapidity of the change on exposure of the body to the environmental temperature, and finally as to the absolute level which the skin temperature reaches after prolonged exposure to an environment varying from 14° to approximately 30°C.

When the subject arrived at the Laboratory, the clothing was loosened and temperature measurements were made at numerous points on the skin under the clothing. Prior to these records, the subject clothed and wearing a heavy coat had been exposed to an external temperature of about 17°C. The results of one series of measurements are recorded in table 1. The extreme range in the skin temperature on this particular day was 6.6°C. Special attention is called to the relatively high value of 31.6°C. on the exposed forehead, the record of approximately 34°C. at the waist, a part well protected by clothing, and values of 30°C. or below on the buttocks, shin, and calf. This series is typical of the skin temperature of a normally-clothed woman.

The method of using isolated points, such as those given in table 1, can be much improved upon by taking advantage of the fact that the thermal junction, when not excessively protected by the cotton batting and hard rubber, almost instantaneously assumes the skin temperature. This is particularly the case if the junction is moved from one part of the skin to another after the hard rubber and cotton have assumed approximately the temperature of the skin. When recording the values of isolated points, we have employed a good grade, direct reading galvanometer, thus making the apparatus compact and portable. The response of the thermal junction is so rapid, however, that when connected with a sensitive aperiodic instrument, i.e., a string galvanometer, it is perfectly possible to move the junction at a moderately rapid rate over the surface of the body and record photographically a continuous and true temperature curve for an infinite number of points on the skin.

TABLE 1

RECORDS OF SKIN TEMPERATURE UNDER THE CLOTHING

MEDIAN LINE		RIGHT SIDE		LEFT SIDE	
Location	°C.	Location	°C.	Location	°C.
Front		*Front*		*Front*	
Forehead	31.6	Neck	32.5	Neck	32.1
Second rib	30.3	Nipple line		Nipple line	
Fourth rib	32.1	Upper chest	33.6	Upper chest	33.7
Lower edge breast bone	33.2	Under breast	33.8	Under breast	33.7
7.0 cm. above umbilicus	34.4	Waist	34.4	Waist	34.7
3.0 cm. below umbilicus	33.4	Groin	34.0	Groin	32.8
11.5 cm. below umbilicus	31.8	Thigh	30.7	Thigh	31.1
		Shin	29.8	Shin	30.9
		Back		*Back*	
		Shoulder blade (above)	33.6	Shoulder blade (above)	33.7
		Shoulder blade (below)	32.5	Waist	33.7
		Waist	33.7	Buttock	29.9
		Buttock	30.5	Thigh	31.4
		Thigh	31.0	Calf	28.2
		Calf	28.1	Hand	32.3
		Hand	32.5		

To illustrate our many results obtained by this latter method, two typical curves are given in figure 1. These were obtained after the nude subject had been exposed to an environmental temperature of 14.6°C. for two and one-half hours, the greater part of the time in the standing position. The upper curve follows the left mammillary line, while the lower curve was taken in a corresponding position on the back. An extraordinary difference in skin temperature at the different points may be noted, the extreme range on the front and back showing differences of 10°C. or more. The rectal temperature taken simultaneously was 36.7°C.

The pronounced influence of the environmental temperature upon the curve for the skin temperature was especially studied by making temperature photographs over exactly the same line at intervals from the moment of disrobing

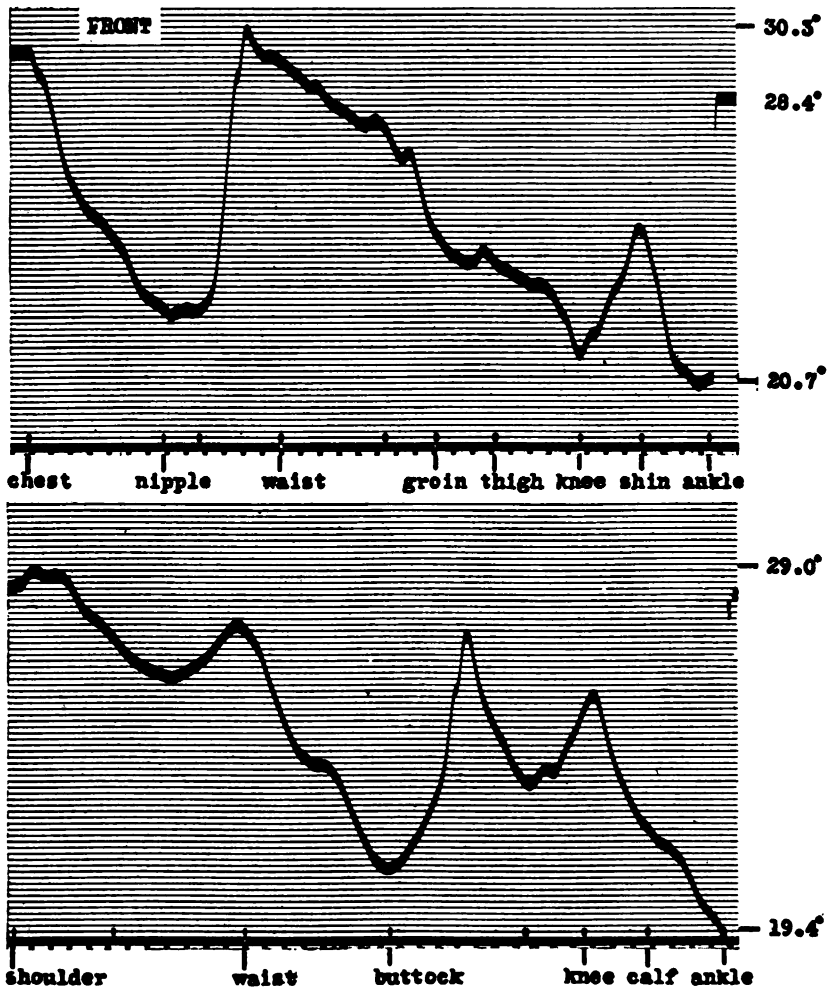


FIG. 1

Photographic records from a string galvanometer for skin temperatures at different parts of the nude body following a 2½ hour exposure to an environmental temperature of 14.6°C. The thermal junction was moved at constant tempo from shoulder to ankle. Time registered at bottom of each curve is in 2-second intervals.

to the end of the period of prolonged exposure to cold. As would be expected, these show a progressive increase in the amplitude of the curve as the time of exposure is prolonged; marked changes occurring even inside of the first fif-

teen minutes. Similar studies made at the end of the exposure to environmental temperatures varying from 14° to 30°C. show a distinct tendency towards a flattening out of the curves at the higher temperatures. Thus when the thermal junction was passed down the front and back of the body over exactly the same lines and at the end of two and one-half hours' exposure, the difference between the highest and lowest points in the curves was 10.6°C. with an environmental temperature of 14.6°C.; 9.8°C. with a temperature of 19°C.; 5.4°C. with a temperature of 25.8°C.; and 4.2°C. with a temperature of 30°C.

This study of the temperature of the skin has two important bearings upon all investigations on the heat production of the human body. First, in all researches on direct calorimetry it has been necessary to corrct the heat actually measured by the calorimeter for the heat gained or lost from the body as the result of changes in temperature. Heretofore it has been assumed that as temperature curves measured either deep in the body trunk or in the artificial cavities are similar, a change of 0.1° in rectal temperature indicates a change of 0.1° in the temperature of the entire body. Our observations, particularly with cold temperature environments, show skin temperatures falling several degrees even when interior body-trunk temperatures may be simultaneously rising slightly. The correction of direct heat measurements by records of the rectal temperature is thus open to grave criticism. Unfortunately no substitute correction can as yet be offered. Secondly, these pronounced differences in skin temperature have great significance in any consideration of the so-called 'law of surface area.' It is still maintained by many physiologists that, practically independent of species, the heat production of the quiet organism is determined by its surface area. Our observations show clearly that, contrary to popular impression, the temperature of the skin, presumably one of the most important factors affecting heat loss, is very far from uniform for we have seen that even with well-clothed individuals differences in the temperature of various localities of 4° to 5°C. are of regular occurrence. These observations bring to light and establish one more serious objection to the legality of the conceptions underlying the 'body-surface law.'

ON A CERTAIN CLASS OF RATIONAL RULED SURFACES

By Arnold Emch

Department of Mathematics: University of Illinois

Communicated by R. S. Woodward, May 19, 1919

As is well known, ruled surfaces, or scrolls as Cayley calls them, may be generated or defined in a number of ways. There exists, for example, a one-to-one correspondence between ruled surfaces and a certain class of partial differential equations, so that the theories of the two classes are abstractly identical.

A much favored method, especially in descriptive geometry, consists in considering ruled surfaces as continuous sets of straight lines, or generatrices, which intersect three fixed curves, the directrices, simultaneously. If these are algebraic curves of orders l, m, n, with no common points, the ruled surface which they determine is, in general, of order $2\, l.\, m.\, n.$

Frequently, ruled surfaces are also defined as systems of elements, either common to two rectilinear congruences, or to three rectilinear complexes.

Of great importance for the following investigation is the definition of ruled surfaces as systems of lines which join corresponding points of an (α, β) — correspondence between the points of two algebraic curves C_m and C_n of orders m and n. If these curves are plane, and if to a point of C_m correspond α points on C_n, and to a point of C_n β points of C_m, then the order of the surface is in general $\alpha\, m + \beta\, n$.

Finally there is the cinematic method in which ruled surfaces are generated by the continuous movement of the generatrix according to some definite cinematical law. In this connection the description of the hyperboloid of revolution of one sheet is well known.

The literature seems to contain but little about this method of generating ruled surfaces. A number of treatises on differential geometry contain chapters on cinematically generated surfaces.

The class of surfaces here considered is obtained as follows: Given a directrix circle C_2 and a directrix line C_1, which passes through the center of C_2 at right angles to the plane of C_2. The generatrix g moves in such a manner that a fixed point M of g moves uniformly along C_2, while g in every position passes through C_1. The plane e through C_1 in which g lies evidently rotates about C_1 with the same velocity $k\theta$ as M. In this plane e, g rotates about M with a uniform velocity $k\mu\theta$. When $\mu = p/q$ is a rational fraction, the surface generated is also rational and belongs to the class of ruled surfaces generated by means of an (α, β) correspondence between C_1 and C_2.

When C_1 coincides with the z-axis, so that C_2 lies in the xy-plane, and we denote by ρ the distance of the projection P' of a point P on the generatrix g from the origin and by θ the angle $P'OX$, the equations of the surface expressed by the parameters ρ and θ are

$$x = \rho \cos \theta,\ y = \rho \sin \theta,\ z = (\rho - a) \cot \frac{p}{q}\, \theta.$$

It is shown that these may be expressed rationally by ρ and another parameter t. Also the implicite cartesian equation of the surface is obtained, as well as are the parametric equations of the double curve of the surface. The following theorems are of interest:

Theorem 1. The surface of the class is rational and of order $2\,(p + q)$ *or* $p + q$, *according as* q *is odd or even.*

Theorem 2. When q *is odd the generatrices of the surface cut* C_1 *and* C_2 *in two*

point sets which are in a $(q, 2p)$ — correspondence. C_1 and C_2 are $2p$-fold and q-fold curves of the surface. The surface has moreover p real and $2pq - 2p - q + 1$ imaginary double generatrices.

When $q = 2s$ is even the generatrices cut C_1 and C_2 in two point sets which are in an also rational (s, p) — correspondence. C_1 and C_2 are respectively p- and s-fold curves of the surface. The surface has no real, but $ps - p - s + 1$ imaginary double generatrices.

In the whole discussion the assumption is made, of course, that p and q are relatively prime.

Theorem 3. When q is odd the order of each of the $(q - 1)/2$ double curves is $4p$ or $2q$ *according as $q \lesseqgtr 2p$. They are rational and each lies on a surface of revolution of order 4 generated by the rotation of an equilateral hyperbola about the z-axis.*

Theorem 4. When $q = 2s$ is even and s odd, there are $(s - 1)/2$ double curves of order $2p$ or q according as $p \gtrless s$, and one double curve of order p or s, according as $p \gtrless s$. When $s = 2\sigma$ is even, there are σ double curves of order $2p$ or q, according as $p \gtrless s$.

The intersections of the double curves with a plane through the z-axis may be arranged according to certain cyclic groups whose orders may be easily determined. One interesting fact is that the surfaces of the class in certain species, are applicable among themselves. The following theorems appertain to this fact:

Theorem 5. Surfaces of the class are applicable to each other when their orders are $2(p + q)$ and $2(mp + nq)$, and the ratio of the radii of their C_2's is m/n, with q odd, p and q, m and n, and m and q as relative primes.

Theorem 6. Surfaces of the class of odd order are applicable to each other when their orders are $p + q$ and $mp + nq$, and the ratio of the radii of their C_2's is m/n. Moreover q is even, and p and m are odd.

As the surfaces of even and odd orders are respectively bifacial and unifacial, we have

Theorem 7. Bifacial and unifacial surfaces of the class are applicable to surfaces of the same type only.

The intersection of a torus with the surfaces of the class yields all so-called cycloharmonic curves. Also the 'bands of Moebius' may readily be cut out from the surfaces.

Among the class here considered are cubic, quartic, and quintic scrolls investigated by Cremona, Cayley, Schwarz and others. Models of these have been and are being constructed in the mathematical model shop of the University of Illinois.

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 5 JULY 15, 1919 Number 7

EVIDENCE OF STREAM MOTION AFFORDED BY THE FAINT STARS NEAR THE ORION NEBULA

By Adriaan van Maanen
Mount Wilson Observatory, Carnegie Institution of Washington

Communicated by G. E. Hale. Read before the Academy, April 28, 1919

The increased accuracy which can be obtained in the determination of proper motions of the stars by long-focus instruments makes it necessary to take into account quantities which otherwise could be neglected. When the probable error of the motion in a coordinate becomes as small as 0."003 or 0."004, we may no longer neglect systematic errors of this size or larger, if they can possibly be determined.

In a discussion of the proper motions of 85 stars in the region of the Pleiades,[1] photographed with the 80-foot focus of the 60-inch Mount Wilson reflector it was found that in the reductions the quadratic terms of the coordinates could not be neglected, notwithstanding the fact that the size of the field was only 24 by 30 minutes of arc. For this and other reasons it was suspected that by neglecting such terms in a former discussion of the proper motions of 162 stars near the Orion nebula,[2] photographed with the 40-inch Yerkes refractor, we had not attained the best possible results. A new solution has therefore been made including the quadratic terms; the necessary corrections are of the same order as those for the Pleiades field and may not be neglected. The comparison reveals the relative efficiency for this kind of work of the long-focus reflector, which seems easily able to withstand a comparison with the refractor.

The fact that we are here dealing with a field in which many of the stars must belong to the nebula, especially in the center of the plates, has its disadvantages, as well as its advantages, for, the motion of the nebula being small, we cannot separate by their proper motion alone the individual stars belonging to the nebular system. This necessitates the exclusion for reference purposes of most of the central stars, whereby the weight of the reductions is considerably lessened.

On the other hand, from the abundance of stars near the Trapezium we can conclude that most of them form a part of the system of the Orion nebula. From their connection with the nebulosity other stars can also be selected as probable members of the group. Except for any internal motion, which, however, can be neglected in a first approximation, all these stars must have the same proper motion; they therefore enable us to drive any magnitude error in case such error does exist.

Another advantage of a field like that discussed here, is that, although the small field measured contains only two or three stars for which meridian observations are available, we can now use for the reduction of our relative to absolute motion all the stars in the catalogues which seem to belong to the system of the Orion nebula. For this purpose twelve stars in Boss's *Preliminary General Catalogue* were found to be available. The motions previously derived were corrected in this way and reduced to absolute motions in Boss's system. The results for different groups of stars are as follows:

13 stars which seem to be enveloped in nebulosity have

$$\mu_\alpha = +0\overset{''}{.}0052 \pm 0\overset{''}{.}0020$$
$$\mu_\delta = -0\overset{''}{.}0024 \pm 0.0024$$

32 stars near the Trapezium have

$$\mu_\alpha = +0\overset{''}{.}0059 \pm 0.0013$$
$$\mu_\delta = -0.0021 \pm 0.0010$$

21 stars, found to be variable by different observers,

$$\mu_\alpha = +0.0046 \pm 0\overset{''}{.}0009$$
$$\mu_\delta = +0\overset{''}{.}0011 \pm 0\overset{''}{.}0011$$

Excluding here three stars for which the variability is uncertain, we find

$$\mu_\alpha = +0.0049 \pm 0\overset{''}{.}0009$$
$$\mu_\delta = -0\overset{''}{.}0003 \pm 0\overset{''}{.}0010$$

From this it is clear that practically all these variables must belong to the system of the Orion nebula and that their variability is therefore due to a physical connection with the nebula.

From the work of Buisson, Fabry and Bourget,[3] we know there is some evidence that the nebula rotates about an axis NW — SE, the NE portion receding, the SW approaching. As their measures are confined to the region within 2′ from the Trapezium, I have investigated whether the stars measured within the same area show the effect of such a rotation. As most of the stars seen in the densest part are likely to be on this side of the nebula, they ought to show a preponderance of motion from SW towards NE. The mean motion in this direction (27 stars) was found to be $0\overset{''}{.}0010 \pm 0\overset{''}{.}0012$, while the mean motion at right angles is $0\overset{''}{.}0000 \pm 0\overset{''}{.}0015$. This small systematic motion is well within the limits of the probable error.

For the background stars, that is, all stars not included in the other groups, the change in the mean values of μ_α and μ_δ with magnitude, and the fair agree-

ment of these values with the parallactic motion, prove that we have been successful in excluding from this group most of the stars which are connected with the nebulosity. As a further test I have constructed the diagrams in Fig. 1. The number of stars and the total motion were determined for sectors of 15°; then to smooth the curves, overlapping means for three successive sectors were formed. In order to avoid the influence of the large proper motions, stars with $\mu > 0''.050$ were excluded. The smaller diagram shows the results for the number of stars, the larger for the total motion in the different directions. If among the background stars we have included a considerable number of nebula stars, we should find a preponderance of both numbers and motion in the direction of $p = 110°$, in which the nebula is moving; the fact

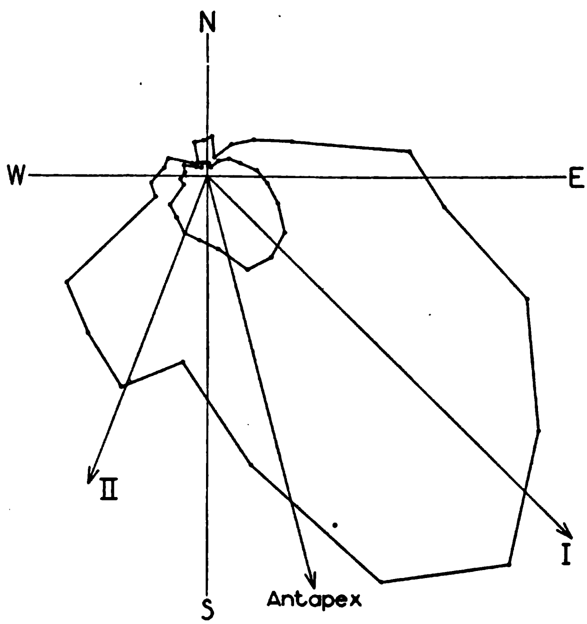


FIG. 1

that no such preponderance appears, is another proof that only a few stars of the nebula system can have been included among the background stars.

The same diagram shows that neither the maximum frequency nor the maximum motion coincides with the direction of the parallactic motion. To investigate whether this fact can be accounted for by stream-motion, I computed the direction of both streams, adopting as coördinates of the vertices:

Stream I, $\alpha = 91°$, $\delta = -15°$
Stream II, $\alpha = 288°$, $\delta = -64°$

We find for

Stream I, $p = 139°$, $\lambda = 13°$
Stream II, $p = 191°$, $\lambda = 108°$

The agreement of the maximum frequency as well as that of the maximum motion with Stream I is quite remarkable, when we take into account the small number of stars available (88), the more so as λ is only 13°; the influence of Stream II also, although not so strong, can be plainly seen.

More material for different parts of the sky will of course be necessary before we can conclude with any certainty that stars as faint as the 14th and 15th magnitudes show the stream-motion.

[1] van Maanen, Adriaan, *Mt. Wilson Contr.*, No. 167, 1919.

[2] van Maanen, Adriaan, *Astr. J.*, *Albany, N. Y.*, 27, 1912 (139–146).

[3] Buisson, H., Ch. Fabry, and H. Bourget, *Astrophys. J.*, *Chicago, Ill.*, 40, 1914 (241–258).

[4] Eddington, A. S., *Steller Movements*, *London*, 1914, page 100.

ON THE USE OF THE SPECTROSCOPIC METHOD FOR DETERMINING THE PARALLAXES OF THE BRIGHTER STARS

By W. S. Adams and G. Strömberg

Mount Wilson Observatory, Carnegie Institution of Washington

Read before the Academy, April 28, 1919

When the method of deriving the luminosities and the parallaxes of stars by means of the intensities of certain lines in their spectra was developed by Adams and Kohlschütter a few years ago the applicability of the method to the stars of highest luminosity and smallest parallax was necessarily somewhat uncertain. This was due to the fact that the method depends upon the calibration of a scale of line-intensities by means of stars of known parallax and magnitude, and that at this time the observational material for stars of small parallax was necessarily scanty and subject to relatively large percentage errors. All that could be done was to select a few stars with the best parallaxes, so far as could be estimated, and base upon them a set of provisional reduction-curves for the spectroscopic determinations. As a result the values for the absolute magnitudes derived in this way while sufficiently accurate to indicate clearly that the parallaxes of certain stars were very small were not of such a quality as to show in all cases the slight differences between individual stars of small parallax.

The situation has improved greatly in recent years. On the one hand the parallaxes of a large number of stars have been measured with high accuracy by various observers with the aid of photographic methods. On the other hand the amount of spectroscopic material for the stars of various magnitudes and proper motions has accumulated to such an extent that use can be made of the extremely valuable method of determining mean parallaxes for groups of stars from the parallactic motion. Accordingly it has now become possible

to subject the spectroscopic method to a rigorous test of its applicability to the stars of high luminosity and small parallax. The importance of this question is evident when we consider that the trigonometric method of measurement with a probable error which is nearly independent of the size of the parallax can give but approximate values of the absolute magnitude for stars of very small parallax. Thus if the measured parallax of a star of the fifth magnitude is $+0\overset{''}{.}010 \pm 0\overset{''}{.}005$ its absolute magnitude may lie anywhere within the limits -1.5 and $+0.9$, and for smaller parallaxes the range is much larger. Since a great majority of the brighter stars in the sky have small parallaxes the use of the spectroscopic method in which the error is more nearly proportional to the size of the parallax is of fundamental importance for such cases.

As a first step in the investigation a new determination of the constants of reduction for the intensities of the spectral lines was made in 1918 with the aid of the large number of measured parallaxes then available. The absolute magnitudes based upon this system have been divided into groups and comparisons have been instituted with the values obtained from parallactic motion and all available measured parallaxes. In this way we can determine whether the absolute magnitude varies continuously with the line-intensity; and it is possible to use a large number of measured parallaxes including stars of different proper motion and apparent magnitude.

The results of the comparison are shown graphically in the accompanying figures. The spectroscopic absolute magnitudes M_s are plotted as abscissae, the line-intensities being indicated on parallel axes. The absolute magnitudes M_c derived from measured parallaxes and from parallactic motion are plotted as ordinates. The full-line curve represents the values computed from parallactic motion; the broken curve those from measured parallaxes. The wide break in the curves for the K_4–K_9 and the M stars are due to the well-known division of these stars into the giant and dwarf classes.

It is at once evident that the computed absolute magnitudes vary continuously with the spectroscopic magnitudes quite as well among the stars of highest luminosity as among the fainter stars. In fact the largest discordance is found not among the brighter stars but among some of the faintest stars of groups A_7 to F_8 where the luminosities as computed from measured parallaxes show but little variation with line-intensity and for the very faintest stars appear to act in the wrong direction. It is to be noted that in this comparison the more accurate result for the more distant stars is given by the parallactic motion and for the nearer stars by the measured parallaxes.

It is clear that if there are no systematic corrections to be applied to the spectroscopic absolute magnitudes used in this comparison the curves should consist of straight lines with an inclination of 45°. The results show that these corrections are relatively small and that the spectroscopic criteria hold throughout the range of magnitude investigated.

It should be realized fully that the method of deriving parallaxes from the intensities of the spectral lines depends for its accuracy upon our knowledge of the parallaxes of the stars used for the determination of the relationship between absolute magnitude and line-intensity. It is a process of approximation in the early stages of which the corrections may be considerable, but which

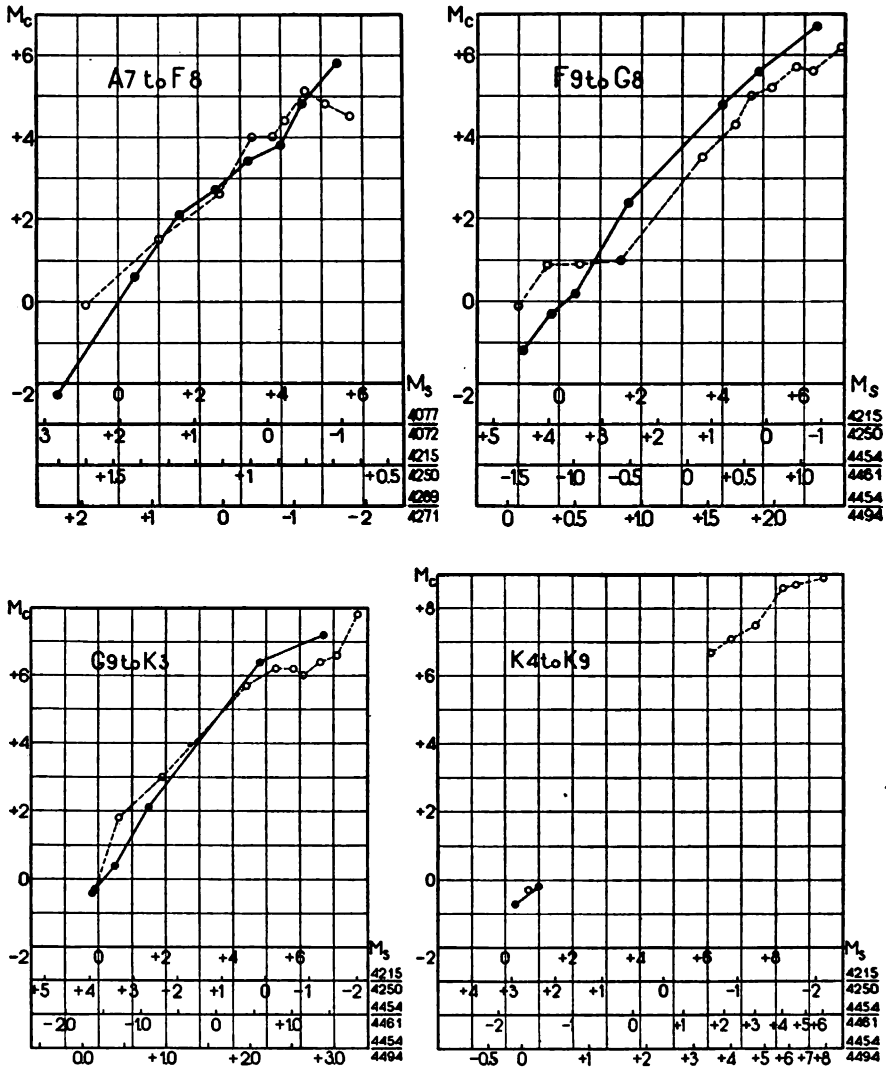


FIG. 1. CURVES SHOWING THE RELATIONSHIP OF THE SPECTROSCOPIC ABSOLUTE MAGNITUDES (M_S) TO THE MAGNITUDES AS DERIVED FROM PARALLACTIC MOTION AND MEASURED PARALLAXES (M_C)

The line intensities upon which the spectroscopic magnitudes are based are also given as abscissæ.

gains more and more in accuracy as the material upon which it is based becomes larger and more reliable.

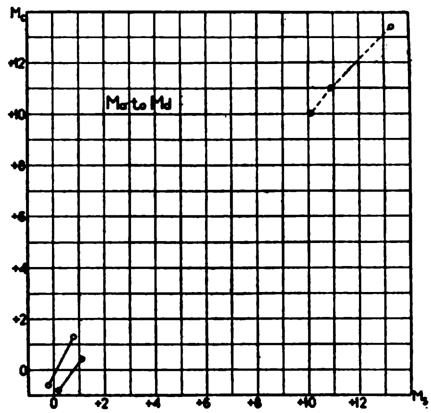


FIG. 2

To illustrate the present state of agreement between the spectroscopic and the trigonometric parallaxes the following table is added for purposes of comparison. The stars are divided into groups and the limits of proper motion for each group are given under the heading μ. The symbols, $\overline{M}$ and $\bar{\pi}$ represent the average absolute magnitude and parallax, respectively. The last column of the table shows the differences between the spectroscopic and the trigonometric parallaxes. The agreement may certainly be regarded as satisfactory in view of the number of stars involved.

Apparent Magnitudes 3.5 to 6.5

μ	No.	$\overline{M}$	$\bar{\pi}_{spec.}$	$\bar{\pi}_{trig.}$	$\bar{\pi}_s - \bar{\pi}_t$
≦0.″020	23	−0.4	0.″009	+0.″008	+0.″001
0.″021–0.″040	19	+0.2	0.013	0.007	+0.006
0.041–0.070	36	+0.3	0.013	0.016	−0.003
0.071–0.100	29	+0.7	0.016	0.021	−0.005
0.101–0.150	38	+0.8	0.021	0.026	−0.005
0.151–0.200	31	+2.3	0.039	0.038	+0.001
0.201–0.300	35	+2.7	0.042	0.043	−0.001
0.300–0.500	51	+3.2	0.042	0.042	0.000

In conclusion a few words may be added about the extent to which the spectroscopic method of determining parallaxes has been applied at Mount Wilson. Between 1500 and 1600 stars have now been investigated in this way including nearly all stars with trigonometric parallaxes and an extensive list of those with very large and very small proper motions. It is of interest to note that in this large amount of observational material hardly a single serious contra-

diction has been found between the spectroscopic and the trigonometric results. This is the more remarkable since we should expect occasional idiosyncracies in the spectra of stars subject to exceptional physical conditions, and the rareness of such cases is interesting evidence for the uniformity of the development of stellar spectra in general.

RELATION OF COLOR TO INTRINSIC LUMINOSITY IN STARS OF THE SAME SPECTRAL TYPE

By Frederick H. Seares

Mount Wilson Observatory, Carnegie Institution of Washington

Communicated by G. E. Hale. Read before the Academy, April 28, 1919

It has long been known that the color of a star is intimately related to its spectral type. More recently it has been found that color is also correlated with intrinsic brightness.[1] Thus, two stars having different luminosities, but the same spectrum so far as the usual criteria are concerned, may differ in color-index by more than half a magnitude.[2]

The phenomenon is perhaps to be accounted for as follows: Similarity in spectrum implies at least approximate equality in surface brightness. Difference in luminosity is therefore mainly a matter of size, the brighter object being the larger. But inequality of dimensions doubtless entails differences in atmospheric constitution, and hence also in the selective absorption occurring in the atmospheres. Since color is determined by the distribution of intensity in the continuous spectrum, while type relates more particularly to the characteristics of the spectral lines, one star may thus appear redder or bluer than another, although both are of the same type.

But whatever the explanation, an extension of the results of Adams, van Rhijn, Monk, and others to include a wide range of luminosity for each spectral type is immediately desirable, because of their bearing upon the difficult problem of stellar constitution. Further, any phenomenon correlated with stellar luminosity should be studied to the utmost, on account of its possible importance for the determination of interstellar distances. If the intrinsic brightness of a star can be found, the problem of its parallax is solved.

The method employed for the measurement of color in securing the preliminary results here described is that of exposure-ratios[3], which is convenient and reliable, and much more expeditious than the direct determination of color-index.

An isochromatic plate exposed behind a yellow filter records the intensity of the 'yellow' light received from the star. The same plate used without filter registers the 'blue' light; the longer wave-lengths are then also active, but the image, nevertheless, is essentially blue, owing to the relatively great blue-sensitiveness of the isochromatic plate. In neither case are we dealing with monochromatic light, or even with a very narrow range of color. That, how-

ever, is of no great consequence; it is only important that the mean wavelengths for 'blue' and 'yellow' should differ by a considerable amount. The relative intensity for the two spectral regions will then vary with the color of the star, and in consequence may be used as an index of its color.

Practically, however, it is simpler to use as a measure of color the ratio of the exposure times which will yield blue and yellow images of equal intensity, or, better still, the logarithm of this ratio. Thus we make a series of exposures to blue, usually 2^s, 4^s and 8^s, and then one or more to yellow, such that the resulting images fall within the limits of size fixed by the first and last blue exposures. For an early-type star the yellow exposures are generally 32^s and 64^s, for an intermediate type 16^s and 32^s, and for a late type 8^s and 16^s. The logarithm of the exposure-ratio for equal blue and yellow images is then estimated directly from the plate. With a normal emulsion the logarithms for the principal spectral types (ratio of blue to yellow for stars of zero absolute magnitude) are approximately as follows: Bo, 8.79; Ao, 8.94; Fo, 9.10; Go, 9.26; Ko, 9.44; Ma, 9.55.

Troublesome photographic difficulties are avoided by using images of constant size produced by a constant series of exposure times, compensation for differences in apparent magnitude being made by varying the aperture of the telescope. With the 60-inch reflector stars as faint as the eleventh magnitude can be observed with the exposures given above.

The photographic emulsion is changed as infrequently as possible, for every change necessitates a systematic correction, and indeed each plate usually requires a small correction. From five to ten stars are ordinarily observed on each plate; and, since the adopted value for a star is based on several plates, the measures themselves afford a means of reducing the individual plates to a homogeneous standard system of colors.

The measures require a correction for atmospheric extinction, but since all the observations on a plate are made at about the same zenith distance, this is assumed to be constant and is determined as a part of the systematic correction for the plate. Errors depending on seeing and atmospheric conditions generally also enter into the plate correction and are thus largely eliminated from the results.

The internal consistency under favorable conditions is illustrated by the accompanying observations of the two components of 61 Cygni which extend over an interval of about two years. The logarithms of the exposure-ratios for the two stars, which were photographed simultaneously, are in the first two columns of the table. The average deviation of the differences in the third column corresponds to about 0.08 magnitude, which indicates the uncertainty in a single determination of the difference in color of two stars on the same plate. This differential result, which is free from systematic plate-errors, is equivalent to an uncertainty of 0.05 or 0.06 magnitude in one measurement of the color of a single star, a quantity of the same order as the average deviations found directly from the measures of each of the two components,

whence we conclude that the plate-errors have been successfully eliminated. The complete observation of a star, including the setting of the telescope, requires seven or eight minutes.

Exposure-ratio—Components of 61 Cygni

LOG R_1	LOG R_2	DIFFERENCES	v_1	v_2	v_3
9.39	9.50	−0.11	+0.03	−0.02	+0.05
9.41	9.47	−0.06	+0.01	+0.01	0.00
9.43	9.45	−0.02	−0.01	+0.03	−0.04
9.39	9.50	−0.11	+0.03	−0.02	+0.05
9.43	9.45	−0.02	−0.01	+0.03	−0.04
9.42	9.48	−0.06	0.00	0.00	0.00
9.42	9.47	−0.05	0.00	+0.01	−0.01
9.42	9.47	−0.05	0.00	+0.01	−0.01
9.44	9.45	−0.01	−0.02	+0.03	−0.05
9.40	9.48	−0.08	+0.02	0.00	+0.02
9.40	9.49	−0.09	+0.02	−0.01	+0.03
9.39	9.50	−0.11	+0.03	−0.02	+0.05
9.41	9.49	−0.08	+0.01	−0.01	+0.02
9.39	9.49	−0.10	+0.03	−0.01	+0.04
9.48	9.51	−0.03	−0.06	−0.03	−0.03
9.44	9.50	−0.06	−0.02	−0.02	0.00
Means 9.416	9.481	−0.065	±0.019	±0.016	±0.028 (log R)
1.09	1.27	−0.18	±0.053	±0.045	±0.078 (magnitude)

Results are now available for about 150 stars whose spectral types and absolute magnitudes have been derived mainly from three sources: the extensive and valuable lists of spectroscopic parallaxes by Adams and Joy, which include types F, G, K, and M;[4] the A stars known to belong to the Taurus Stream;[5] and the B stars whose parallaxes have been determined by Kapteyn.[6]

The weight of the results is very unequal; some of the stars used as standards have been observed a dozen or more times, while for others only a single observation is available. The nature of the correlation between color and luminosity is nevertheless clearly enough indicated, although quantitatively it will be subject to considerable revision.

The discussions thus far undertaken are very simple. The stars having absolute magnitudes between +1 and −1 were separated fom the others and arranged according to spectral type. By plotting spectrum against exposure-ratio a curve was obtained (circles, fig. 1;) which shows the relation of these two factors for stars of zero absolute magnitude. The dwarf stars of the later spectral types, whose mean absolute magnitudes are 6 or fainter, were similarly treated, thus giving the second curve (points) shown in fig. 1.

For $M = 0$ the variation is nearly linear up to K5; but beyond this point there seems to be no further increase in color with advancing spectrum, the latest K's (K6) and the various sub-classes of the M stars all having approxi-

mately the same color. Further, the result noted by earlier observers is at once evident, namely, that the giant G and K stars are redder than the dwarfs of the same spectrum. The difference becomes zero for F_5, and is again zero, or at least very small, for the M stars, although the difference in luminosity between the giants and dwarfs is here about 10 magnitudes with a corresponding range in intensity of 1 to 10,000.

For further study of the relation of color to luminosity we consider groups of stars having the same spectral type. As they stand, the data are insufficient; but provisionally we proceed by reducing all the stars within any spectral sub-division to the mean spectrum with the aid of the curves of fig. 1. Thus all the B stars are reduced to B5 by applying to the logarithms of the

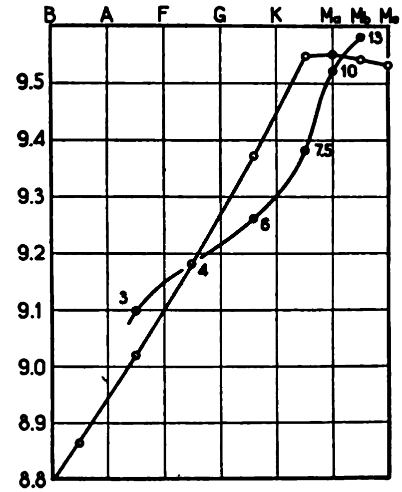


FIG. 1. ORDINATES: COLOR (LOGARITHM OF EXPOSURE-RATIO)
ABSCISSÆ: SPECTRAL TYPE

The open circles represent mean valves for stars of zero absolute magnitude. The dots refer to stars of fainter luminosity, the respective mean absolute magnitudes being indicated by the accompanying figures.

exposure-ratio of each a small correction which makes the colors very nearly what they would have been had the types actually been B5 throughout. The six groups of results thus obtained have been plotted separately—logarithms of exposure-ratios as ordinates against absolute magnitudes as abscissae—and are shown in figs. 2 and 3.

The results for the B stars are to be regarded as altogether provisional. They suggest that the color decreases, that is, becomes bluer, as the intrinsic brightness decreases. The A stars, all of which belong to the Taurus stream, increase in color with increasing absolute magnitude. This result seems definite, and is confirmed by the Fo and F1 stars derived from the lists of Adams and Joy, which are shown by the crosses in fig. 2. When reduced to A5 they agree well with the A stars of the Taurus stream. A further confirmation is

found in the fact that these A stars, whose absolute magnitudes, with one exception, lie within the range $M = +1.2$ to $+3.0$, agree with the curve for $M = 0$ determined by the adjacent spectral types (see fig. 1,) only after their colors have been reduced to $M = 0$ in accordance with the relation shown in fig. 2.

The giant and dwarf F stars in the mean have sensibly the same color, but when the individual results are plotted we find pretty definite evidence of a change in color as indicated in fig. 2, first an increase and then a decrease in color with increasing M, the maximum lying somewhere near $M = +2$.

The results for the G's and K's (fig. 3;) are qualitatively certain beyond any question, the giants as before remarked being very much redder than the dwarfs. For the K stars the difference in the corresponding color indicates for $M = -1$ and $+7$ is more than 0.6 magnitude. This large difference is

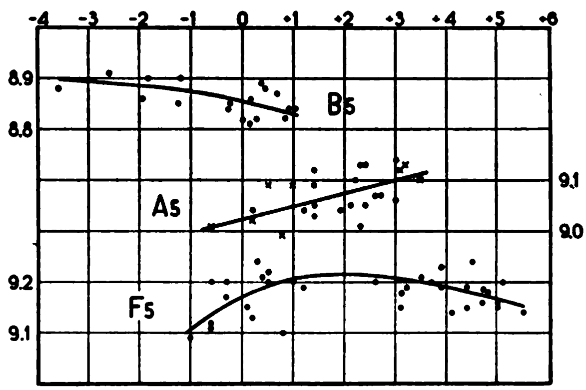


FIG. 2. VARIATION OF COLOR (LOGARITHM OF EXPOSURE-RATIO, AS ORDINATE) WITH ABSOLUTE MAGNITUDE (ABSCISSA) FOR B, A AND F STARS

The points represent individual stars whose colors have been reduced by the curves of Fig. 1 to the mean spectra, B5, A5, etc., for the respective classes. The crosses in the A5 group are F0 and F1 stars reduced to A5.

immediately suggestive of the errors that may occur in attempting to infer spectral type from color, or to determine visual magnitude from photographic with the aid of the spectral type, as is not infrequently done.

Among the dwarfs themselves, neither for the G's nor the K's, does there appear to be much change of color, although the faintest K's suggest that a minimum occurs at $M = +7$ and that from there on the color increases with increasing M.

The last curve of fig. 3 illustrates the similarity of color for the giant and dwarf M stars.

In spite of the diversity of the curves, relationships for adjacent spectral classes can usually be traced. Thus the curve for the A stars seems to be related to the ascending branch of that for the F's, while the descending branch of the latter curve reappears in the results for the G's and K's. The behavior

of the M's is peculiar, and owing to the lack of late K types the transition cannot certainly be traced. The interrelation of the B and A stars is also open to question. From the present results we should infer that for Ao stars there is little or no dependence of color upon absolute magnitude. This indeed seems to be the case. For example, Vega and the companion to o^2 Eridani, both Ao and differing intrinsically by more than ten magnitudes, are very nearly of the same color.

The change in the relation of color to luminosity from type to type is so rapid that a closer subdivision of the spectral classes is desirable, but this must await the accumulation of further data.

Aside from the bearing of these results upon the problem of stellar constitution, which it would be premature to consider at present, it is perhaps worthy

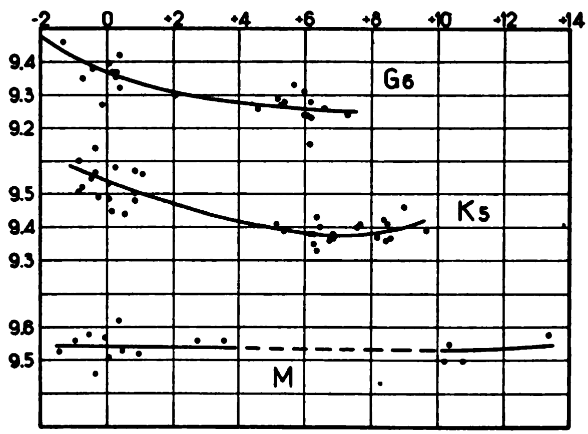


FIG. 3. VARIATION OF COLOR (LOGARITHM OF EXPOSURE-RATIO, AS ORDINATE WITH ABSOLUTE MAGNITUDE (ABSCISSA) FOR G, K, AND M STARS

All the G and K stars have been reduced in color G6 and K5, respectively. Ma. Mb, and Mc stars have been combined without correction for color.

of remark that they may occasionally be useful for determinations of distance Thus in the present case the results afford a useful control on the parallax of the Taurus Stream. As previously mentioned, it is only when reduced to zero absolute magnitude with the aid of the curve for the A5 stars that the colors of the Taurus stars are brought into agreement with the smooth curve for zero absolute magnitude defined by the adjacent spectral types (see fig. 1). Conversely, had the parallax of the Taurus Stream been unknown, it could have determined from the data now available with an indicated uncertainty of about 20 per cent, or $\pm 0''.005$, which is comparable with the precision of the result usually adopted.

The use of any such method, however, presupposes a knowledge of the spectral types of the stars in question. But if spectra are available, the spectroscopic method of determining distance can immediately be employed; at least

this will be true if stars of intermediate or late type are present. The results afforded by observations of color would however be useful as a control on spectroscopic or trigonometric determinations of parallax, although they might not add greatly to our knowledge of stellar distances. But if the method depending on color measurements could be extended in such a way as to obviate the necessity of direct spectral observations, the advance would be of importance, for it could then be applied to faint stars beyond the reach of both trigonometric and spectroscopic methods of measuring distance.

Theoretically such a procedure seems not impossible. Suppose, for example, that we observe three spectral regions instead of two, and that we combine the results in such a way as to yield two exposure-ratios, $\log R\ (\lambda_1, \lambda_2)$ and $\log R\ (\lambda_1, \lambda_3)$, λ_1, λ_2, and λ_3 representing the effective wave-lengths of the three regions in question. Presumably both these quantities would be functions of the spectral type S and the absolute magnitude M; hence, we could write

$$\log R\ (\lambda_1, \lambda_2) = F_1\ (S, M)$$
$$\log R\ (\lambda_1, \lambda_3) = F_2\ (S, M)$$

from which it might be possible to determine both S and M, and therefore, finally, the parallax, with the aid of the usual formula

$$5 \log \pi = M - m - 5$$

in which m is the apparent magnitude.

It must be admitted, however, that the trials thus far made hold out no great promise of success, owing to the fact that the coefficients in the two equations involving S and M are so nearly proportional as to render the solution indeterminate. The spectral regions used were in the violet, blue, and yellow. These were tried first because of photographic considerations; but others may prove better adapted to the requirements of the problem, and it is proposed to give the question further attention.

[1] Adams, W. S., *Mt. Wilson Contr.* No. 78, *Astrophys. J., Chicago*, **39**, 1914 (89–92).

Also results by van Rhijn; quoted by Kapteyn in *Mt. Wilson Contr.* No. 83; *Astrophys. J.*, **40**, 1914 (187–204); and later given in detail in "Derivation of the Change of Color with Distance and Apparent Magnitude," Groningen, 1915.

Adams, W. S., and Arnold Kohlschütter, *Mt. Wilson Contr.* No. 89, *Astrophys. J. Chicago*, **40**, 1914 (385–398).

Monk, G. S., *Mt. Wilson Contr.* No. 119, *Astrophys. J.*, **44**, 1916 (45–50).

Seares, F. H., *Pub. A. S. P., San Francisco, Calif.*, **30**, 1918 (99–133).

Lindblad, Bertil, *Arkiv för Mathematik, Astronomi och Fysik, Stockholm*, **13**, No. 26, 1918 (1–75).

[2] Seares, F. H., loc. cit.

[3] Seares, F. H., These Proceedings, **2**, 1916 (521–525).

[4] Adams, W. S., and A. H. Joy, *Mt. Wilson Contr.* No. 142, *Astrophys. J.*, **46**, 1917 (313–339).

[5] Kapteyn, J. C., *Mt. Wilson Contr.* No. 147, p. 76, *Astrophys. J.*, **47**, 1918 (104–133, 146–178, 255–282), p. 266.

[6] Kapteyn, J. C., *Mt. Wilson Contr.* Nos. 82, 147, *Astrophys. J.*, **40**, 1914 (43–126), **46**, 1918 (104–133, 146–178, 255–282).

THE MOTIONS IN SPACE OF SOME STARS OF HIGH RADIAL VELOCITY

By W. S. Adams and A. H. Joy

Mount Wilson Observatory, Carnegie Institution of Washington

Read before the Academy, April 28, 1919

The motion in space of the stars of very high radial velocity is of exceptional interest because such stars are usually of comparatively low intrinsic brightness and probably of small mass as well. It might, therefore, be expected that these stars would show the effects of stream motion strongly and that the components of their velocities might be related to the fundamental plane of the stellar galaxy.

As a basis for the study we selected all stars with radial velocities exceeding 80 km. per second for which proper motions and parallaxes are available. This gave a total of 37 stars with velocities ranging between 81 and 339 km., the latter the highest stellar radial velocity so far known. It is of interest to note that the largest velocities of approach and of recession are very nearly equal. The parallaxes of all but five of these stars have been derived by the spectroscopic method, the trigonometric values being used for the remainder.

After elimination of the solar motion the velocity-components and the apices of the stars relative to the centroid of the stellar system were determined by the aid of their proper motions, parallaxes and radial velocities. These components were in turn reduced to the plane of the galaxy and the galactic coördinates of the apices and the total velocities were calculated for all the stars.

Some of the results of the investigation are shown graphically in the accompanying figure. The plane is that of the galaxy with the apices shown in projection. Vectors drawn to the origin represent the direction and amount of motion in this plane.

A few of the more important conclusions may be indicated briefly.

1. The highest velocity in space found for any star is 494 km. for the ninth magnitude star A. G. Berlin 1366. Several other stars with larger parallaxes show values of nearly this amount. In all cases the components of motion in the plane of the galaxy exceed greatly those at right angles to it.

2. For the stars as a whole we find the two components in the galactic plane very nearly equal and more than two and one-half times as great as the component perpendicular to it. We find a similar result from a consideration of the latitudes of the apices. Only 6 out of the 37 stars have apices with latitudes exceeding 30°. It is clear, therefore, that the influence of the galactic condensation is very marked upon the motion of these stars.

3. Nearly an entire hemisphere in longitude is devoid of apices, the values all lying between 131° and 322°.

4. The velocity of the centroid of these stars is remarkably high. Thus even if we omit total velocities greater than 300 km. we find a value for the centroid of 74 km. This motion is almost wholly in the galactic plane.

5. The effect of stream motion among these stars is very marked. The axes for which the sum of the squares of the projected velocities is a maximum and a minimum have longitudes of 141° and 61°, respectively, and latitudes of +9° and −49°. The projections of these axes upon the galactic plane are

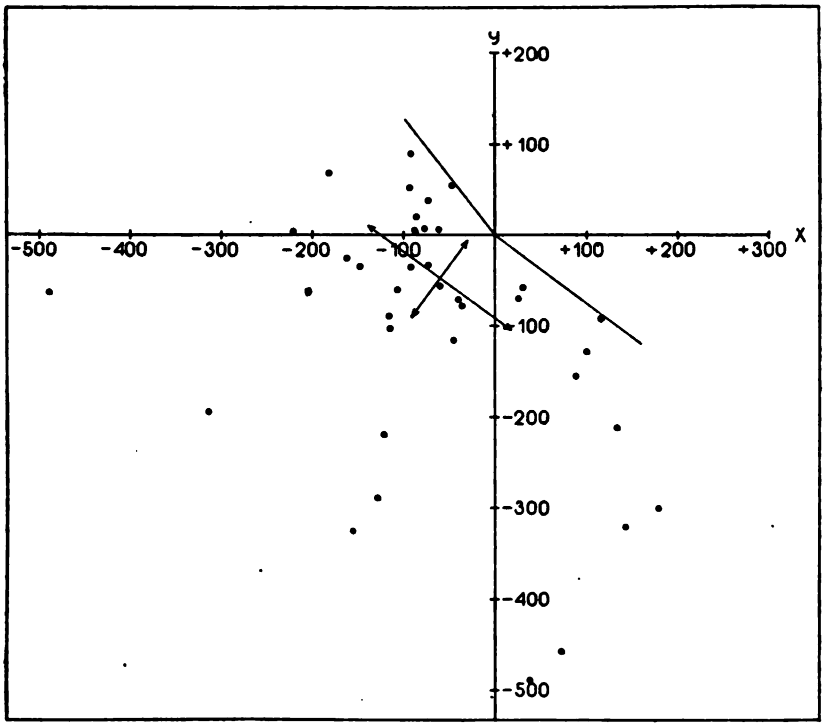


FIG. 1. PROTECTION ON GALACTIC PLANE OF APICES OF MOTION OF 37 STARS OF HIGH RADIAL VELOCITY

The vectors drawn from the origin represent the projected velocities in km per sec. The axes of the ellipse of intersection of the velocity-ellipsoid with the galactic plane are indicated by the two arrow-headed lines.

indicated in the figure. The mean square dispersion along the major axis is over twice that along the minor axis. The value for the major axis is in close agreement with that found by Strömberg from a discussion of the radial velocities of 260 dwarf stars, and with that of Raymond from 559 stars of large proper motion. All of these investigations indicate that the galactic longitude of the principal vertex for the stars of high velocity is considerably less than that for stars in general.

6. There appears to be some tendency for the stars to move along a line of galactic longitude about 260°, allowance being made for the motion of the centroid. This direction coincides nearly with that of the greatest star density as determined by various observers.

7. The stars of highest velocity, over 300 km., also appear to move along a line parallel to that of the major axis when the motion is referred to their common center. This is shown in the figure by the positions of the apices.

8. The average galactic latitude of the apices of the stars of high luminosity is nearly twice as great as that of the fainter stars, the values being 26° and 14°. The latitudes of the apices of only four of the fainter stars exceed 26°.

9. The average space-velocity of the stars of low luminosity is much larger than that of the brighter stars. Twenty-eight stars of average absolute magnitude 5.9 show a velocity of 216 km. Nine stars of absolute magnitude 0.4 a velocity of 130 km.

10. An extraordinarily large proportion of the stars in this list, 26 out of 37, are of types F and G. The successive types F, G, K and M show average space-velocities of 307, 156, 122 and 121 km. respectively, the weight of the determination for the last two types being rather low. Among the stars of type F those of earlier spectral type show the larger velocities. Thus the six stars with spectra between F_0 and F_5 inclusive have an average velocity of 365 km. as against 307 km. for all stars of the F type.

The most important result of this brief investigation is the evidence for the marked influence of the condensation of matter in the plane of the stellar galaxy upon the motions of these relatively faint stars. Their susceptibility to stream motion is perhaps another result of the same general influence which is, no doubt, gravitational in character. Probably the most peculiar fact in connection with these stars is the spectral type of the stars of highest velocity. That a type which we are accustomed to consider as intermediate in the scale of stellar development should contain so large a proportion of the most rapidly moving stars is difficult of explanation unless we may assume that these stars are of exceptionally small mass. Since the relationship of velocity to absolute magnitude seems to be fairly well established this hypothesis may be worthy of consideration.

DEVIATIONS OF THE SUN'S GENERAL MAGNETIC FIELD FROM THAT OF A UNIFORMLY MAGNETIZED SPHERE

By F. H. Seares, A. van Maanen, and F. Ellerman
Mount Wilson Observatory, Carnegie Institution of Washington

Communicated by G. E. Hale. Read before the Academy, April 28, 1919

A communication to the Academy in 1917[1] describes the method used for the location of the sun's magnetic axis and gives values for the inclination, the period of revolution about the solar axis of rotation, and the epoch when the magnetic pole was on the central meridian. These results were based upon observations made on 63 days between June 8 and September 25, 1914. In the meantime the spectrograms of 11 additional days within the limits of this interval have been measured and reduced, and the elements defining the position of the axis have been revised with the following results (appended quantities are probable errors):

$$i = 6^{\circ}\!.0 \pm 0^{\circ}\!.4, \quad P = 31.52 \pm 0.28 \text{ days}$$
$$t_0 = 1914, \text{ June } 25.38 \pm 0.42 \text{ days, G. M. T.}$$
$$k = 0.99 \text{ (a constant inversely proportional to the polar field-strength)}$$

The modifications produced by the revision are unimportant, and, so far as the 1914 series of observations is concerned, these values may be accepted as final.

The uncertainty in the period is naturally large, for it has been derived from data covering less than four complete revolutions of the magnetic axis. Moreover, this limitation prevents any conclusion as to the constancy of the period. An improvement of the results in these particulars therefore requires additional observations, which should be distributed over a long interval. A beginning in this direction was made with a short series in September, 1916, which should reduce the uncertainty in the period well below a tenth of a day and make it possible to carry the longitude of the pole forward, without ambiguity as to the number of revolutions, to the coming sun-spot minimum, when further observations can be undertaken wihout risk of interference from the magnetic fields of spots.

The 1916 observations are now under discussion, and the indications are that the longitudes to be derived from them will agree closely with those calculated from the 1914 results. The period given above is therefore probably near the true value.

All discussions thus far described rest upon the assumption that the sun's general field is that of a uniformly magnetized sphere. The example of the earth, however, suggests that this hypothesis may be justified only to some rough degree of approximation, and that the field may not possess the uniformity hitherto presupposed.

We have therefore rediscussed the data from this standpoint, with results which seem to be conclusive. The investigation has had to meet the difficulty that the percentage errors in the data, in the nature of the case, are large; but there seems no doubt that there are irregularities in the field-strength at least, which cannot be accounted for on the basis of systematic or accidental error.

In order that the numerical quantities involved in the above solution—which we may refer to as the uniform-field solution—might be utilized as completely as possible, we have based the discussion upon the formulae there used, namely,

$$\left.\begin{array}{c} Ax + By = \Delta \\ x = k^{-1}\cos i, \qquad y = k^{-1}\sin i \cos\lambda \\ Y = y/x = \tan i \cos\lambda \end{array}\right\} \tag{1}$$

A and B are known quantities, functions of the heliographic latitudes of the sun's center and the points observed; Δ is the observed displacement of the spectral line; and i and λ are the inclination of the magnetic axis and the longitude of the magnetic pole. Y is a function of i and λ, whose value is calculated for each day from all the measured Δ's for that day. The discussion of the resulting series of Y's then leads to the required values of i, t_0, P, and finally to k, which is connected with H_p, the field-strength at the ple, by the relation

$$k = \frac{4}{CH_p}, \tag{2}$$

in which C is a constant depending on the spectral line employed.

For the uniform-field solution, the values of Y were calculated separately for three different lines. For the investigation of the deviations from the uniform field, the measures of the three lines were combined (this is possible since the values of x for the three lines are sensibly equal) and the entire collection of data then subdivided according to zones of heliographic latitude as follows:

$$\begin{array}{rl} \text{I} & \varphi > +10^\circ \\ \text{II} & +10^\circ \gtreqless \varphi \gtreqless -10^\circ \\ \text{III} & \varphi < -10^\circ \end{array}$$

The outer limites for forces I and III vary somewhat, but are approximately 45°.

To the data within these limits we have applied equations (1), much as described above. In other words, we have derived separate uniform-field solutions for each of the zones. Had the three series of results agreed within the uncertainties affecting their determination, we could only have concluded that there is no evidence for the existence of appreciable deviations from the uniform field originally presupposed. As a matter of fact, we have found a large difference for k, which leads to the conclusion expressed above.

Certain details require comment. The fundamental equation—the first of (1)—cannot be applied directly to either zones I or III, because of the numeri-

cal values of the coefficients A and B. There is no change in algebraic sign and the range in their values is so small that the simultaneous determination of x and y becomes practically indeterminate. For the middle zone the conditions are more favorable, but even here it has not seemed advisable to attempt a direct solution. Since the quantity x, as shown by the uniform-field solution, is apparently constant throughout the entire interval, we have preferred to solve by successive approximations, beginning with an assumed value of x.

We therefore write the fundamental equation in the form

$$B \tan i \cos \lambda = \Delta x' - A, \tag{3}$$

where for convenience $x' = k/\cos i$ has been substituted for $1/x$. The value of x' from the uniform-field solution is 1.00, and this we use for the first approximation in deriving $Y = \tan i \cos \lambda$ from (3). We have for each zone, from all the data for each day, the normal equation

$$[BB] \tan i \cos \lambda = [B\Delta]\, x' - [AB] \tag{4}$$

The individual values of Y are presumably in error, because of the assumed value for x'. It is easily shown, however, that neither the phase nor the period of the curve $Y = \tan i \cos \lambda$ is thereby affected. The entire effect goes into the amplitude. Using the first approximation for x', we can therefore determine t_0 and P for each zone, free from error, exactly as in the case of the uniform-field solution. The curves for Y show that the results in each case are sensibly the same as those previously found, and we therefore adopt for each zone the values given above.

Since the weight of $\tan i \cos \lambda$ given by (4) is $[BB]$ the normal equation for $\tan i$ is

$$\tan i\, \Sigma\, [BB] \cos^2 \lambda = x'\, \Sigma\, [B\Delta] \cos \lambda - \Sigma\, [AB] \cos \lambda \tag{5}$$

where the outer summation symbol covers all the separate days. With this equation we find for the three values of i

$$\text{I} \quad 4^\circ\!.9 \pm 0^\circ\!.7; \qquad \text{II} \quad 6^\circ\!.7 \pm 0^\circ\!.5; \qquad \text{III} \quad 3^\circ\!.7 \pm 1^\circ\!.0 \tag{6}$$

With the aid of these values we now determine from the measures of each day a new approximation for x', using the normal equation

$$[\Delta\Delta]\, x' = [A\Delta] + [B\Delta] \tan i \cos \lambda \tag{7}$$

which is at once derived from the fundamental equation. Noting that the weight of each x' is $[\Delta\Delta]$ and combining the different days, we have for the mean x'

$$x'\Sigma[\Delta\Delta] = \Sigma[A\Delta] + \tan i\, \Sigma[B\Delta] \cos \lambda \tag{8}$$

The results of the second aproximation for x' are

$$\text{I} \quad 0.97 \pm 0.009; \qquad \text{II} \quad 0.60 \pm 0.018; \qquad \text{III} \quad 0.96 \pm 0.008 \tag{9}$$

It thus appears that for zones I and III the original assumption was nearly correct; for the second zone, however, the change in x' is large.

For the further approximations we have from (5) and (8), respectively,

$$d(\tan i) = \alpha dx' \tag{10}$$
$$dx' = \beta d(\tan i) \tag{11}$$

where

$$\alpha = \frac{\Sigma\,[B\Delta]\cos\lambda}{\Sigma\,[BB]\cos^2\lambda}, \qquad \beta = \frac{\Sigma\,[B\Delta]\cos\lambda}{\Sigma\,[\Delta\Delta]} \tag{12}$$

For zones I and III, α and β are small quantities of the order of 0.01 or 0.02, and since dx', the differences between the first and second approximations for x', are only -0.03 and -0.04, respectively, equations (10) and (11) show that the corresponding values of i and x' given in (6) and (9) are final.

For zone II, $\alpha = 0.120$ and $\beta = 1.42$. We have found already, in order, $x'_1 = 1.00$, $\tan i_1 = 0.118$, $x'_2 = 0.60$, whence $dx' = -0.40$; and by successive substitution into (10) and (11) we derive $\tan i_2 = 0.070$, $x'_3 = 0.53$, $\tan i_3 = 0.062$, $x'_4 = 0.52$, $\tan i_4 = 0.061$. As no further change is produced by additional substitutions, x'_4 and $\tan i_4$ are adopted as the final values.

Calculating k from x' and collecting results, we have

$$\begin{array}{llll} \text{For } \varphi = 45^\circ\text{ N to }10^\circ\text{ N}, & i = 4^\circ\!.9, & k = 0.96 \\ \phantom{\text{For } \varphi} = 10\text{ N to }10\text{ S}, & = 3.5, & = 0.52 \\ \phantom{\text{For } \varphi} = 10\text{ S to }45\text{ S}, & = 3.7, & = 0.96 \end{array}$$

$$P = 31.52 \pm 0.28 \text{ days}$$
$$t_0 = 1914,\ \text{June } 25.38 \pm 0.42 \text{ days, G. M. T.}$$

The uncertainty in the inclinations averages about $0^\circ\!.7$, while that in the k's is one or two units of the second decimal. Since the inclinations, periods, and epochs for the three zones are sensibly the same, the deviations from a uniform spherical field within the region from 45° N. to 45° S. are apparently restricted to the polar field-strength, which is inversely proportional to k. Here the variation with latitude is unexpectedly large—far greater than the uncertainty of the calculation. Unless the observations are affected by a large systematic error, which is itself a rather complicated function of the magnitude of the observed displacement Δ, the change must be real.

The measures have all been made with the parallel-plate micrometer and it is difficult to believe that errors of the kind required to make the result illusory can have entered. Our experience would lead us to believe that important variations in the field-strength of the character indicated actually exist.

It is unexpected to find the i's for all the zones smaller than that from the uniform-field solution. It is easily seen, however, that the two series of results stand in the proper relation to each other. In the uniform-field solution the displacements within the limits of zones I and III contribute about fifteen

times as much to the weight of k as do those in zone II. The mean result for k should therefore be sensibly the same as the values for zones I and III when treated separately, as proves to be the case (k = 0.99, 96, and 96, respectively). The weight of the inclination, on the other hand, is derived largely from the observations of the equatorial zone. Moreover, i is calculated after k has been found, so that the conditions in the uniform-field solution are analogous to those of the first approximation of zone II. In this approximation the equatorial observations combined with $x' = 1.00$—the equivalent of $k = 0.99$—gave $i = 6\overset{\circ}{.}7$, which is nearly the same as the value found from all the data. The measures of zones I and III combined with $k = 0.99$, as we know from the zonal analysis, must lead to relatively small values for i, amounting to about 4°. The mean therefore lies between these two extremes, with a preponderance of weight in zone II. This accounts for the value of $6\overset{\circ}{.}0$ originally found—a mean result which is in excess of all the inclinations found by treating the zones separately. It is only when the equatorial observations are discussed by themselves that the true value of k for this region reveals itself or has any appreciable influence upon the solution; but when once found the inclination is necessarily decreased.

We are under great obligation to Miss Wolfe of the Computing Division who has rendered much assistance with the extensive numerical calculations required for the discussion of the data.

[1] Seares, F. H., van Maanen, A., and Ellerman, F., these PROCEEDINGS 4, 1918, (4–9).

ON THE PROBLEM OF THE PRODUCTION OF FAT FROM PROTEIN IN THE DOG

BY H. V. ATKINSON AND GRAHAM LUSK

CORNELL UNIVERSITY MEDICAL COLLEGE, NEW YORK CITY

Read before the Academy, April 28, 1919

In eight experiments in a series of thirteen, after giving meat in large quantities (700 to 1300 grams) to a dog weighing 11 kgm., the respiratory quotients during the height of protein metabolism were between 0.793 and 0.800. A calculation showed that under these conditions the retained carbon residue of the protein metabolized was held back in such a form that, had it been oxidized, the respiratory quotient of this retained pabulum would have been 0.85. This would represent the oxidation of material half of whose calories were composed of fat and half of carbohydrate. The dog showed quotients of 0.82 and above only after the larger quantities of meat were given (1000 grams or more). It was extremely difficult to induce the dog to take meat in sufficient quantity to indicate a considerable production of fat from protein. Incidentally it was observed that the basal metabolism of a dog fed with meat in large quantity for a time and then caused to revert to a 'standard diet'

(meat, 100 grams; lard, 20 grams; and biscuit meal, 100 grams) remained persistently (even after 2½ weeks) at a higher level than had obtained prior to the meat ingestion. This confirms F. G. Benedict's idea of a higher basal metabolism in the presence of 'surplus' cellular nitrogen, or the 'improvement quota' of protein according to Rubner's suggestive terminology. When meat was given after the partial depletion of the body cells of their 'improvement quota', protein was retained in greater measure, less protein was metabolized and the heat production was therefore lower than on a subsequent day (experiments 46, 47; 51, 54).

The following table gives the method of calculation followed:

Experiment 55. N in urine per hour = 1.58 grams

	grams		grams		
$N\text{-}CO_2$	14.77	$N\text{-}O_2$	13.35	N-cals	41.89
Resp. CO_2	11.42	Resp. O_2	10.42	Deposit cals	9.92
Dif	3.35		2.93	Indirect cals	31.97
				Direct cals	31.98

R. Q. of deposit = 0.83

R. Q. per hour—0.81, 0.74, 0.80, 0.84. R. Q. for whole period = 0.797

In the above table:

$N\text{-}CO_2$ is the amount of CO_2 derivable from the protein metabolism during an hour.

$N\text{-}O_2$ is the amount of O_2 necessary to oxidize the protein metabolized in one hour.

Resp. CO_2 and Resp. O_2 are the amounts of CO_2 and O_2 which were actually respired during an hour.

The difference represents (1) the CO_2 which would have been expired had all the retained carbon of the protein metabolism been oxidized and (2) the O_2 which would have been employed in that process. The relation between the volumes of these two gases indicates that the material retained and unoxidized would have yielded a respiratory quotient of 0.83, which indicates the retention of a pabulum, approximately half of the calories of which were derived from fat and half from glucose.

N-cals. is the quantity of heat which would have been eliminated by the dog had all the protein metabolized by the dog been completely oxidized. From this is subtracted the number of calories estimated to have been retained as a mixture of fat and glucose aforesaid. The difference represents the calories as calculated by *indirect calorimetry*, which in this case agrees exactly with those directly measured by the calorimeter by the method of *direct calorimetry*.

The following summary of results is appended.

Effect of meat ingestion on hourly metabolism

EXP. NO.	DATE	FOOD	NUMBER OF HOURS	URINE N	R. Q.	CALORIES Indirect	CALORIES Direct	C DEPOSITED	R. Q. OF DEPOSIT	HOURS AFTER FOOD
	1919									
27	Feb. 6	Basal	2	0.15	0.84	15.92	16.08			(Weight = 11.24 kgm.)
29	Feb. 17	Meat, 700 g.	3	1.40	0.820	29.97	30.01	0.60	0.73	4, 5, 6
30	Feb. 18	Meat, 800 g.	3	1.46	0.800	31.47	32.75	0.66	0.83	4, 5, 6
31	Feb. 19	Meat, 900 g.	3	1.47	0.787	34.33		0.48	0.96	4, 5, 6
32	Feb. 20	Meat, 1000 g	2	1.46	0.797	34.27	34.50	0.42	0.87	4, 5
			2	1.46	0 820	35.50	37.20	0.23	0.62	6, 7
33	Feb. 21	Meat, 1100 g.	2	1.45	0.831	31.65	31.36	0.56	0.68	4, 5
			2	1.45	0.843	35.28	34.54	0.25	0.49	6, 7
34	Feb. 24	Meat, 1080 g.*	2	1.57	0.800	34.00	34.12	0.70	0.83	4, 5
35	Feb. 26	Basal	3	0.27	0.82	19.74	19.59			(Weight = 12.07 kgm.)
36	Feb. 27	Basal	3	0.20	0.83	18.25	17.16			
37	Feb. 28	Basal	2	0.17	0.85	17.30	16.95			
38	Mar. 1	Basal	2	0.15	0.82	18.21				
39	Mar. 3	Basal	3	0.15	0.85	17.57	17.22			
43	Mar. 12	Basal	2	0.15	0.81	17.08	16.99	0		(Weight = 11.50 kgm.)
46	Mar. 17	Meat, 1200 g.	3	1.02	0.796	26.57	28.10	0		5, 6, 7 after 1 day's fast
47	Mar. 18	Meat, 800 g.	3	1.44	0.795	29.90	30.77	0.77	0.84	5, 6, 7
48	Mar. 19	Meat, 800 g.	4	1.35	0.793	29.37	30.27	0.61	0.86	5 to 8
49	Mar. 22	Basal	2	0.23	0.79	17.72	17.54			
50	Mar. 24	Basal	2	0.16	0.84	17.26	16.87			
51	Mar. 28	Meat, 800 g.	4	1.02	0.795	27.04	27.52			5 to 8 after 4 days' fast
54	Apr. 15	Meat, 800 g.	4	1.41	0.794	31.07	30.57	0.59	0.86	5 to 8
55	Apr. 16	Meat, 1000 g.	4	1.58	0.797	31.97	31.98	0.91	0.83	5 to 8
56	Apr. 19	Meat, 1300 g.	4	1.47	0.826	31.62	33.25	0.59	0.71	5 to 8 after 1 day's fast

* Standard diet at 5 p.m. and thereafter daily until March 15.

ON THE TWIST IN CONFORMED MAPPING

By T. H. Gronwall

Range Firing Section, Aberdeen Proving Ground

Communicated by E. H. Moore, April 29, 1919

Note II on Conformal Mapping under aid of Grant No. 207 from the Bache Fund.

Let $w = w(z)$ be a power series in z, convergent for $|z|<1$ and such that the circle $|z|<1$ is mapped conformally on a *simple* (that is, simply connected and nowhere overlapping) region in the w-plane. By a linear transformation $w^1 = aw + b$, we may reduce $w(z)$ to the form $z + a_2z^2 + \ldots + a_nz^n +$

. . . . For the comparison of maps of different regions, two geometrical concepts are of fundamental importance: the *distortion* = $|dw/dz|$, or the ratio of the lengths of corresponding line elements in the w- and z-planes, and the *twist* = imaginary part of log (dw/dz), or the angle between corresponding line elements, this angle being always taken between $-\pi$(excl.) and π (incl.). Koebe[1] has shown that on the circle $|z| = r$, $0 < r < 1$, both $|dw/dz|$ and $|z|$ lie between positive bounds which depend on r alone, and the writer[2] has determined the exact values of these bounds.

It is the purpose of the present note to state the corresponding result in regard to the twist:

When the analytic function

$$w = z + a_2z^2 + \ldots + a_nz^n + \ldots$$

maps the circle $|z| < 1$ *on the interior of a simple region D in the w-plane, the twist* τ *satisfies the following inequalities for* $|z| = r$ *and* $0 < r < 2^{-\frac{1}{2}}$

$$-4 \arcsin r < \tau < 4 \arcsin r, \tag{1}$$

except when

$$w = \frac{z(1 - \cos\beta \cdot e^{\alpha i}z)}{(1 - e^{(\alpha+\beta)i}z)^2}, \tag{2}$$

where α *and* β *are real, and* $\cos\beta = r$. *In this case,* τ *attains the upper or lower bound in* (1) *for* $z = re^{-\alpha i}$ *according as* $\beta = +\text{arc cos } r$ *or* $\beta = -\text{arc cos } r$. *For* $2^{-\frac{1}{2}} \leqq r < 1$, *we have only the inequality included in the definition of* τ

$$-\pi < \tau \leqq \pi, \tag{3}$$

and no single class of functions analogous to (2) *reaching the upper and lower bounds can be assigned.*

When the region D is CONVEX, *we have for* $|z| = r$ *in the whole interval* $0 < r < 1$

$$-2 \arcsin r < \tau < 2 \arcsin r \tag{4}$$

except when

$$w = \frac{z}{1 - e^{(\alpha+\beta)i}z}, \quad \cos\beta = r, \tag{5}$$

the upper and lower bounds being then attained as above.

The proof is similar to that of the distortion theorem outlined before.[2] The region D is approximated by rectilinear polygons, for which we have the formula of Schwarz

$$\frac{dw}{dz} = (1 - e^{\alpha_1 i}z)^{\mu_1}(1 - e^{\alpha_2 i}z)^{\mu_2} \ldots (1 - e^{\alpha_m i}z)^{\mu_m} \tag{6}$$

whence, for $z = re^{\theta i}$,

$$\tau = \Sigma_1^m - \mu_\nu \arctan \frac{r \sin(\theta + \alpha_\nu)}{1 - r\cos(\theta + \alpha_\nu)} + 2k\pi i, \tag{7}$$

where the arctangents are all taken between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$, and the integer k is chosen so as to make $-\pi<\tau\leqq\pi$. Since the polygon is not overlapping, we have

$$\Sigma\mu_\nu = -2, -3\leqq\mu_\nu\leqq 1 \quad (\nu= 1,2, \ldots,m)$$

For a fixed value of m, we consider the set of all the values of $\alpha_1, \ldots, \alpha_m, \mu_1, \ldots, \mu_m$ for which (6) maps $|z|<1$ on a *simple* polygon, and it is first shown that this set is *closed*. On this set, and for $-\pi\leqq\theta\leqq\pi$ the expression (7) has therefore a maximum and a minimum, which are found by observing that

$$\arctan\frac{r\sin\varphi}{1-r\cos\varphi}$$

has a maximum = arcsin r for φ = arccos r and a minimum = − arcsin r for φ = − arccos r. In the discussion of (7), it is necessary to distinguish the case where all μ are negative, corresponding to a convex polygon, from the general case where some μ are positive; for this reason, the convex regions appear separately in the statement of the theorem. By considerations of continuity, and the use of elementary properties of harmonic functions, it is finally shown that the upper and lower bounds in (1) and (4) are reached in the cases (2) and (5) only, and that no closer bounds than (3) can be found for $2^{-\frac{1}{2}}\leqq r<1$ in the non-convex case.

Regarding (2) and (5), we observe that by rotating the z- and w-planes through the angle $-\alpha$ about their origins, we may make $\alpha = 0$; in this case, the circle $|z|<1$ is mapped by (2) on the w-plane slit along the straight line segment

$$w = \frac{i}{4\sin\beta} - i e^{-2\beta i}t, \qquad (0\leqq t\leqq + \quad),$$

and by (5) on the half plane in which the real part of $we^{\beta i}$ is greater than $-\frac{1}{4}$.

[1] Koebe, *Göttingen, Nachr. Ges. Wiss.*, 1909, (73).
[2] Gronwall. *Paris, C. R. Acad. Sci.*, 162, 1916, (249).

SEVENTEEN SKELETONS OF MOROPUS; PROBABLE HABITS OF THIS ANIMAL

By Henry Fairfield Osborn
American Museum of Natural History, New York City

Read before the Academy, April 29, 1919

Moropus is an aberrant perissodactyl, closely related to the family of the Titanotheres and more remotely to that of the Horses. It occurs in the Lower Miocene age in France and North America, and its ancestors have been

traced back to the Upper Eocene in both countries; it is thus of Holarctic distribution, and while very rare, it must have been perfectly adapted to its environment, because it survived the majority of perissodactyls and occurs in the Pliocene of Europe and England and will not improbably be found in the North American Pliocene.

The habits and habitat of the animal have always presented a very difficult problem. The skeleton presents the most noteworthy exception to Cuvier's law of correlation. All the foot bones which were discovered since Cuvier's

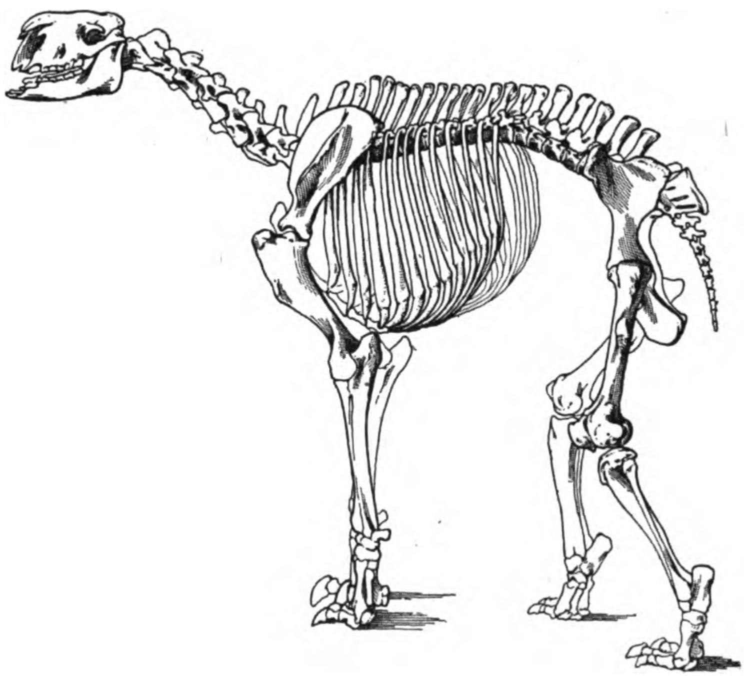


FIG. 1

Mounted skeleton of *Moropus cookei* in The American Museum of Natural History. One of the seventeen. Drawing one twenty-sixth natural size.

time consisted of large deeply-cleft terminal phalanges and were grouped with the edentates, especially the plantigrade sloths. All the teeth which were discovered, on the other hand, were grouped with the perissodactyl ungulates. It was not until H. Filhol discovered the nearly complete skeleton of *Macrotherium* that he was enabled to prove that the chalicotheres were of composite adaptive structure, with the teeth of perissodactyls and the claws of edentates. *Macrotherium* is very similar to the American *Moropus*.

Great light was thrown upon the structure of *Moropus* through the explorations of the Carnegie Museum by Holland and Peterson, described in 1914, from materials collected in the famous Agate Spring Quarry, Sioux County,

Nebraska, discovered by James H. Cook in 1897. After the lapse of the Carnegie researches and explorations, the American Museum entered this quarry and through five years of continuous exploration (1911–1916) an irregular area within a square of about 36 feet yielded nearly complete skulls of ten individuals and skeletal parts of seventeen individuals all together. From this wonderful material it has been possible to supplement the descriptions of Holland and Peterson and to present for the first time the proportions and pose, by which we may estimate the habits of this animal. We reach the conclusion that the Moropus type was not plains living, but forest living; that it was the seclusion of the forests which protected this type and which accounts for its great rarity in fossil deposits, for it is characteristic of forest-living forms that they are not readily entombed. We form an entirely different conception of the habits of the animal when we observe the extremely long fore limbs, which are not of the digging or fossorial type, and which thus belie the apparently fossorial or digging structure of the terminal phalanges. It appears more probable that these terminal claws were used partly for purposes of offense and defense, but largely for the pulling down of the branches of the trees. The animal was probably forest living like the African okapi, with which in its general body and head proportions it has many analogies. Like the okapi it survived through retreat to the recesses of the forests.

THE STRUCTURE OF ATOMS AND THE OCTET THEORY OF VALENCE

By Irving Langmuir

Research Laboratory, General Electric Company, Schenectady, New York

Read before the Academy, April 29, 1919

In a paper soon to be published in the *Journal of the American Chemical Society*, I will give a new theory of the structure of atoms and molecules based upon chemical data. This theory, which assumes an atom of the Rutherford type, and is essentially an extension of Lewis' theory of the 'cubical atom,'[1] may be most concisely stated in terms of the following postulates.

1. The electrons in atoms are either stationary or rotate, revolve or oscillate about definite positions in the atom. The electrons in the most stable atoms, namely, those of the inert gases, have positions symmetrical with respect to a plane called the equatorial plane, passing through the nucleus at the center of the atom. No electrons lie in the equatorial plane. There is an axis of symmetry (polar axis) perpendicular to the equatorial plane through which four secondary planes of symmetry pass, forming angles of 45° with each other. These atoms thus have the symmetry of a tetragonal crystal.

2. The electrons in any given atom are distributed through a series of concentric (nearly) spherical shells, all of equal thickness. Thus the mean

radii of the shells form an arithmetric series 1, 2, 3, 4, and the effective areas are in the ratios $1:2^2:3^2:4^2$.

3. Each shell is divided into cellular spaces or cells occupying equal areas in their respective shells and distributed over the surface of the shells according to the symmetry required by postulate 1. The first shell thus contains 2 cells, the second 8, the third 18, and the fourth 32.

4. Each of the cells in the first shell can contain only one electron, but each other cell can contain either one or two. All the inner shells must have their full quotas of electrons before the outside shell can contain any. No cell in the outside layer can contain two electrons until all the other cells in this layer contain at least one.

5. When the number of electrons in the outside layer is small, these electrons arrange themselves over the underlying ones, being acted on by magnetic attractive forces. But as the charge on the kernel or the number of electrons in the outside layer increases, the electrostatic repulsion of the underlying electrons becomes predominant and the outer electrons then tend to rearrange themselves so as to be as far as possible from the underlying ones.

6. The most stable arrangement of electrons is that of the pair in the helium atom. A stable pair may also be held by: (*a*) a single nucleus; (*b*) two hydrogen nuclei; (*c*) a hydrogen nucleus and the kernel of another atom; (*d*) two atomic kernels (very rare).

7. The next most stable arrangement of electrons is the octet; that is, a group of eight electrons like that in the second shell of the neon atom. Any atom with atomic number less than twenty, and which has more than three electrons in its outside layer tends to take up enough electrons to complete its octet.

8. Two octets may hold one, two, or sometimes three pairs of electrons in common. One octet may share one, two, three or four pairs of its electrons with one, two, three or four other octets. One or more pairs of electrons in an octet may be shared by the corresponding number of hydrogen nuclei. No electron can be shared by more than two octets.

The inert gases are those having atoms in which all the cells in the outside shell have equal numbers of electrons. Thus according to the first four postulates the atomic numbers of the inert gases should be 2, 10, 18, 36, 54, and 86 in agreement with fact.

All atoms with an atomic number greater than that of helium, have as their first shell a pair of electrons close to the nucleus. The line connecting the two electrons establishes the polar axis for the atom. Neon has in its second shell eight electrons, four in each hemisphere (i.e., above and below the equatorial plane), arranged symmetrically about the polar axis. The eight electrons are thus nearly at the corners of a cube. In argon there are eight more electrons in the second shell.

The eight electrons in the third shell of the atom of iron are arranged over the underlying electrons in the second shell. The two extra electrons in the

atom of nickel are placed in the polar axis. Beyond nickel the electrons in the atom cannot be held by magnetic forces, and thus tend to rearrange themselves so as to be placed as far as possible from the underlying electrons. This leads to an explanation of the chemical and magnetic properties of copper, zinc, etc.

Krypton has in its third shell nine electrons in each hemisphere, symmetrically placed with respect to the polar axis and to the four electrons in the second shell. The ninth electron in each hemisphere goes into the polar axis. Xenon is like krypton, except that it has twice as many electrons in its third shell. Beyond Xenon eighteen electrons in the fourth shell can be held by magnetic forces over the eighteen cells of the third shell, so that lutecium, the eighteenth element beyond Xenon marks the last of the rare earth elements. The electrons in the outside shell of the atoms beyond this element are arranged as far as possible so as to leave eighteen empty spaces over the underlying electrons. In this way it is possible to explain the chemical and magnetic properties of tantallum and tungsten as contrasted to those of the rare earths.

Niton has sixteen electrons in each hemisphere of its fourth shell. These are placed symmetrically with respect to the polar axis and the eight underlying electrons.

This theory of atomic structure explains in a satisfactory way most of the periodic properties of all the elements including those of the eighth group and the rare earths. It lends itself especially well to the explanation of the so-called physical properties, such as boiling-points, freezing-points, electric conductivity, etc. For the details of its application to specific elements the paper in the *Journal of the American Chemical Society* should be consulted.

Postulates 6, 7 and 8 lead directly to a new theory of valence which we may call the Octet Theory. This theory may be stated in terms of the equation

$$e = 8n - 2p \tag{1}$$

where e is the total number of available electrons in the shells of all the atoms in a molecule; n is the number of octets forming the outside shells of the atoms and p is the number of pairs of electrons held in common by the octets (Postulate 8). If we let E be the number of electrons in the 'shell' of an atom then $e = \Sigma\,(E)$. The value of E for a given atom, at least in case of the first twenty elements, corresponds to the ordinal number of its group in the periodic table. Thus we have the following values of E:—one for hydrogen, lithium, sodium, etc., two for magnesium, three for boron, aluminum, etc.; four for carbon and silicon, five for nitrogen and phosphorus; six for oxygen and sulphur; seven for the halogens, and zero for the inert gases.

The above equation expresses the fact that every pair of electrons held in common between two octets results in a decrease in two in the total number of electrons needed to form the shells of the atoms in the molecules. It also implicitly expresses the fact that *all the electrons* in the shells of the atoms forming a molecule form part of one or two of the octets in the molecule.

It seems that this simple equation is a practically complete statement of a theory of valence that applies with very few exceptions to all compounds formed from the first twenty elements. With some modifications it applies also to most compounds of other elements. In the case of organic compounds it is found that each pair of electrons held in common between atoms corresponds exactly to the valence bond used in the ordinary theory of valence. It is therefore proposed to define valence as the number of pairs of electrons which a given atom shares with others. In view of the fact known that valence is very often used to express something quite different, it is recommended that the word *covalence* be used to denote valence defined as above.

Equation (1) expresses the fact that the number of covalence bonds in a molecule must be related to the number of available electrons in the molecule. A simple mathematical analysis[2] shows that all structural formulas written according to the ordinary valence theory in which the valence for each element is taken equal to 8–E, will satisfy Equation (1). Thus the Octet Theory requires no modification in any formula written with the following valencies; carbon-four, nitrogen-three; oxygen-two; chlorine-one and hydrogen-one. In some cases, however, the Octet Theory suggests that other formulas besides those usually adopted may be possible. Whenever the old theory of valence has assumed valencies other than those mentioned above, such as five for nitrogen or phosphorus; four or six for sulphur; three, five or seven for chlorine, etc., the Octet Theory gives quite different structural formulas from those usually assumed. This is readily seen when it is considered that the covalency of an element according to the Octet Theory can never exceed four, since there are only four pairs of electrons in an octet.

A careful examination of the cases showing a discrepancy between the old and new theories furnishes the strongest kind of evidence in support of the Octet Theory. The non-existence of such compounds as H_4S, H_6S, SCl_6, NCl_5, NH_5, etc., is in full accord with the theory as is also the existence of SO_2, SO_3, N_2O_5, HNO_3, NH_4Cl, etc. In these latter cases, however, the formulas written are different from those usually adopted. For example, the covalence of sulphur is three in SO_2, four in SO_3; that of nitrogen is four in N_2O_5, HNO_3, and NH_4Cl. These covalencies are, however, not *assumed* as in the ordinary valence theory, but are *derived* from Equation (1), which is the *same* equation as that which applies to all ordinary organic compounds. In a similar way it is found that the Octet Theory fully explains the structures of such compounds as N_2O, N_2O_3, N_2O_4, HN_3, $N(CH_3)_4Cl$, H_3PO_3, H_3PO_4, $HClO$, $HClO_2$, $HClO_3$, $HClO_4$, H_2O_2, and even so-called complex compounds such as $B(CH_3)_3$, NH_3, $K_2PtCl_2.2NH_3$, KBF_4, Na_2S_5, etc.

From this viewpoint a very large number of compounds previously considered by Werner are found to be typical primary valence compounds not essentially different in their structures from organic compounds. It is especially significant that the structure of these compounds is found from Equation (1), without any additional assumptions. Thus we are lead to a *single theory* of

valence which explains and coördinates the separate valence theories that we have needed in the past.

There are many facts not previously well understood which are very readily explained by the new theory. For example, the fact that we have weak and strong acids, weak and strong bases, but no 'weak' and 'strong' salts, is automatically explained.

The theory indicates that all salts consist of negative and positive ions even when in the solid condition, and that no pair of electrons are held in common between the negative and positive groups. Thus in sodium chloride the covalence of both sodium and chlorine is zero, and this fact explains the non-existence of molecules of sodium chloride shown by the X-ray crystal analysis. When sodium chloride is dissolved in water the water does not cause ionization, but simply causes the separation of atoms already ionized. This direct result of the Octet Theory is in full accord with experiment and with Milner's recent theory of strong electrolytes. *London, Phil. Mag.*, **35**, 1918, (214, 354).

In the field of organic compounds the theory fits the facts particularly well. Although in the case of compounds like SF_6, H_2SiF_6, etc., there is very definite evidence that the *kernels* of the atoms of sulphur and silicon are cubical in shape, there is the strongest evidence that in organic compounds the carbon atom has the eight electrons of its octet drawn together into four pairs, arranged at the corners of a tetrahedron. This is in full accord with the fact that in SF_6, and H_2SiF_6, the central atom has given up electrons to the surrounding atoms, so that the cubical kernels do not share any electrons with the other atoms, while in organic compounds the carbon atoms always share four pairs of electrons with adjacent atoms. From this we must conclude that a pair of electrons held in common by two octets acts as if it were located at a point between the two atoms. This conclusion, which can be reduced from the properties of a very few simple organic compounds is found to apply apparently without exception to compounds of nitrogen, sulphur, phosphorus, and even cobalt compounds, etc. It seems to explain all the cases of stereoisomerism that I am familiar with. For example, in the amine oxides, $NR_1R_2R_3O$, nitrogen is quadricovalent, so that these substances exist as optical isomers, just as in the case of a carbon atom attached to form different groups.

The isomerism of compounds of tervalent nitrogen such as ketoximes, hydrazones, ozazones, and diazo-compounds, etc., is readily accounted for, as well as the absence of isomers among tertrary amines, etc. Not only are the substituted ammonium compounds fully explained, but also the sulfonium, phosphonium, and oxonium compounds. Thus the structures of $S(CH_3)_3OH$, $S(CH_3)_4Cl_2$, $(C_2H_5)_2O.HCl$, etc., are readily found from the Octet Theory and their salt-like character is explained. The covalence of the central atom in the above compounds is three, four and three respectively.

When the values of *e* and *n* are both the same for two or more compounds it is evident according to the Octet Theory that these may have practically identical structures. An example of this kind is found in N_2 and CO. The total

number of electrons in each molecule (including those in their kernels) is fourteen. Evidence is given in the paper in the *Journal of the American Chemical Society* that the structures of these two molecules is identical, except for the fact that in one case there are two nuclei of seven positive charges each, while in the other there are nuclei of six and eight charges, respectively. These molecules are, however, exceptional, in that the molecule consists of a *single octet* arranged around a complex kernel.

Another example of a pair of compounds which according to the Octet Theory should have similar structures occurs in the case of CO_2 and N_2O. For each of these molecules $n = 3$, $e = 16$ and therefore $p = 4$. The best method of testing this conclusion lies in comparing the '*physical*' properties of the two substances. The 'chemical' properties depend primarily on the ease with which the molecules can be broken up, and thus is a measure of the internal forces within the molecule, which depend to a large extent on the charges on the kernels. The so-called 'physical' properties on the other hand depend on the stray field of force outside of the molecule, and this naturally depends rather on the arrangement of the outside electrons.

As a matter of fact we find that most of the physical properties of these two gases are practically identical.

The following data taken from Landolt-Börnstein tables and Abegg's hand book illustrate this.

	N_2O	CO_2
Critical pressures	75	77 atmos.
Critical temperatures	35.4°	31.9°
Viscosity at 20°C.	148×10^{-6}	148×10^{-6}
Heat Conductivity at 100°C.	0.0506	0.0506
Density of Liquid at −20°	0.996	1.031
Density of Liquid at +10°	0.856	0.858
Refractive index of liquid. D line, 16°C	1.193	1.190
Dielectric constant of liquid at 0°	1.598	1.582
Magnetic susceptibility of gas at 40 atmos. 16°C.	0.12×10^{-6}	0.12×10^{-6}
Solubility in water 0°	1.305	1.780
Solubility in alcohol at 15°	3.25	3.13

Both gases form hydrates $N_2O.6H_2O$ and $CO_2.6H_2O$. The vapor pressure of the hydrate of N_2O is 5 atmospheres at −6°C. while the hydrate of CO_2 has this vapor pressure at −9°C. The heats of formation of the two hydrates are given respectively as 14,900 and 15,000 calories per gram molecule.

The surface tension of liquid N_2O is 2.9 dynes per cm. at 12.°2, while CO_2 has this same surface tension at 9.0°.

Thus N_2O at any given temperature has properties practically identical with those of CO_2 at a temperature 3° lower.

These results establish the similarity of outside structure of the molecules.

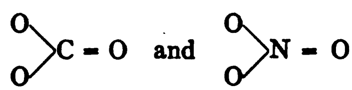
There is one property however, which is in marked contrast to those given above. The freezing-point of N_2O is $-102°$ while that of CO_2 is $-56°$. This fact may be taken as an indication that the freezing-point is a property which is abnormally sensitive to even slight differences in structure. The evidences seem to indicate that the molecule of CO_2 is slightly more symmetrical, and has a slightly weaker external field of force than that of N_2O. Such differences could easily be produced by the difference in the charges on the kernels.

There are many other examples of compounds having similarly formed molecules. It will be convenient to call these *isosteric compounds* or *isosteres*. These may be defined as compounds whose molecules have the same number and arrangement of electrons.

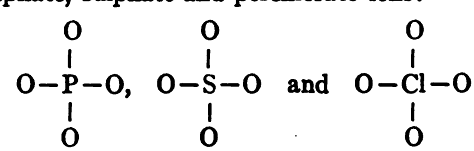
Another example of a pair of isosteres is that of HN_3 and HCNO. The similarity of properties should be most marked in the salts of these acids. The available data on solubilities and crystalline form seem to show that the salts of these two acids are very closely similar in physical properties.

This relationship of compounds may be carried much further. Thus, according to the Octet Theory, we should regard CH_4 as an isostere of the NH_4 ion. The electric charge on the ion prevents a direct comparison of physical properties. Other examples are:

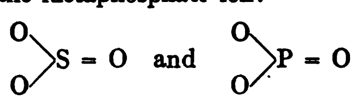
The carbonate and nitrate ions:



The orthophosphate, sulphate and perchlorate ions:



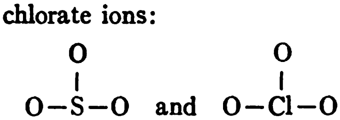
Sulfur trioxide and the metaphosphate ion:



The hydrofluoric acid molecule and the hydroxyl ion:

H – F and O – H

The sulfite and the chlorate ions:



We may attribute the differences in physical properties in all these cases to the effect of the differences in the electric charges.

Lewis has already pointed out that a theory of the kind outlined in this paper explains satisfactorily the facts which have led many chemists to assume polar valence. For example, the chlorine atom in chlor-acetic acid, because of the relatively large charge on its kernel, as compared for example with a carbon atom, tends to displace towards itself the electrons holding it to the carbon atom. This effect is transmitted with gradually decreasing intensity to the further end of the molecule, where it results in drawing the pair of electrons which holds the hydrogen nucleus to the octet of the oxygen atom, away from the hydrogen nucleus. Another way of looking at the effect is to consider that the positive kernel of the oxygen atom is displaced toward the hydrogen nucleus, and thus tends to weaken the force holding it. This effect makes it easier for the hydrogen nucleus to separate from the rest of the molecule as a positive ion. It is felt that this explanation can be applied in general to explain nearly all cases where polar valence bonds have been assumed in the past. This question will be discussed in more detail in the second paper to be published in the *Journal of the American Chemical Society*.

[1] Lewis, G. N., *J. Amer. Chem. Soc.*, 38, 1916, (762–785).

[2] This will be published in full in a paper soon to be submitted to the *Journal of the American Chemical Society*. This second paper will deal in some detail with the application of the Octet Theory to organic chemistry, particularly to nitrogen, sulphur, compounds, etc. The stereoisomerism of such compounds will be discussed.

A NEW INSTRUMENT FOR MEASURING PRESSURES IN A GUN

By A. G. Webster and L. T. E. Thompson

Ballistic Institute, Clark University, Worcester, Massachusetts*

Read before the Academy, April 29, 1919

It is now over fifty years since the crusher gauge was invented for measuring the maximum pressure developed in a gun. This apparatus has probably gone through fewer changes than almost any physical instrument except the telegraphic sounder. It is looked upon by all experts as inaccurate, and should be superseded. We have developed an apparatus which shows not only maximum pressure, but also the pressure at any time while the projectile is in the barrel; that is, it gives the curve which represents the pressure as a function of the time. Attention is called to the fact that this curve is not obtained by a series of points, and that no part of the curve is missing. The success of this instrument is due to its being designed in accordance with the principles of dynamics, and of optics.

The general nature of the apparatus is shown in figures 1 and 2. The success of such an apparatus that is to be free from vibrations of its own is brought about by using an extremely stiff spring. Such a spring is obtained by a short, steel girder, or a circular plate, the girder being shown in

* Contribution from the Ballistic Institute, Clark University No. 4.

figure 1, *c*. The piston *p* is that used in the ordinary gauge gun and bears upon the middle of the girder. From the mass of the piston and the dimensions of the spring we may calculate the natural frequency as about 16,000 per second. It is evident, however, that the damping is so great that no vibrations of the instrument appear. The upper side of the spring is held by two knife edges, on the end of a strong screw held in a massive frame attached to the gun. The arrangement is easy to see from figures 1 and 2. The registration is by means of the image of a spot of light illuminated by an arc lamp and focused by a good photographic lens upon a film carried by a rotating drum, after being reflected upon a small concave mirror attached to the end of the bar.

Gauge

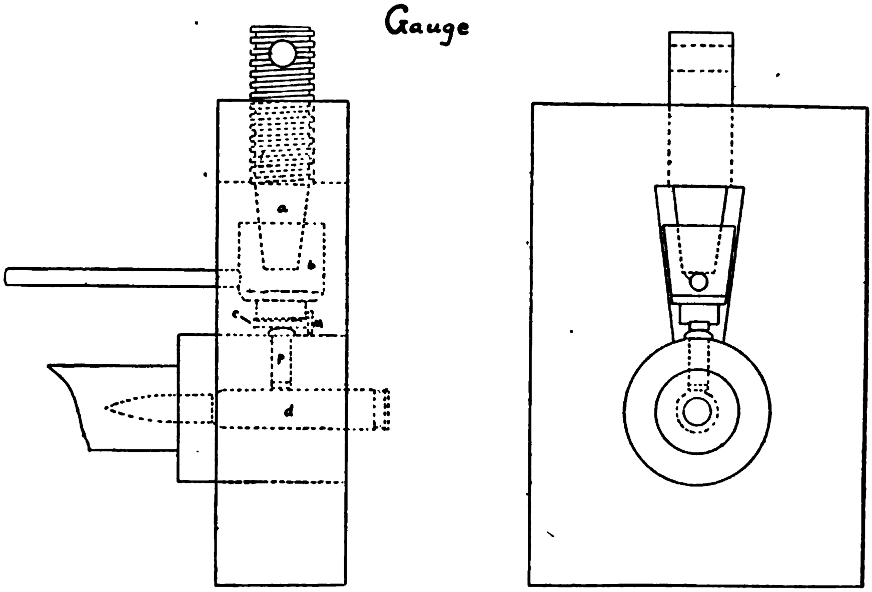


FIG. 1

Instead of rotating the drum it is possible to rotate a totally reflecting prism, so that the film does not need to move. When used to give the *p*, *t* curve (from which the *p*, *s* curve, or ordinary indicator diagram may be obtained by integration—figure 4) the drum is rotated at high speed, say 30 to 50 meters per second peripheral velocity. For maximum pressures it is necessary to move the drum only about 1/100 of a turn as the individual exposures are made. The film when developed shows the series of straight lines the height of which represents the maximum pressure in the barrel. One set of these is shown in figure 3. The information which is available from the *p*, *t* curve is of great importance, and the success of the instrument in giving these curves accurately and with certainty is very gratifying. For example, in

comparing the properties of various powders (which we had occasion to do, see figure 3) the rapidity, duration of burning, maximum pressure, maintenance of pressure, total impulse, and the point and pressure at which the bullet leaves the barrel, and the time in the barrel can all be shown with the p, t curve. By mechanical integration we find the curve of velocity and distance and by combining the distance with the pressure we obtain the p, s or indicator curve, shown in figure 4. A method is now under experiment to obtain the p, s curve directly whether on a small or large gun.

The advantages of the gauge used for maximum pressure measurement are also evident:

1. After the calibration of the spring only one measurement is necessary; namely, the length of the straight line on the film or paper which measures the pressure.

2. The length of this line is usually 50 mm. or more, permitting quick and accurate measurement.

FIG. 2

3. The same instrument is used for every measurement; that is, the flexure of the spring is only temporary. The spring returns absolutely to the zero position there being no permanent set or elastic hysteresis.

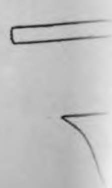
4. The operation can be carried out quickly and inexpensively.

5. Permanent record is provided, reading pressures directly on the film or print, and if desired the calibration can be photographed directly on the film.

In order to show experimentally that the static calibration would give accurate results which could not be used in measuring impulsive forces, a calibration was carried out by the use of shock or impulsive forces. The f, t curve was obtained on a rotating film as in the ordinary indicator experiment, when a known weight dropped from a known height struck a piston bearing on the spring of the gauge.

In figure 3 are shown a number of curves indicating the properties of different powders furnished by the Winchester Repeating Arms and the DuPont Companies, together with a sample of German anti-tank gun powder furnished by Major Anthony Fiala of the Springfield Armory.

figure 1, *c*. T
upon the midd
sions of the sp
second. It is
tions of the in
knife edges, o
the gun. Th
tion is by m
and focused
drum, after
of the bar.



I
pri
cu
ta
50

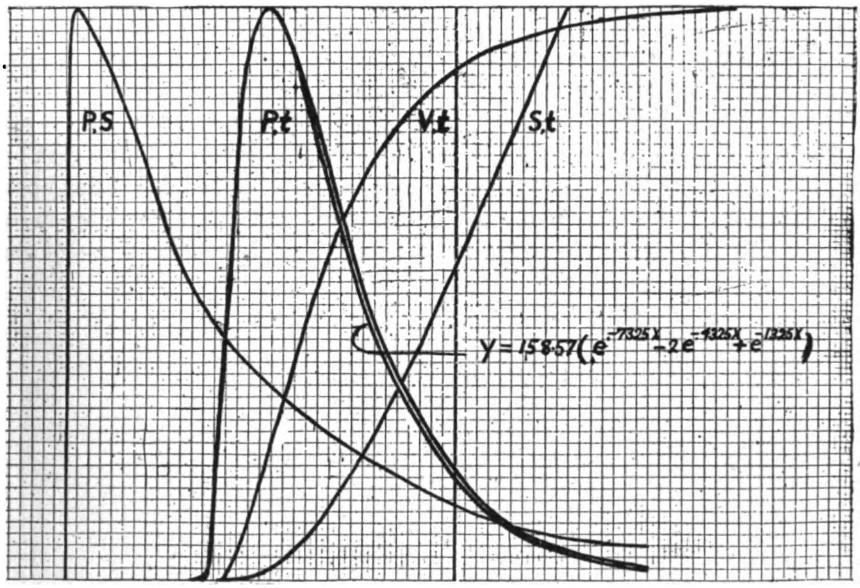


FIG. 4

The emergence of the bullet from the barrel is shown by the interruption of a beam of light thrown upon the film by a separate mirror.

In a subsequent paper the theory of the apparatus and the conclusions that may be obtained from it, including the resistance in the barrel and the variation of specific heats will be described.

ON THE ANGLE OF REPOSE OF WET SAND

By A. G. Webster

Ballistic Institute, Clark University, Worcester, Massachusetts*

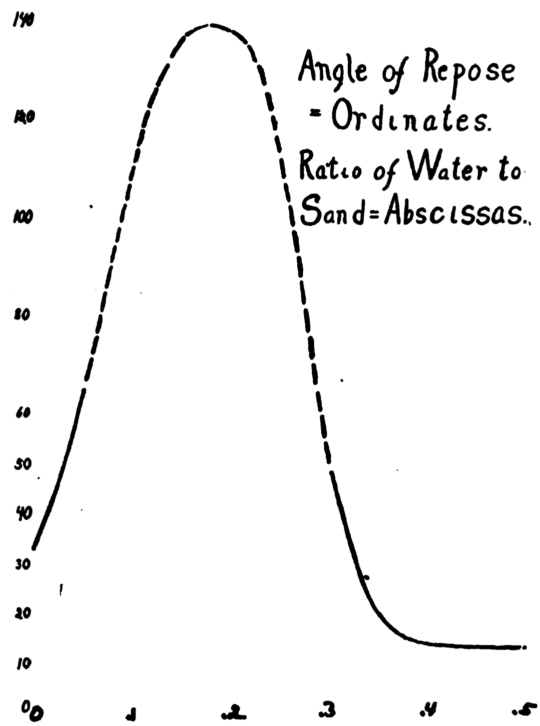
Read before the Academy, April 29, 1919

It is well known that sand, gravel, broken stones, grain, sugar or any pulverulent substance has a definite angle of limiting steepness which is called the angle of repose. In Dr. Breasted's lecture the pile of debris in front of the Temple of Thebes showed such a definite angle. In driving to Washington last week I passed along the Cape Cod Canal and noticed a very definite angle of repose for the sand alongside the Canal. Passing by the beach at Narragansett Pier I measured its slope and found it to be very uniform and about one in fourteen.

*Contribution from the Ballistic Institute, Clark University, No. 6.

A few years ago a committee of this Academy was sent to Panama to examine into the cause of the Culebra slides. My interest was excited at that time, and a year ago by my being consulted as an expert on the collapse of a house.

A few days ago it occurred to me, having a load of sand on the floor of my ballistic laboratory, to wet it and make an artificial beach. I found that on scraping it up with a board a very definite slope was obtained depending upon the wetness. I therefore requested my assistant Dr. E. A. Harrington to make a few quantitative experiments to determine the angle in terms of the wetness. This is shown in the figure.



Beginning with absolutely dry sand which was weighed in a tray a certain amount of water was added and the whole weighed. At first the added water is quickly absorbed and on account of the work done by the surface tension and the cohesion of the water a certain positive amount of cohesion is obtained by the sand, and it will remain in equilibrium vertically and even overhang. Of course these experiments are not extremely accurate. When a certain degree of wetness is passed the sand then acts like a plastic substance, the degree of plasticity depending upon the relative amounts of water and sand.

The status of the mixture is described in the table.

Professor Harry F. Reid informs me that a continental shelf of the Atlantic Ocean has a very definite slope which is very nearly that obtained for the extremely wet sand in these experiments.

I reserve the mathematical theory for a future paper.

SAND	WATER	ANGLE OF REPOSE	REMARKS
pounds	*pounds*		
10	0	33°	Dry
10	0.5	65°	Not hard
10	1.0	120°	Not accurate, but large obtuse angle, hard
10	1.5	120°–140°	Not accurate, but large obtuse angle, hard
10	2.0	120°–140°	Not accurate, but large obtuse angle, hard
10	2.5	120°	Not accurate, but large obtuse angle, hard
10	3.0	48°	Fairly hard
10	3.5	19°	All mixes
10	3.75	14.5°	Very slight excess of water
10	4.0	13°	Water not all absorbed
10	5.0	12°	Excess of water

The sand has been meshed by Professor Roys of the Worcester Polytechnic Institute with the following results:

SIZE OF SCREEN	DIAMETER OF OPENING	PER CENT OF SAND WHICH PASSED THROUGH
meshes	*inches*	
200	0.0029	7.3
100	0.0055	29.55
50	0.011	84.60
30	0.022	99.45
20	0.034	99.85
10	0.073	99.95
4	0.20	100.00

PALÆOMASTODON, THE ANCESTOR OF THE LONG-JAWED MASTODONS ONLY

By Henry Fairfield Osborn

American Museum of Natural History, New York City

Read before the Academy, April 29, 1919

In 1900[1] the author predicted that the ancestors of the Proboscideans, as well as of the Hyracoidea and some other orders of mammals, would be discovered in Africa. Two years later the members of the British Geological Survey of Egypt discovered in the Oligocene of the Fayûm remains of *Palæo-*

mastodon and of *Mœritherium*, which were at once regarded as the solution of the ancestry of the Proboscideans. These animals took their place in all literature as two steps in the early evolution of this remarkable group.

In 1909[2] Osborn pointed out that *Mœritherium* is to be regarded as a terrestrial form of the Sirenians (manatees and dugongs) in no way directly related to the Proboscideans. It now appears that *Palæomastodon* must also be removed from its generalized position and be regarded as the ancestor of the long-jawed mastodons only; it is far too much specialized in the longirostral direction to be ancestral to the Proboscidea in general. These long-jawed mastodons are distinguished by the peculiar use of the front teeth of the lower jaw, which together made a broadly flattened, spoon-shaped tooth,

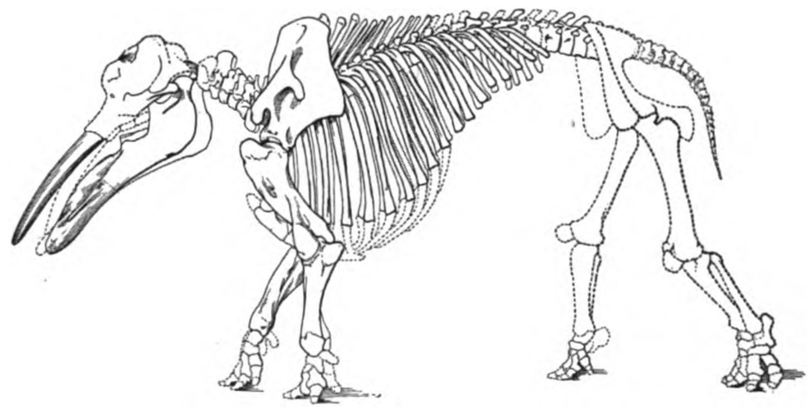


FIG. 1

Outline of the mounted skeleton of *Megabelodon* in The American Museum of Natural History, discovered by E. L. Troxell in the Middle Pliocene of Texas. Drawing one forty-fifth natural size.

almost entirely enamel covered. Phases of the evolution of this long-jawed phylum are seen in the classic *Trilophodon angustidens* of Cuvier, in the lower Miocene of France. A branch reached Texas in the Upper Miocene (*Trilophodon productus* of Cope), and Florida as well as Texas in the *Trilophodon serridens* of Cope. It attained gigantic proportions in the Middle Pliocene. The *Megabelodon* of Barbour, a superb skeleton of a long-jawed and extremely short-limbed Proboscidean, recently discovered in Texas by Mr. E. L. Troxell, has been mounted in The American Museum of Natural History. It represents one of the culminating stages in the evolution of the long-jawed mastodons. In these animals we find proof of nearly direct linear descent from the *Palæomastodon* of the Fayûm, e.g., the long enamel band on the upper tusks, the broadly spoon-shaped arrangement of the lower tusks with enamel covering. In massiveness these animals parallel and even surpass the true mastodons of the Pleistocene, to which they are only indirectly related.

[1] Osborn, 1900, 182. [2] Osborn, 1909, 332.

THE ELONGATION DUE TO MAGNETIZATION

By C. Barus

Department of Physics, Brown University

Communicated, April 30, 1919

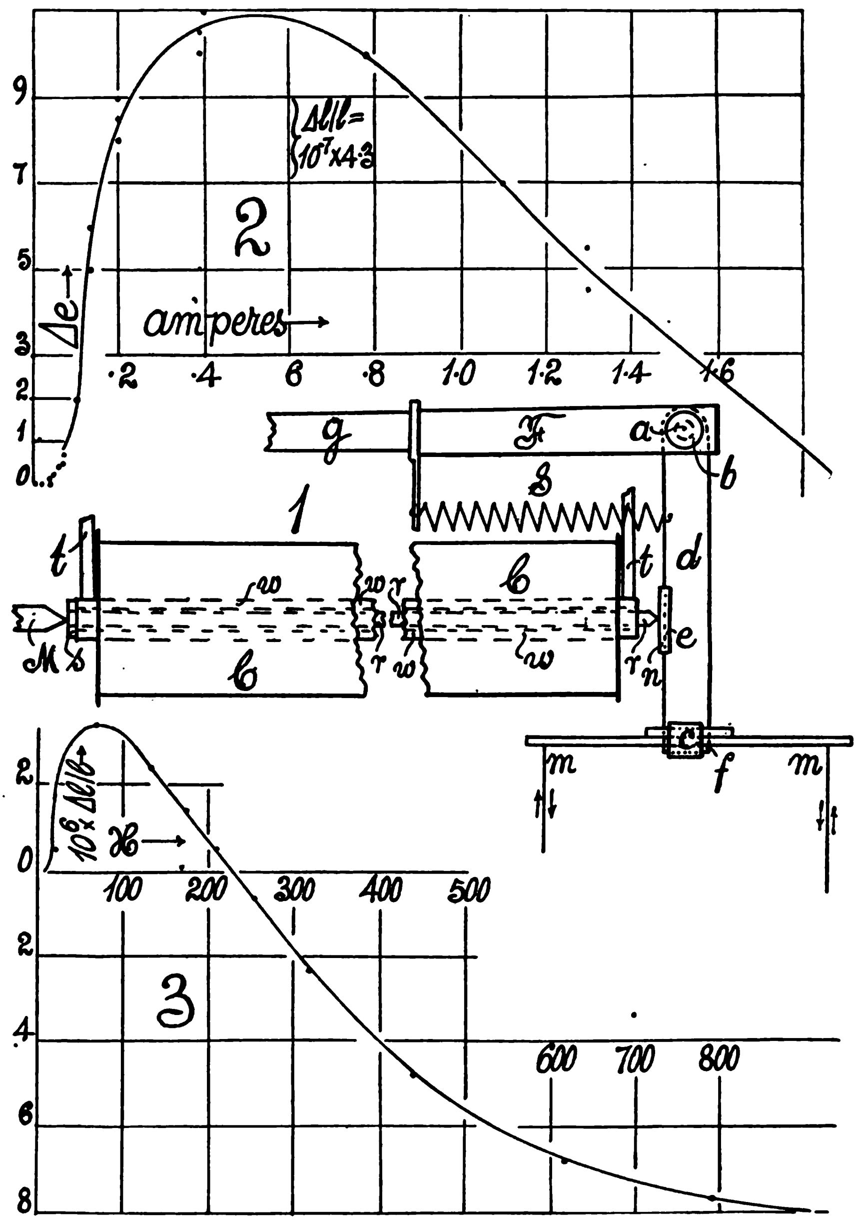
1. Introductory.—The small longitudinal displacements due to magnetostriction have been frequently subjected to investigation and an excellent summary is given in Winkelmann's Handbuch, vol. 5, p. 307, et seq., 1908. The measurements of Prof. C. G. Knott and his students Nagaoka and Honda are particularly noteworthy.[1] In 1911[2] my son, Mr. Maxwell Barus, and I used these phenomena for the purpose of testing a peculiar type of interferences then under discussion.

The present purpose is similar, being a test of the contact lever recently[3] described.

The elongation (and contraction) phenomena are necessarily complicated by the occurrence of hysteresis loops to which the present paper (in which the measurements are not made by the continuous variation of currents and field, but by successively making and breaking the circuit) will give no attention. This subject has been adequately explored by Professor Knott and the authors cited. The chief interest in this paper is rather the continued increase of the contractions due to magnetization, not only after the latter has practically reached a maximum, but in a marked degree (so far as I have gone, fields up to 800), indefinitely. There is no sure indication of an abatement of the contraction. Hence the magnetic contribution of the present paper is to lie in the treatment in strong fields.

2. Apparatus.—The contact lever shown in figures 1 and 2 of the paper cited was modified as indicated in figure 1, where *F* is the semicircular fork in a vertical plane, rigidly attached to the bed plate of the interferometer by a strong clutch (not shown), holding the cylinder, *g*, the handle of the fork. The vertical axis *a* of the contact lever is secured between the screw pivots *b* of the fork. The horizontal strip of brass *d*, rigidly fastened to the center of the axle *a*, carries at its end the auxiliary mirror *mm'* of the quadrâtic interferometer. For this purpose, a short length, *f*, at the end of *d* has been bent upward at right anglès to *d*, so that *mm'* may be held between plates of brass by the yoke-shaped steel clip *c*. At the side of the lever is a vertical brass plate inset *c*, to which a small glass plate *n* has been fastened with cement. It is against this that the conical end of the iron rod *rr* to be examined, pushes. The spring *S* attached to the blade *d* and a lateral projection from the fork *F* insures continuous contact at a constant pressure.

The iron rod *rr* originally about 43 cm. long and 67 cm. in diameter, is enveloped by a tubular water jacket *ww*. Through this a current of water entering at *t* and leaving at *t'* is kept flowing from a large copper Mariotte flask about 50 cm. high and 30 cm. in diameter, and containing water at the tempera-


2
ampere
Δe
Δl/l = 10⁷ × 4.3
·2
·4
·6
·8
1·0
1·2
1·4
1·6
9
7
5
3
2
1
0
1
g
F
a
b
s
d
t
t
w
w
r
r
w
w
M
s
c
c
e
r
n
c
f
m
m
3
10⁶ × Δl/l
H
100
200
300
400
500
600
700
800
2
0
2
4
6
8

ture of the room. Two such flasks were at hand to be used alternately. The coil *CC* 26 cm. long and 3.7 cm. in external diameter, is wound immediately on the tubular water jacket.

The rod *rr* fits the tube *ww* loosely and is centrally detached at the remote end by aid of a bushing *s* and a small bolt. The front end is free. The coil *CC* is held in position by a large clutch (not shown) encircling it at the middle and attached to the bed plate of the interferometer. It is additionally attached at the tubulures *t* and *t'*. Finally the conical end *M* of a micrometer screw (also rigidly attached to the bed plate) gives the remote end of the rod *rr* any desirable fiducial position. This micrometer *M* has the further advantage of permitting an independent standardization of the contact lever, as there is obviously always sufficient elastic yielding in the apparatus to considerably shift the interference fringes.

In the experiments made, the breadth of the ray rectangle *mm'* of the interferometer was $b = 9.7$ cm.; the normal distance between the rod *rr* and the axis *a*, 7 cm.; the length of the contact lever *a* to *mm'*, 10.6 cm.; and the length of the axle *a*, 10 cm.

3. *Observations*.—The helix *C* in figure 1 was slender in shape, the length being 37 cm. and the diameter within being about 1.5 cm. There were about 11.2 turns per centimeter per layer and 8 layers of wire so that the field within may be estimated at $H = 110\ i$ gauss, i being the current in amperes. The current 0.01 to over 8 amperes thus corresponded to field from 1 to over 800 gauss.

The first rod selected was of low carbon shop steel 43 cm. long, so that it projected a few centimeters beyond either end of the helix.

The displacement of fringes observed was characteristic, being (in the smaller fields) slow and deliberate on closing the circuit (so that their motion could almost be followed by the eye), but very rapid on breaking the circuit.

The experiments were begun with small fringes (about 0.1 mm. in the ocular), and the readings Δe were made in terms of an ocular micrometer scale which was a centimeter divided into 0.1 mm. This was compared with the datum, ΔN, of the displacement micrometer normal to one of the mirrors of the interferometer.

If ΔN corresponds to the angular displacement, $\Delta\theta$, of the contact lever and to Δl of elongation of the iron rod r in the helic C (figure 1) we may write as above

$$2b\Delta\theta = 2\Delta N \cos i \qquad (1)$$

if i is the angle of incidence (45°) at the mirrors of the interferometer and b the breadth of the ray parallelogram. But

$$\Delta l = r\Delta\theta \qquad (2)$$

if r is the normal distance of the line of thrust of the rod *rr* from the axis of the contact lever. Thus

$$\Delta l = (r \cos i/b)\ (\Delta N/\Delta e)\ \Delta e \qquad (3)$$

If l is the length of the iron rod $\Delta l/l$ will be the datum required.

It was quite possible to ascertain $\Delta e = 0.1$; i.e., elongations of but $\Delta l/l = 5 \times 10^{-8}$, equivalent to $\Delta l = 2.3 \times 10^{-5}$ cm. The current must exceed 0.02 am. before any elongation can be detected, after which, however, the elongations abruptly begin and increase rapidly to a maximum, which is reached before saturation.

The experiments, figure 2, were made with somewhat greater care and with larger fringes. The standardization of the ocular micrometer showed

$$\Delta N/\Delta e = 10^{-5} \times 1.82, \quad \Delta l/l = 10^{-7} \times 2.16\,\Delta e$$

But for incidental difficulties (tremors, etc,), the results in figure 2 would probably be very smooth.

A number of supplementary experiments (see figure 2) were made to see whether the observed $\Delta l = 0$ for currents below 0.02 amperes might not be equivalent to an initial small minimum. But Δl remained persistently zero, while currents decreased from 0.02 to 0.001 amperes. At 0.004 am. the field was reversed, but no significant Δl could be observed. The fringes just moved ($\Delta e = 0.1$) when i was about 0.035 amperes, indicating a field of 3 or 4 gauss.

A rough test made of the equation by pushing the rod rr forward by the backstop screw M, figure 1, gave corroborative results.

If $\Delta\varphi$ refers to the turns of the screw M

$$\Delta l = (r \cos i/b)\,(\Delta N/\Delta e)\,(\Delta e/\Delta\varphi)\,\Delta\varphi$$

$(\Delta N/\Delta e)$ was found to be $10^{-5} \times 3.3$ cm. per scale part and $(\Delta e/\Delta\varphi)$ scale parts per degree of turn. Hence with the above data

$$\Delta l = 10^{-4} \times 1.8$$

The back screw having 40 turns to the inch, i.e., a pitch of 0.0635 cm. gives us $10^{-4} \times 1.76$ cm. per degree of turn.

Another feature may be mentioned here. The expansion of the coil when carrying very large currents is a thrust on the back stop M, which is quite appreciable and appears as an apparent contraction of the rod.

4. *Vibration telescope.*—To test the surmise that initial elongations always precede the final contraction, the vibration telescope heretofore described[4] was installed. It was then found that the even band of fringes drawn out by the vibrating objective broke up into strongly sinuous lines on making and particularly on breaking the circuit through the helix. When the circuit was made and broken alternately, the waves broke up further into a succession of discontinuous pulses of more than double the amplitudes of the waves. With the field properly adjusted by passing 1.8 to 2 amperes through the coil, there was no further observable displacement after the strong waves lines, produced immediately after closing the circuit, had subsided.

5. *Further observations.*—The data of figure 3 were investigated in a single series. To reduce the heat discrepancy a brisk current of water was passed through the tubular water jacket. This seemed the safer plan, even though the fringes were shaken. The observations were made in triplets and largely confined to the higher fields.

The curve is quite as clearly indicated as may be expected owing to the difficulties cited; but the higher observations ($H > 200$) are decreasing contractions. The reason of this is partly owing to the method of observation in triplets, where (curiously enough) the third reading (field zero) was a contraction in relation to the first reading in the absence of the field.

The apparatus in these experiments was therefore suspected of being faulty in design, inasmuch as the clutch of the contact lever and of the coil were attached to the same rigid standard. This arrangement was now modified, so that the two mountings were quite independent, whereupon the anomalous results specified largely receded.

As a second test a rod 28 cm. long of Swedish iron was inserted, the extra length being pieced out by brass tubing soldered to each end, so that the iron lay quite within the coil. The data obtained closely resembled figure 3. Tests made with other metals gave no positive results.

6. *Coefficient of expansion.*—To arrive at a definite reason for the occurrence of the anomalous contraction mentioned above, it seemed desirable to modify the magnetic apparatus for the purpose of measuring the coefficient of expansion of a given metal. This could easily be done by using the coil merely as a heater.

A number of experiments were made, using either the ocular micrometer (here the temperature increments must lie within 2°) and the Fraunhofer micrometer at the mirror of the interferometer. It was observed that all the expansions were apt to begin with a contraction immediately after the heating current had been closed. Hence there is an initial expansion of the coil itself. It was soon found that the consequent flexure of the table was the ultimate cause of the interferometer discrepancy.

A modification of the apparatus was made therefore by allowing the end C of the coil to recline on a large grooved wheel, which by slight rotation would admit of any expansion of the kind in question. With this improvement the anomalous contractions vanished and the work thereafter proceded smoothly.

7. *Theoretical observations.*—To account for such a graph as figure 3, as a whole, one may argue that the initial elongations are to be referred to the rotations of the molecular magnets. For these elongations are coextensive in field variations with the marked increase of magnetization. It would then seem plausible that thereafter the attractions between the oriented molecular magnets may be instanced to account for the persistent contractions in continually increasing fields. Thus it seems worth while to endeavor to ascertain whether such a supposition would conform with any reasonable value of the susceptibility k of the iron, which one may estimate as decreasing from over

100 to less than 10 as the rod approaches saturation ($H = 150$ gauss) and to decrease thereafter asymptotically to zero.

If p is the force per square centimeter of section of the rod and E Young's modulus,

$$p = E(\Delta l/l) \tag{1}$$

regarding the magnetic stress as traction.

Using the expression for the potential of a disc, the field F in a narrow crevasse normal to F, between molecular layers of magnetic surface density of magnetization kH

$$F = 4\pi kH + H$$

where k is the susceptibility of the metal.

Hence the force per square centimeter should be $p' = FkH$, or

$$p' = H^2(4\pi k^2 + k) \tag{2}$$

Equating $p = p'$ in equations (1) and (2)

$$\frac{\Delta l}{l} = \frac{4\pi k^2 + k}{E} H^2 \tag{3}$$

If the data in figure 3 are taken above 800 gauss, supposing that these are far enough removed from the initial complications, the estimate would be ($E = 2 \times 10^{12}$), $k = 1.6$.

An order of mean susceptibility of 1.6 (which seems not an unreasonable assumption) would thus account for the observed contractions. Naturally as k is essentially variable with H a better statement of the case might be given by postulating such a relation.

[1] *London, Phil. Mag.*, 37, 1894, (131).
[2] *Carnegie Inst., Washington, Pub.*, No. 149.
[3] These PROCEEDINGS, 5, 1919, (39).
[4] These PROCEEDINGS, 4, 1918, (328).

GROUPS INVOLVING ONLY TWO OPERATORS WHICH ARE SQUARES

By A. G. Miller

Department of Mathematics, University of Illinois

Communicated by E. H. Moore, May 2, 1919

The abelian group of order 2^m and of type (1, 1, 1, . . .) is completely characterized by the fact that all of it operators have a common square. When we impose the condition that the operators of a finite group G have two and only two distinct squares then G must belong to one of three infinite systems of groups whose characteristic properties we proceed to determine.

All the operators of G besides the identity must be of order 2 or of order 4 and G must involve operators of each of these two orders. Hence the order of G is of the form 2^m. When G is abelian it is of type (2, 1, 1, . . .) and it will therefore be assumed in what follows that G is non-abelian. The octic group and the quaternion group constitute well known illustrations of such a group and have the smallest possible order.

When the operators of order 2 contained in G together with the identity constitute a subgroup this subgroup is the central of G and hence G belongs to the system of groups called Hamiltonian by R. Dedekind.[1] In this case it is known that G is the direct product of the quaternion group and an abelian group of order 2^α and of type (1, 1, 1, . . .). Hence it will be assumed in what follows that G involves non-commutative operators of order 2.

Every operator of order 4 contained in G is transformed either into itself or into its inverse by every operator of G and an operator of order 2 contained in G has at most two conjugates under the group.[2] Let H_1, H_2 represent subgroups composed respectively of all the operators of G which are commutative with two non-commutative operators of order 2 s_1, s_2. The cross-cut K_1 of H_1 and H_2 is of index 4 under G and includes the central of G. A set of independent generators of G can be so selected as to include s_1, s_2 and operators from K_1.

Exactly one-half of the operators of G which are not also in K_1, are of order 2 since the quotient group G/K_1 is abelian. If K_1 involves non-commutative operators of order 2 two such operators s_3, s_4 may be selected from K_1 in exactly the same way as s_1 and s_2 were selected from G. The remaining operators of a set of independent generators including s_1, s_2, s_3, s_4 may be selected from an invariant subgroup of index 4 under K_1 and of index 16 under G all of whose operators are commutative with each of the four operators already chosen.

As G is supposed to be of finite order we arrive by this process at a subgroup K_m in which all the operators of order 2 are commutative. Hence K_m belongs to one of the following three well known categories of groups. Abelian and of type (1, 1, 1, . . .), abelian and of type (2, 1, 1, . . .), or Hamiltonian of order 2^a. The commutator subgroup of G is of order 2.

In each case, G may be constructed by starting with K_m, forming the direct product of K_m and an operator t_1 of order 2, and then extending this direct product by means of an operator t_2 of order 2 which is commutative with each of the operators of K_m and transforms t_1 into itself multiplied by the commutator of order 2 contained in G. When K_m is Hamiltonian or abelian and of type (2, 1, 1, . . .) this commutator is determined by K_m. In the other possible case it may be selected arbitrarily from the operators of order 2 found in K_m.

When $m>1$, we use the group K_{m-1} just constructed in exactly the same way as K_m was used in the preceding paragraph. The commutator of order 2 is completely determined for each of the categories by K_{m-1}, $m>1$. When $m>2$

we proceed in the same manner with K_{m-2}, etc. It may be noted that in each of the groups belonging to one of the three categories thus constructed more than one-half of the operators are of order 2, in those belonging to the second category the number of operators of order 2 is one less than one-half of the order of G, while in those belonging to the third category the number of operators of order 2 is obtained by subtracting from one-half the order of G one plus one-fourth the order of K_m.

Some of these results constitute a proof of the following theorem: *If only two of the operators of a group G are the squares of operators contained in G then the non-invariant operators of G have only two conjugates, each cyclic subgroup of order 4 is invariant, and G belongs to one of three categories of groups of order 2 which can be separately generated by a set of operators such that each of these operators is commutative with each of the others except at most one of them.*

When m is sufficiently large there is one and only one group belonging to each of these three categories and having a give number γ of pairs of non-commutative operators of order 2 in its set of independent generators when this set is obtained in the manner described above. The smallest values of m for these categories are $2\gamma + 1$, $2\gamma + 2$, and $2\gamma + 3$ respectively. When m has a larger value G must be the direct product of an abelian group of type $(1, 1, 1, \ldots)$ and of the minimal group having γ such pairs of generators and contained in the category to which G belongs.

By means of these facts it is very easy to determine the number of the groups of a given order 2^m which belong to each of these three categories. This number is the largest integer which does not exceed $\frac{m-1}{2}$, $\frac{m-2}{2}$, and $\frac{m-3}{2}$ for the three categories respectively. In particular, the number of the distinct groups of order 128 belonging to each of these categories is 3, 2, 2 respectively, it being assumed that each of the groups in question contains at least two non-commutative operators of order 2.

In each one of these groups every two non-commutative operators of order 2 generate the octic group and every two non-commutative operators of order 4 generate the quaternion group. Moreover, every non-abelian subgroup is invariant. In two of the categories the central is composed of operators of order 2 in addition to the identity, while the central of the remaining category is of type $(2, 1, 1, \ldots)$. Every one of these groups is generated by its operators of order 2. From the standpoint of definition and structure these categories rank among the simplest known infinite systems of non-abelian groups

[1] Dedekind, R., *Math. Ann., Leipzig*, **48**, 1897, (548–561).

[2] Miller, G. A., *Trans. Amer. Math. Soc., New York*, **8**, 1907, (1–13).

ACOUSTICAL IMPEDANCE, AND THE THEORY OF HORNS AND OF THE PHONOGRAPH

By Arthur Gordon Webster

Department of Physics, Clark University

Communicated, May 8, 1919*

The introduction more than thirty years ago of the term 'impedance' by Mr. Oliver Heaviside has been productive of very great convenience in the theory of alternating currents of electricity. Unfortunately, engineers have not seemed to notice that the idea may be made as useful in mechanics and acoustics as in electricity. In fact, in such apparatus as the telephone one may combine the notions of electrical and mechanical impedance with great advantage. Whenever we have permanent vibrations of a single given frequency, which is here denoted, as usual, by $n/2\pi$, the notion of impedance is valuable in replacing all the quantities involved in the reactions of the system by a single complex number. If we follow the convenient practice of denoting an oscillating quantity by e^{int} and taking its real part (as introduced by Cauchy) all the derivatives of e^{int} are obtained by multiplication by powers of in, or graphically by advancing the representative vector by the proper number of right angles.

If we have any oscillating system into which a volume of air X periodically enters under an excess pressure p, I propose to define the impedance by the *complex* ratio $Z = p/X$. If we call $dX/dt = I$ the current as in electricity, if we followed electrical analogy we should write $Z = pI$ so that the definition as given above makes our impedance lead by a right angle the usual definition. I believe this to be more convenient for our purposes than the usual definition and it need cause no confusion.

If we have a vibrating piston of area S as in the phonometer, we shall refer its motion to the volume $S\xi$ it carries with it and the force acting on it to the pressure, so that $F = Sp$. The differential equation of the motion is

$$m\frac{d^2\xi}{dt^2} + \kappa\frac{d\xi}{dt} + f\xi = F = Sp, \qquad X = S\xi, \tag{1}$$

we have

$$Z_1 = (f - mn^2 + i\kappa n)/S^2, \tag{2}$$

where m is the mass, κ the damping, f the stiffness. The real part of S^2Z, $f - mn^2$, is the uncompensated stiffness, which is positive in a system tuned too high, when the displacement lags behind the force, by an angle between zero and one right angle, negative when the system is tuned too low, when the

* This article was read in December 1914 at the meeting of the American Physical Society at Philadelphia, and has been held back because of the continual development of the experimental apparatus described in a previous paper in these Proceedings.

lag is between one and two right angles, as shown in figure 1. If we force air into a chamber of volume V, the compression $s = X/V$ will be related to the excess pressure p by the relation $p = es$, where e is the modulus of elasticity of the air $e = \rho a^2$, ρ being the density and a the velocity of sound. Consequently we have

$$Z_0 = \frac{e}{V} = \frac{\rho a^2}{V}, \tag{3}$$

and the analogy is to a condenser. If we have air passing through an orifice or short tube of conductivity c its inertia gives an apparent mass ρ/c, and if it

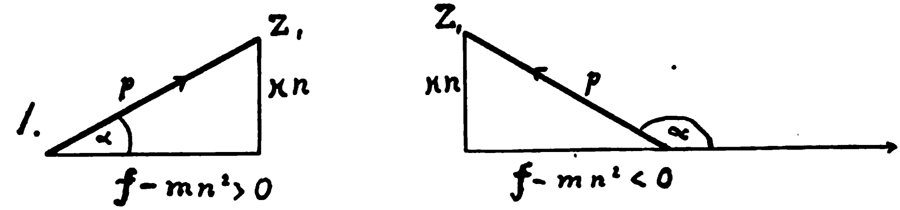


FIG. 1

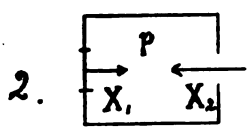


FIG. 2

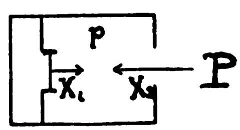


FIG. 3

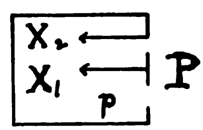


FIG. 4

FIG. 5

escapes from a circular hole in an infinite plane it dissipates energy so that the whole impedance credited to the hole is

$$Z_2 = -\frac{\rho n^2}{c} + \frac{\rho n^3}{2\pi a}\, i = ek^2 \left\{ \frac{k}{2V}\, i - \frac{1}{c} \right\}, \text{ where } k = \frac{n}{a}. \tag{4}$$

These three typical impedances will be at constant use in acoustics. It is to be remembered that systems in series have their impedances added and in parallel have the reciprocals of impedance added. Also that the free vibrations of a system are obtained by equating the impedances to zero.

As a simple example consider the phone described in the previous article, figure 3.

Let $X_1 = S\xi$ be the volume introduced by the piston X_2 that entering by the hole. Then

$$p = Z_0(X_1 + X_2) = -Z_2X_2, \tag{5}$$

$$X_2 = \frac{-Z_0}{Z_0 + Z_2}X_1,$$

and inserting values,

$$X_2 = -\frac{S\xi}{1 + Vk^2\left(\frac{ki}{2\pi} - \frac{1}{c}\right)} \tag{6}$$

Disregarding phase by taking the modulus and putting $k = n/a$ we have the phone formula for the strength of source.

$$A = \left|\frac{dX}{dt}\right| = S\Psi|\xi|, \tag{7}$$

where

$$\Psi = \frac{n}{\left\{\left(1 - \frac{Vk^2}{c}\right)^2 + \frac{V^2k^6}{4\pi^2}\right\}^{\frac{1}{2}}} \tag{8}$$

If instead of sending the air out through a hole it goes into a cone or any other horn, we must use for the impedance Z_2 that given below, and we arrive at the theory of the phonograph, and are thus able to answer the question as to the function of the horn in persuading the sound to come out of the phonograph when the motion of the diaphragm is given (it is well known that very little sound is emitted by the phonograph or the telephone with the horn taken off, although in the former case the motion of the diaphragm is exactly the same).

The phonometer was formerly arranged with the back of the diaphragm protected from the sound, figure 3. Let P be the external pressure, then, as before,

$$p = Z_0(X_1 + X_2) \tag{9}$$

and in addition,

$$\begin{aligned} -p &= Z_1X_1, \\ P - p &= Z_2X_2, \end{aligned} \tag{10}$$

from which

$$X_1 = \frac{-PZ_0}{Z_0Z_1 + Z_1Z_2 + Z_2Z_0}, \tag{11}$$

giving the formula for the measurement of the pressure,

$$P = \varphi\xi/S \tag{12}$$

$$\varphi = \frac{\gamma}{[\{uv - (\alpha\beta + \gamma^2)\}^2 + \{\beta u + \alpha v\}^2]^{\frac{1}{2}}} \tag{13}$$

and φ may be termed the sensitiveness of the phonometer. Where

$$\gamma = S^2 Z_0 = S^2 \rho a^2/V, \qquad \alpha = \kappa n, \qquad \beta = S^2 \rho n^3/2\pi a,$$

$$u = f - n^2 m + S^2 \rho a^2/V, \qquad v = S^2\{\rho a^2/V - \rho u^2/c\} = S^2 Z_0 (1 - k^2 V/c) \quad (14)$$

As described in my recent article the back of the piston is exposed to the sound, figure 4. Then

$$\begin{aligned} P - p &= Z_1 X_1 = Z_2 X_2 \\ p &= Z_0 (X_1 + X_2) \end{aligned} \quad (15)$$

from which

$$X_1 = \frac{Z_2 P}{Z_0 Z_1 + Z_1 Z_2 + Z_2 Z_0} \quad (16)$$

$$\varphi = \left[\frac{(v - \gamma)^2 + \beta^2}{\{uv - (\alpha\beta + \gamma^2)\}^2 + (\alpha v + \beta u)^2}\right]^{\frac{1}{2}}, \quad (17)$$

Tubes and Horns.—Beside the above described phone and phonometer, the theory of which assumed a resonator so small that the pressure is supposed to be the same at every internal point, I have made use of many arrangements employing tubes or cones, in which we must take account of wave-motion. The familiar theory of cylindrical pipes may be included in the following generalized theory, which I have found experimentally to serve well.

Let us consider a tube of infinitesimal cross section σ varying as a function of the distance x from the end of the tube. Then if q is the displacement of the air, p the pressure, s the compression, we have the fundamental equations

$$p = es = \rho a^2 s = -e \operatorname{div} q = -\frac{e}{\sigma}\frac{d(q\sigma)}{dx}, \quad (18)$$

$$\frac{d^2p}{dt^2} = a^2 \Delta p = a^2 \operatorname{div} \operatorname{grad} p = a^2 \left\{\frac{1}{\sigma}\frac{d}{dx}\left(\sigma \frac{dp}{dx}\right)\right\} \quad (19)$$

$$\frac{d^2q}{dt^2} = -\frac{1}{\rho}\frac{dp}{dx} = a^2 \frac{d}{dx}\left\{\frac{1}{\sigma}\frac{d}{dx}(q\sigma)\right\}. \quad (20)$$

For a simple periodic motion we put p, q proportional to e^{int}, and obtain

$$\frac{d^2p}{dx^2} + \frac{d \log \sigma}{dx}\frac{dp}{dx} + k^2 p = 0, \quad \frac{d^2q}{dt^2} + \frac{d \log \sigma}{dx}\frac{dq}{dx} + \frac{d^2 \log \sigma}{dx^2} q + k^2 q = 0. \quad (21)$$

Both these linear equations may be solved by means of series, and if we call $u(kx)$, $v(kx)$ two independent solutions we have

$$p = Au + Bv, \quad \beta q = Au' + Bv', \quad \beta = \rho a^2 k,$$

where the accents signify differentiation according to kx. If we denote values at one end $x = x_1$ and at the other end $x = x_2$ by suffixes 1, 2, respectively, and form the determinants

$$D_1 = \begin{vmatrix} u_1, & v_1 \\ u_1', & v_1' \end{vmatrix}, \quad D_2 = \begin{vmatrix} u_2, & v_2 \\ u_2', & v_2' \end{vmatrix}, \quad D_3 = \begin{vmatrix} u_1, & v_1 \\ u_2', & v_2' \end{vmatrix},$$
$$D_4 = \begin{vmatrix} u_2, & v_2 \\ u_1', & v_1' \end{vmatrix}, \quad D_5 = \begin{vmatrix} u_1, & v_1 \\ u_2, & v_2 \end{vmatrix}, \quad D_6 = \begin{vmatrix} u_1', & v_1' \\ u_2', & v_2' \end{vmatrix}, \tag{22}$$

which satisfy the relation,

$$D_1D_2 = D_3D_4 + D_5D_6$$

we may determine the constants A, B in terms of any two out of p_1, q_1, p_2, q_2, so that we obtain

$$\begin{aligned} p_2 &= (p_1D_4 + \beta q_1D_5)/D_1, & \beta q_2 &= (-p_1D_6 + \beta q_1D_3)/D_1, \\ p_1 &= (p_2D_3 - \beta q_2D_5)/D_2, & \beta q_1 &= \quad (p_2D_6 + \beta q_2D_4)/D_2. \end{aligned} \tag{23}$$

As it is more convenient to deal with the volumes $X_1 = \sigma_1 q_1$, $X_2 = \sigma_2 q_2$ we shall have in general

$$p_2 = ap_1 + bX_1, \quad X_2 = cp_1 + dX_1, \tag{24}$$

where

$$a = \frac{D_4}{D_1}, \quad b = \frac{\beta}{\sigma_1}\frac{D_5}{D_1} \quad c = -\frac{\sigma_2}{\beta}\frac{D_6}{D_1}, \quad d = \frac{\sigma_2}{\sigma_1}\frac{D_3}{D_1}, \quad ad - bc = \frac{\sigma_2}{\sigma_1}\frac{D_2}{D_1},$$

and for the impedances belonging to the ends of the tube

$$Z_2 = \frac{aZ_1 + b}{cZ_1 + d_1}, \qquad Z_1 = \frac{dZ_2 - b}{-cZ_2 + a}, \tag{25}$$

so that the impedance at either end of the tube is a linear fractional function of the other. According to the apparatus attached to an end the impedance attached to that end is known. A tube for which a, b, c, d are given may be replaced by any other tube having the same constants.

Examples.—Cylindrical tube, σ constant. Put $x_2 - x_1 = Z_1$,

$$\frac{d^2p}{dx^2} + k^2p = 0,$$

$$u = \cos kx, \qquad r = \sin kx, \qquad u' = -\sin kx, \qquad v' = \cos kx, \tag{26}$$

$$D_1 = D_2 = 1, \qquad D_3 = D_4 = \cos kl, \qquad D_5 = D_6 = \sin kl, \tag{27}$$

$$a = d = \cos kl, \qquad b = \frac{\beta}{\sigma}\sin kl, \qquad c = -\frac{\sigma}{\beta}\sin kl,$$

$$Z_2 = \frac{\beta}{\sigma}\,\frac{Z_1\cos kl + \frac{\beta}{\sigma}\sin kl}{-Z_1\sin kl + \frac{\beta}{\sigma}\cos kl,}, \quad Z_1 = \frac{\beta}{\sigma}\,\frac{Z_2\cos kl - \frac{\beta}{\sigma}\sin kl,}{Z_2\sin kl + \frac{\beta}{\sigma}\cos kl} \tag{28}$$

Conical tube, $\sigma = \sigma_0 x^2$

$$\frac{d^2p}{dx^2} + \frac{2}{x}\frac{dp}{dx} + k^2x = 0 \tag{29}$$

$$u = \frac{\cos kx}{kx}, \quad v = \frac{\sin kx}{kx}, \quad u' = -\left(\frac{\sin kx}{kx} + \frac{\cos kx}{k^2x^2}\right), \quad v' = \frac{\cos kx}{kx} - \frac{\sin kx}{k^2x^2}$$

$$D_1 = \frac{1}{k^2{x_1}^2}, \quad D_2 = \frac{1}{k^2{x_2}^2}, \quad D_2 = \frac{\cos kl}{k^2x_1x_2},$$

$$D_4 = \frac{\cos kl}{k^2x_1x_2} + \frac{\sin kl}{k^3x_1{x_2}^2}, \quad D_5 = \frac{\sin kl}{k^2x_1x_2}$$

and if we introduce two lengths ϵ_1, ϵ_2, defined by the equations

$$\tan k\epsilon_1 = kx_1, \qquad \tan k\epsilon_2 = kx_2,$$

we easily get

$$a = \frac{x_1}{x_2}\frac{\sin k(l+\epsilon_1)}{\sin k\epsilon_1}, \qquad b = \frac{\beta}{\sigma_1}\frac{x_1}{x_2}\sin kl, \tag{30}$$

$$c = -\frac{\sigma_2}{\beta}\frac{x_1}{x_2}\frac{\sin k(l+\epsilon_1-\epsilon_2)}{\sin k\epsilon_1 \sin k\epsilon_2}, \qquad d = \frac{\sigma_2}{\sigma_1}\frac{x_1}{x_2}\frac{\sin k(l-\epsilon_2)}{\sin k\epsilon_2}, \tag{31}$$

$$Z_2 = -\frac{\beta}{\sigma_2}\,\frac{Z_1\dfrac{\sin k(l+\epsilon_1)}{\sin k\epsilon_1} + \dfrac{\beta}{\sigma_1}\sin kl,}{Z_1\dfrac{\sin k(l+\epsilon_1-\epsilon_2)}{\sin k\epsilon_1 \sin k\epsilon_2} + \dfrac{\beta}{\sigma_1}\dfrac{\sin k(l-\epsilon_2)}{\sin k\epsilon_2}}$$

$$Z_1 = -\frac{\beta}{\sigma_1}\,\frac{Z_2\dfrac{\sin k(l-\epsilon_1)}{\sin k\epsilon_2} + \dfrac{\beta}{\sigma_2}\sin kl}{Z_2\dfrac{\sin k(l+\epsilon_1-\epsilon_2)}{\sin k\epsilon_1 \sin k\epsilon_2} + \dfrac{\beta}{\sigma_2}\dfrac{\sin k(l+\epsilon_1)}{\sin k\epsilon_2}} \tag{32}$$

The formulae (31), (32) were used by Professor G. W. Stewart in designing horns to be used during the war.

It is not true, as is frequently stated in books on musical instruments, that the brass instruments of the orchestra are hyperbolic in profile, but I have found for all practical purposes the bell of every instrument may be represented by one of the three formulae

$$\sigma = \sigma_0 x^n, \qquad \sigma = \sigma_0 e^{-mx}, \qquad \sigma = \sigma_0 e^{-mx^2}$$

Even if an equation cannot be given to the profile the differential equation may be easily integrated graphically, or the length may be divided up into sections and different values of n used for different sections, as is customary in the theory of ballistics.

Case 1. $\sigma = \sigma_0 x^n$. (Change units so that $k = 1$)

$$\frac{d^2p}{dx^2} + \frac{n}{x}\frac{dp}{dx} + p = 0, \qquad \frac{d^2X}{dx^2} - \frac{n}{x}\frac{dX}{dx} + X = 0 \tag{34}$$

We have

$$p = J_{\pm\frac{n-1}{2}}(x)/x^{\frac{n-1}{2}}, \qquad X = J_{\pm\frac{n+1}{2}}(x)x^{\frac{n+1}{2}}, \tag{35}$$

Examples.

$$n = 0, \qquad n = 2, \qquad n = -2$$

$$J_{\frac{1}{2}}(x) = \sin x/\sqrt{x}, \qquad J_{\frac{3}{2}}(x) = \sin x/x^{\frac{3}{2}} - \cos /x\sqrt{x},$$

$$J_{-\frac{1}{2}}(x) = \cos x/\sqrt{x}, \qquad J_{-\frac{3}{2}}(x) = -\sin x/\sqrt{x} - \cos x/x^{\frac{3}{2}}$$

These include the straight cylinder, the straight cone, and the purely hyperbolic horn. In the latter case we have figure 5, where x_1, is the bell. If the horn is closed at x_2 we have

$$Z_2 = \infty$$

$$Z_1 = ck^2\left\{\frac{1}{c} - \frac{k}{2\pi}i\right\} = -\frac{d}{c} = \frac{(\sin kl + kx_1\cos kl)\,\beta x_1}{\sigma_0 k\sin kl_1}$$

and if we put $\xi = kl$

$$\operatorname{ctn}\xi = \frac{\sigma_1}{lc}\xi - \frac{l}{x_1}\frac{1}{\xi}, \tag{36}$$

which may be easily discussed graphically.

On the other hand if the horn is open at x_2 we have

$$\tan\xi/\xi = \left(1 - \frac{\sigma x_1 x_2}{cl^3}\xi^2\right)\Big/\left(1 + \xi^2\left\{\frac{x_1x_2}{l^2} - \frac{\sigma_1 x_1}{cl^2}\right\}\right). \tag{37}$$

These formulae were confirmed experimentally by my then assistant Dr. H. K. Stimson in 1915 on a coach-horn, a trombone, and a phonograph horn, with the following results:

		CALCULATED	OBSERVED
For the coach-horn	Closed	177	181
	Open	254	202
For the trombone	Closed	286	305
	Open	418	432
For the phonograph	Closed	311	304
	Open	329	415

These results give a fair agreement considering that we have used for the conductivity of the mouth the simple formula $c = 0.6\,R$ which is true only for cross-sections infinitesimal compared with the wave-length, whereas in the case of the wooden phonograph horn, the actual radius is nearly one-fourth of the wave-length.

A paper on the subject of the impedance of such an end will shortly appear.

In the case of an exponential section we have

$$\sigma = \sigma_0 e^{-mx}$$

$$\frac{d^2p}{dx^2} + m\frac{dp}{dx} + p = 0, \qquad \frac{d^2X}{dx^2} - m\frac{dX}{dx} + X = 0,$$

$$p = e^{-\sqrt{4-k^2x}}\left\{A\cos kx + B\sin kx\right\},$$

$$X = e^{-\sqrt{4-k^2x}}\left\{C\cos kx + D\sin kx\right\}.$$

and it is noticeable that the pressure vanishes at the same cross-section as for a straight tube.

Finally, in the case

$$\sigma = \sigma_0 e^{-mx^2}$$

we may solve the equation by means of the confluent hyper-geometric function.

It is to be noticed that in none of these cases, except the straight tube, are the different overtones harmonic. Thus, the characteristic tone of the "brass" is not due to the substance, but is entirely a matter of geometry as is shown by the heavy casting in plaster of Paris of a trombone bell used by the writer, the tone of which cannot be distinguished from that of the brass bell. I believe this phenomenon is well known.

Inasmuch as all musical instruments are composed either of resonators combined with strings, bars, plates, and horns, I feel that the above theory, while merely an approximation as to accuracy, will go far toward enabling us to complete the theory of musical instruments. Of course, the actual tones emitted by a brass instrument will depend upon the dynamics of the lips which is reserved for a future paper.

PHENOMENA OF CELL DIVISION IN THE CAMBIUM OF ARBORESCENT GYMNOSPERMS AND THEIR CYTOLOGICAL SIGNIFICANCE

By Irving W. Bailey

Bussey Institution for Research in Applied Biology

Communicated by W. M. Wheeler, May 8, 1919

In connection with a series of investigations upon the variations in size and structure of tracheary cells in vascular plants, the writer had occasion to search for information concerning the processes of growth and cell division in the cambium of arborescent plants. He was unable to find satisfactory descriptions of these phenomena in the botanical literature, and was assured by several cytologists that comparatively little was known about them; in all probability, owing to the difficulty of sectioning the lateral meristem (cambium) of woody plants. It seemed advisable, accordingly, to initiate a special histological and cytological investigaton of this important tissue.

During 1917 and 1918, specimens of the cambium of *Pinus Strobus* L. were collected—from trees of different ages and varying environments—at frequent intervals during the growing season, April to September. Methods were perfected for fixing, sectioning and staining this material, which was compared with that secured from 14 other genera of Coniferae. The results of this investigation and of a similar study of arborescent dicotyledons will be published *in extenso* at a later date, but certain of the cytological phenomena encountered are so significant as to merit a brief preliminary description in these pages.

The cambium, although essentially an undifferentiated or 'embryonic' type of tissue, is composed of cells which are considerably elongated. In *Pinus Strobus* L., as in other arborescent gymnosperms, the cambial initials are commonly from one hundred to several hundred times as long as they are wide (radial diameter). From the point of view of Sachs' and Strasburger's theories of the 'working sphere of the nucleus' and the observations of Treub, Kallen, Buscalioni, Pirotta and Buscalioni and others upon the occurrence of many nuclei in elongated protoplasts of the higher plants, it might have been expected *a priori* that these long meristematic cells would contain more than one nucleus each.[1]

This did not prove to be the case, however, in any of the material examined by the writer. Each cell contains a single nucleus, which is centrally located and placed so that its longest axis is approximately parallel to the long axis of the cell.

During mitosis, on the other hand, the polar axis of the division figure—late prophase, metaphase, anaphase, early telophase—*does not stand*, in most cases at least, *at right angles to the long axis of the cell.*[2]

As is shown in figure 1 (*A* and *D*) the mitotic figure is placed diagonally across the cell, at an angle of from 20 to 40 degrees. That this phenomenon

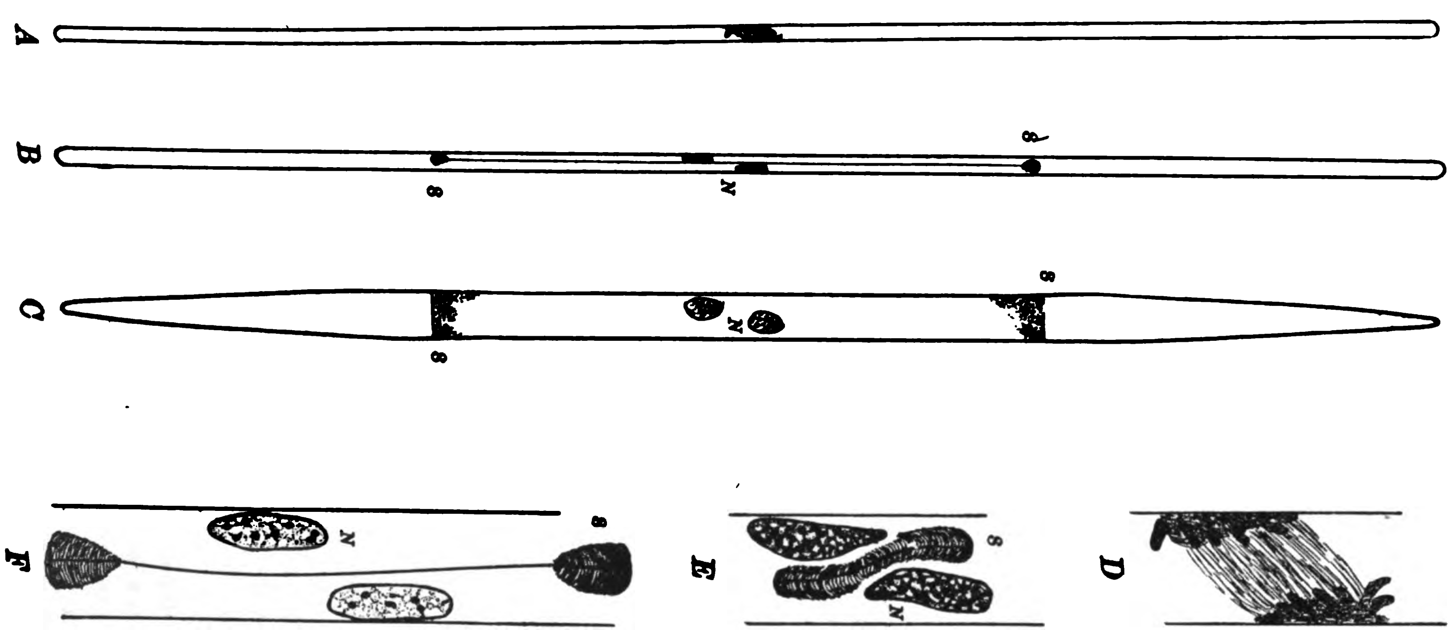


FIG. 1. PINUS STROBUS L.

A, Cambial initial, figured in radial and longitudinal extensions, showing oblique position of karyokinetic figure; *B*, the same, showing daughter nuclei and process of cell-plate formation; *C*, cell B, figured in tangential and longitudinal extensions; *D*, highly magnified view of karyokinetic figure shown in *A*; *E*, central portion of cambial initial, figured in radial extension, showing the beginning of cell-plate formation; *F*, the same, showing later stage of cell-plate formation; *n*, daughter nuclei; *s*, aggregations of kinoplasmic fibers, kinoplasmosomes, which are concerned in the formation of the division membrane.

is not an artifact, i.e. due to the displacement of an ordinary spindle, is indicated by the fact that the whole mitotic figure is asymmetrically developed, in conformity with its diagonal position (*D* and *E*).

Of course it is well known that the longitudinally dividing cells of the cambium form an exception to Sachs' law of 'rectangular intersection of successive division-planes' and Hertwig's modification of Sachs' hypothesis, but it has been assumed by Giesenhagen and others that, in the cambium, the karyokinetic figures lie with their polar axes perpendicular and their equatorial planes parallel to the long axes of the cells.

The formation of a cell-plate starting from one of these obliquely placed spindles is a very interesting phenomenon. The spindle becomes greatly extended laterally by the addition of peripheral fibers and gradually assumes the curved form shown in (*E*). As more peripheral fibers are successively added the remains of the central fibers disappear from about the cell-plate, leaving two separate aggregations of fibers which are connected by the first formed portion of the cell-plate (*F*). These aggregations of kinoplasmic fibers, which may be called *kinoplasmasomes*, have a very characteristic form and structure. They extend across the cell—at right angles to its longitudinal axis—from one radial wall to the other (*C*), and are located in the centre of the protoplast midway between its tangential surfaces (*F*). In sectional view (*F*), they have a somewhat wedge-shaped outline, bluntly convex in front and tapering to a point at the rear along the cell-plate. The *kinoplasmasomes* move in opposite directions towards the ends of the cell (*B* and *C*). As they move forward, the cell-plate is extended until it eventually reaches the two ends of the cell, thus dividing the protoplast into two similar portions each of which contains one of the daughter nuclei. The latter remain close together near the centre of the cell during the process of cell-plate formation. The writer has been unable to demonstrate any visible connection between the daughter nuclei and the kinoplasmasomes or their constituent fibers. Except at the beginning of cell-plate formation the daughter nuclei are in a semi-'resting condition' and are provided with a clearly defined nuclear membrane and numerous nucleoli. Not infrequently the distance traversed by the kinoplasmasomes, in passing from the vicinity of the daughter nuclei to the ends of the protoplast, may be from one to several millimeters.

This type of cell division, in which the process of cell-plate formation is so greatly extended—both as regards space and time—and so clearly dissociated from the usual phenomena of karyokinesis, promises, upon further analysis, to be of some significance in the discussion of the dynamics of cell division.

[1] Schacht and Russow claimed to have seen several nuclei in the cambial cells of Pinus.

[2] The writer is dealing with the normal longitudinal divisions of the cambial initials.

ON THE POSSIBLE FORM OF THE EQUATION OF STATE OF POWDER GASES

By A. G. Webster

Ballistic Institute, Clark University, Worcester, Massachusetts*

Communicated May 14, 1919

It has been customary for ballisticians to make use of the equation proposed by Clausius,

$$\left\{p+\frac{a}{T(v+\beta)^2}\right\}(v-\alpha)=RT, \tag{1}$$

in the simplified form, suitable for the high temperatures concerned,

$$p\ (v-\alpha) = RT. \tag{2}$$

At the same time it is customary to make use of the experimental results of Mallard and le Chatelier and of Berthelot and Vieille on the specific heats which state that C_v is a linear increasing function of the temperature. While apparently no experiments have been made on C_p it is assumed that the difference of the specific heats is constant, as in the case of an ideal gas.

It has occurred to me to examine the question of the most general form possible for the equation of state that shall permit of variability of the specific heats, but maintain the constancy of their difference. This question does not appear to have been treated,

By an application of the two laws of thermodynamics we obtain the well-known equation

$$(C_p-C_v)\frac{\partial T}{\partial p}\frac{\partial T}{\partial v}=T. \tag{3}$$

If we use the usual letters for differential equations, putting x for v, y for p, z for T divided by $C_p - C_v$ supposed constant, and as usual p for $\partial z/\partial x$, q for $\partial z/\partial y$ we have the very simple partial differential equation,

$$F = p\,q - z = 0. \tag{4}$$

This may be very simply integrated by Cauchy's method, which consists in integrating the system

$$\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{Pp+Qq}=-\frac{dp}{X+pZ}=-\frac{dq}{Y+qZ}=\frac{du}{q},$$

where the capital letters represent the derivatives of F with respect to the corresponding small letters, and u is an extraneous parameter. Having found

* Contribution from the Ballistic Institute, Clark University, No. 5.

five integrals, with five arbitrary constants x_0, y_0, z_0, p_0, q_0 we make the latter functions of a second parameter v satisfying the equations

$$p_0 q_0 - z_0 = 0, \quad \frac{\partial z_0}{\partial v} = p_0 \frac{\partial x_0}{\partial v} + q_0 \frac{\partial y_0}{\partial v}. \tag{5}$$

We easily obtain the five integrals,

$$x - x_0 = u, \quad y - y_0 = \frac{p_0}{q_0} u, \quad \frac{p}{p_0} = \frac{q}{q_0}, \quad q - q_0 = u,$$

$$z - z_0 = 2\, p_0 u + \frac{p_0}{q_0} u^2, \text{ with } p_0 = \frac{z_0}{q_0}.$$

Instead of adopting Cauchy's form for the introduction of the arbitrary function, we will attempt to pass the integral surface through the plane $z_0 =$ *const.*, representing an isothermal. We put

$$\begin{aligned} &x_0 = v, \quad y_0 = \varphi(v), \quad p_0 + q_0 \varphi'(v) = 0, \quad \frac{z_0}{q_0^2} = -\varphi'(v), \\ &y = \varphi(v) - u\varphi'(v), \\ &z = z_0 \pm 2\sqrt{-z_0 \varphi'(v)}\; u - \varphi'(v) u^2, \\ &x = u + v. \end{aligned} \tag{6}$$

If we adopt the Clausius equation for the form of one particular isothermal, we may put

$$\begin{aligned} \varphi(v) &= \frac{Rz_0}{v - \alpha} - \frac{a}{z_0(v+\beta)^2}, \\ \varphi'(v) &= -\frac{Rz_0}{(v-\alpha)^2} + \frac{2a}{z_0(v+\beta)^3}. \end{aligned} \tag{7}$$

We thus obtain finally

$$\begin{aligned} &x = u + v, \\ &y = \frac{Rz_0}{v-\alpha} - \frac{a}{z_0(v+\beta)^2} + u\left\{\frac{Rz_0}{(v-\alpha)^2} - \frac{2a}{2_0(v+\beta)^3}\right\} \\ &z = z_0 \pm 2u\sqrt{z_0\left(\frac{Rz_0}{(v-\alpha)^2} - \frac{2a}{z_0(v+\beta)^2}\right)} + u^2\left\{\frac{Rz_0}{(v-\alpha)^2} - \frac{2a}{z_0(v+\beta)^3}\right\} \end{aligned} \tag{8}$$

so that we have the parametric equation of the surface. It may be noted that putting $u = 0$, $z_0 = T$ we fall back on the ordinary Clausius equation (1) as a particular case, with (2) and the ideal gas equations as still more particular.

In order to obtain the expression for the energy for such a gas, we make use of the equation

$$U = \int\left[\left(T\frac{\partial p}{\partial T} - p\right)dv + C_v dT\right], \tag{9}$$

in which we have now to put

$$\frac{\partial p}{\partial T} = \frac{\partial(p, v)}{\partial(x, y)} \Big/ \frac{\partial(T, v)}{\partial(x, y)}$$

We have now to make use of equations (6) in which, replacing the usual thermal notation, and now using x and y for the arbitrary parameters,

$$\begin{aligned} T &= T_0 \pm 2x\sqrt{-T_0\varphi'(y)} - x^2\varphi'(y), \\ p &= \varphi(y) - x\varphi'(y), \\ v &= x + y, \end{aligned} \tag{10}$$

$$\frac{\partial p}{\partial x} = -\varphi'(y), \quad \frac{\partial p}{\partial y} = \varphi'(y) - x\varphi''(y), \quad \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} = 1,$$

$$\frac{\partial T}{\partial x} = \pm 2\sqrt{-T_0\varphi'(y)} - 2x\varphi'(y),$$

$$\frac{\partial T}{\partial y} = \varphi''(y)\sqrt{-\frac{T_0}{\varphi'(y)}} - x^2\varphi''(y),$$

so that finally

$$U = \int\left[\left(T_0 \pm 2x\sqrt{-T_0\varphi'(y)} - x^2\varphi'(y)\right)\left\{\frac{-2\varphi('y) + x\varphi''(y)}{2\sqrt{-T_0\varphi'(y)} - 2x\varphi'(y) - \varphi''(y)\sqrt{-\frac{T_0}{\varphi'(y)}}}\right\}\right.$$
$$\left.\times\left\{\frac{\partial v}{\partial x}dx + \frac{\partial v}{dy}dy\right\} + C_v\left\{\frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy\right\}\right] \tag{11}$$

I have also integrated the equation for the case that the difference of the specific heats is a linear function of the temperature, but this seems not necessary in the light of present experimental data.

THE RELATIVE ADSORPTION OF MIXTURES OF OXYGEN AND NITROGEN IN COCOANUT SHELL CHARCOAL[1]

By Harvey B. Lemon and Kathryn Blodgett

Ryerson Physical Laboratory, University of Chicago

Communicated by A. A. Michelson, May 19, 1919

It is a well known fact that gases are adsorbed in charcoal with rates and in total amounts that vary in a manner closely related to the boiling temperatures of the gases. A mixture of gases may accordingly have its proportions entirely altered by adsorption. This is the principle of the method of Gehloff for the isolation of atmospheric neon.[2] Dewar has mentioned it as a convenient means of extracting a high percentage of oxygen from the air.[3] A quantitative knowledge of the manner in which the presence of one gas to saturation affects the adsorption of another is of great importance since these are the conditions of use under which charcoal has sprung into prominence in modern warfare.

The experiments herein described deal with the relative adsorption of mixtures of oxygen and nitrogen in varying proportions by a highly activated charcoal prepared in the manner described in a previous report by one of the writers. Relatively large amounts of charcoal are employed with respect to the quantity of gas used so that saturation is in all cases reached in the course of thirty minutes or so. The charcoal which weighed 6.5 grams when saturated with dry air at 20° and 750 mm. pressure was contained in Pyrex glass bulbs which could be outgassed by a diffusion pump. Outgassing was for four and a quarter hours at 582°C. After outgassing the tubes were immersed in liquid air of definite age and temperature. While immersed they were exposed to the given gas mixture contained in a constant volume of 975 cc. The initial pressure of the mixture was 73.95 cm. and observations were taken of it at intervals while the adsorption was going on and until it had ceased to fall. A barometer and McLeod gauge formed a part of the above mentioned volume for this purpose.

In figure 1 is given the data in graphical form where log p in cm. is plotted against log t in minutes after the adsorption began. The same sample always is included in this record, two others were used as controls. It is noted that the logarithm of the pressure reached by saturation is almost in a linear relation with the percentage of oxygen in the mixture.

This is shown in figure 2 as the line NO. Here log final pressure is plotted against percentage of oxygen.

The other lines of this diagram NN' and OO' show the final pressures of amounts of pure nitrogen and pure oxygen equal to those existing in the mixture at corresponding values of abscissa but adsorbed separately.

It is important to note that in no case does the final pressure of a mixture fall as low as the sum of the final pressures of the two components when adsorbed separately. In other words, each component in the mixture hinders the adsorption of the other. This is not in agreement with an earlier paper by Bergter[4] who concludes that "in the range of pressure investigated the ability of charcoal to adsorb nitrogen must be increased by the presence of oxygen." Bergter's pressure range was entirely below 1 mm. and the discrepancy may be due to this difference of experimental conditions.

In the course of the work it became of importance to know if the final adsorption would be the same if a given amount of pure gas were adsorbed all at once as it would if half were adsorbed at one time and then the second half

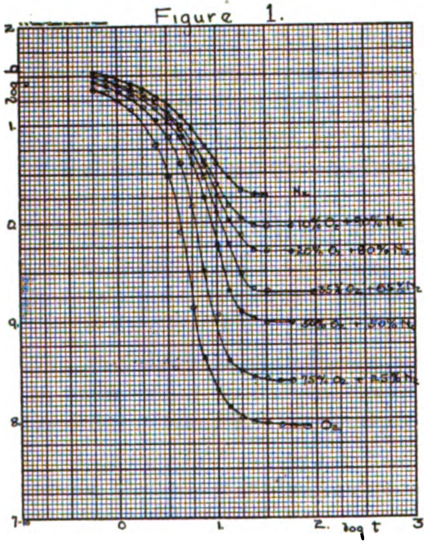


Figure 1.

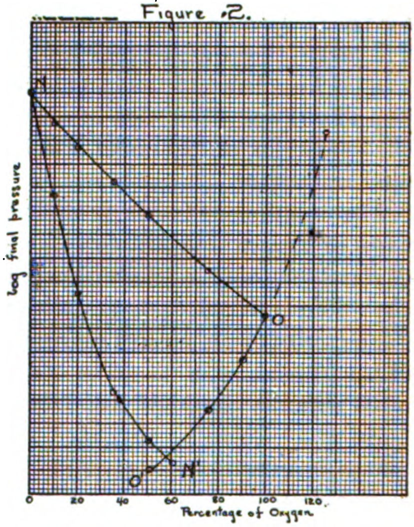


Figure 2.

admitted later. It was found that adsorption is much more complete in the second case, the quantitative pressures being as follows:

	Final pressure	Log p
a—Oxygen 100 per cent	0.00797	7.902–10
b—Oxygen 50 per cent, 50 per cent	0.00604	7.781–10
a—Nitrogen 100 per cent	1.86	0.270
b—Nitrogen 50 per cent, 50 per cent	0.649	9.812–10

In adsorbing equal parts oxygen and nitrogen it seems to be immaterial which is admitted first, the final pressure being within experimental limits the same.

Oxygen 50 per cent, Nitrogen 50 per cent	0.0451	8.654–10
Nitrogen 50 per cent, Oxygen 50 per cent	0.0481	8.682–10
Oxygen and nitrogen mixed each 50 per cent	0.0924	8.966–10

If a mixture of the two gases is adsorbed then as above the final pressure reached is considerably higher.

[1] This article is published with the approval of Major General William L. Siebert, Director Chemical Warfare Service, U. S. A.

[2] Gehloff, *Leipzig, Physik, Zs.*, 7, 1913, (838).

[3] Dewar, *London, Proc. Roy. Soc.*, 74, 1904, (122).

[4] Bergter, *Leipzig, Ann. Physik*, 37, 1912, (606).

VARIATIONS, DUE TO HEAT TREATMENT, IN THE RATE OF ADSORPTION OF AIR BY COCOANUT SHELL CHARCOAL[1]

By Harvey B. Lemon

Ryerson Physical Laboratory, University of Chicago

Communicated by A. A. Michelson, May 19, 1919

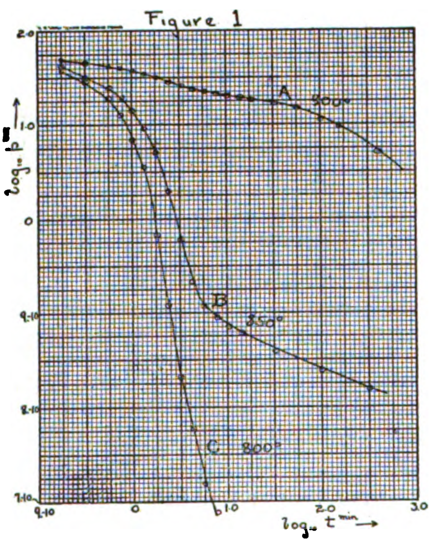
This paper is an abstract of results of experiments most of which were performed prior to January 1918, but the publication of which has been withheld during the war.[1]

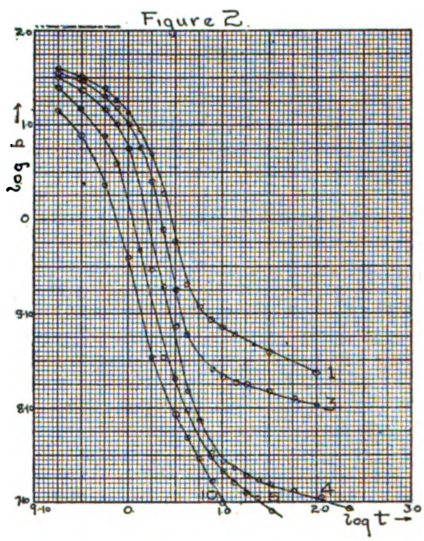
If charcoal contained in a bulb of glass or iron is heated to 600°C for about four hours and the gases which are freed pumped off into the low vacuum furnished by a mercury diffusion pump,[2] it is a well known fact that when the charcoal is then subsequently cooled to the temperature of liquid air it possesses a tremendous capacity for adsorbing gas, and will adsorb it at an extremely rapid rate. This adsorption power, however, has been found to be very different with different specimens of charcoal made from the same material, in this case cocoanut shell, and also to be very different when a single sample is used repeatedly. The magnitude of these differences is of no mean order but may be as large as the ratio of 10,000:1. It is shown in what follows that the heat treatment of the specimen during carbonization and also during successive 'outgassings' is a decisive factor in the control of the efficiency of the material as an adsorbent.

The experimental method was one of extreme simplicity. The shell was carbonized in an enclosed electric furnace having a vent for escape of gases and vapors. The temperature was indicated on a Leeds Northrup potential point resistance thermometer. The charcoal was then ground up to particles of from 1 to 3 mm. diameter and cleansed from all smaller fragments and dust. A definite weight (25.7 grams in most of the experiments, determined in dry air with which the charcoal was saturated) was sealed up in a tube of iron, quartz or Pyrex glass depending on the temperatures to be subsequently used. From this tube cocks communicated, (1) to the diffusion pump, and (2) to a fixed volume that could be filled with dry air at any desired pressure. This fixed volume included a McLeod gauge and a mercury barometer column so that the pressure in it could be read to within a few per cent over a range

of from 100 cm. to 0.00001 cm. A small Geissler tube was in communication with the charcoal bulb for a rapid means of observing the character of the gas content and, qualitatively, the pressure. After outgassing through cock, *1*, this was closed and cock, *2*, opened. The charcoal was thus exposed to a constant volume of 873 cc. initially filled with air at about 90 cm. pressure. This initial pressure was varied slightly as the room temperature varied so that the enclosed mass of air was constant, i.e. 1.2 grams. The air was immediately adsorbed and the rate of fall of the pressure observed as long as it was appreciable.

Results are expressed graphically by plotting the logarithm of the pressure in cm. against the logarithm of the time in minutes elapsing after the cock, *2*, was opened. Figure 1 shows curves for three different samples under





identical conditions. *A* was carbonized at 900°C., *B* at 850°C. and *C* at 800°C. They were all outgassed simultaneously for 6 hours at 425°C. and tested in rapid succession on the same apparatus. The enormous difference in rate is obvious in view of the logarithmic scales. The initial pressure of 90 cm. is reduced in 10 minutes to 20 cm., 0.71 cm. and 0.0003 cm. for *A*, *B* and *C* respectively.

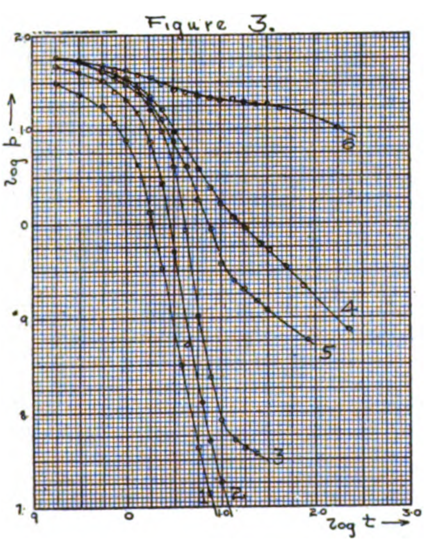
Figure 2 shows the results of successive 'runs' on the same specimen. This specimen was carbonized at 850°C. and the outgassings were all identical at 600° for 4½ hrs. It shows a systematic increase in rate after each run and subsequent outgassing becoming equal after 10 consecutive runs to the former sample shown as *C* in figure 1.

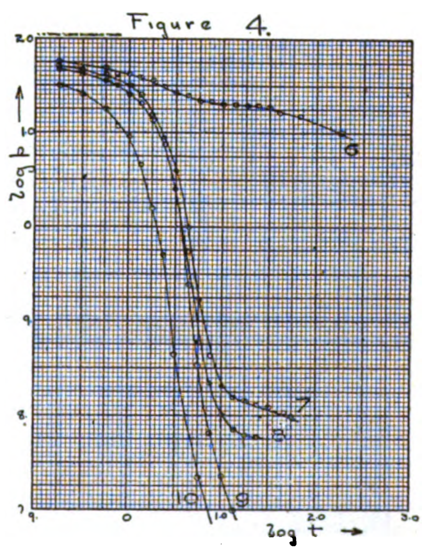
Figure 3 shows an effect which was controlled so as to be the converse of that illustrated in figure 2. It depicts also successive runs on a single speci-

men, the carbonization of which was for 3 hours at 670°C. The first run shows an extremely active adsorption. Successive outgassings were not now identical but were as indicated below:

First	4.5 hours at 633°C.
Second	0.5 hours at 800°C.
Third	0.5 hours at 850°C.
Fourth	0.5 hours at 875°C.
Fifth	4.0 hours at 640°C.
Sixth	0.7 hours at 905°C.

The first four runs show a cumulative loss of activity as result of high outgassing temperature. In the fifth the activity is in part restored by prolonged outgassing at the lower temperature and in the sixth it is again almost





totally destroyed. Quartz tubes are unsuitable for use for the high temperature work because of their devitrification by the hot carbon vapor.

Figure 4, repeating the sixth run shown in the preceding, carried the same sample back to high activity in four subsequent outgassings which were as follows:

Seventh	22.0 hours at 650°C.
Eighth	22.0 hours at 500°C.
Ninth	44.0 hours at 650°C.
Tenth	1.0 hours at 840°C.

A sluggishness of behavior is apparent after this much use as indicated in the long times necessary to produce a change in quality and also in the failure to respond as before to a temperature higher than 800.

Figure 5 shows continuation of the attempt for the second time to destroy the activity. Repeating curve 10 of the preceding and outgassing

Eleventh..0.5 hours at 950°C.
Twelfth...1.5 hours at 985°C.

the activity is again reduced. Recovery is now as before sluggish and was not carried to completion.

Thirteenth......................................3.0 hours at 650°C.
Fourteenth......................................8.0 hours at 625°C.
Fifteenth.......................................12.0 hours at 645°C.
Sixteenth.......................................1.5 hours at 650°C.
Seventeenth.....................................14.0 hours at 650°C.

Figure 5.

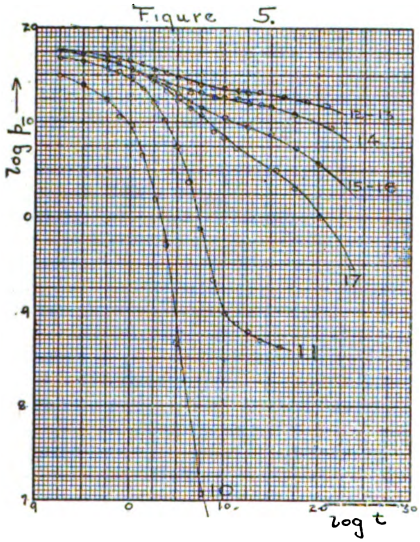
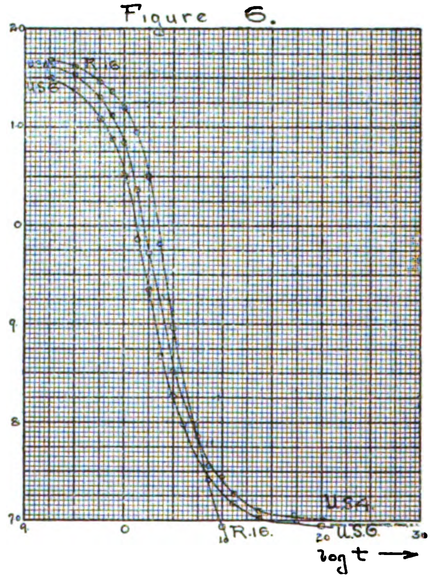


Figure 6.



The characteristic form of the family of curves shown is undoubtedly of generic quality. These curves have been reproduced on a wide variety of samples of cocoanut shell charcoal many times; and when the results are assembled exhibit almost every degree of gradation from a central, closely linear (after the first minute) relationship. Similar experiments on pure gases rather than air are in progress in the hope of simplifying this family of curves somewhat so as to obtain an interpretation expressible in analytic form.

Two hypotheses have been advanced for these phenomena. One ascribes the changes in the sense of increased activity to the gradual distillation out of the material of heavy nonvolatile hydrocarbons, many of which in the form of tars and gums come off during the initial carbonization. The other is based on the conception advanced by Miss Ida Homfray[3] that these phe-

nomena are to be regarded as saturated solutions of a rigid phase, carbon, in a fluid phase, gas. It suggests that repeated solution and subsequent evaporation, here taking the form of adsorption at low temperatures and outgassing at high, produces a gradual modification of the character of the carbon with respect to its fineness of division, something similar to precipitation occurring.

Loss of activity seems difficult to account for on the first hypothesis. It is to be noted that permanent loss of activity can always be produced on any sample by heating to 1200°C. This is usually ascribed to a partial destruction of porosity and has been observed before. Attempts to extract heavy hydrocarbons by the use of the lightest liquid solvents, ligroin alcohol and acetone were inconclusive. Considerable amounts of tarry material were removed, but not by the solvents directly. They distilled out during outgassing after treatment with the solvent. This treatment in all cases causes temporary loss of activity which is renewed in the usual manner with repeated use after low temperature outgassings. Experiments by others however[4] which have shown that activation is possible by other methods than the ones here outlined seems to favor the hydrocarbon hypothesis.

Those other methods for activation of field material for the adsorption of complex vapors have been used in conjunction with the above described process. A comparison of material activated by use and low temperature outgassings alone as herein described, with the most highly activated charcoals produced elsewhere under the conditions of these experiments is given in figure 6. The crossing of the curves may be very significant. The laboratory charcoal, R. 16, is less active initially but ultimately runs to lower values than the field material U. S. 4 and U. S. 6. The differences at either end are not large in comparison with the range of the phenomena discussed above.

Experiments are in progress on saturation values for adsorptions of mixtures of varied proportions. An hypothesis originally advanced by McBain[5] that there is a distinction to be made between surface condensation and interior diffusion is also being subjected to experimental scrutiny with modern materials now at our disposal. Both of these lines of work it is hoped will shed light on the mechanism of the process.

A more detailed account of this work will shortly appear in the Physical Review as a series of papers under the general title of Studies in Charcoal Adsorption.

[1] This article is published with the approval of Major General William L. Siebert, Director Chemical Warfare Service, U. S. A.

[2] Shrader, *Ithaca, N. Y. Physic. Rev.*, 12, 1918, (70).

[3] Homfray, *Liepzig, Zs. Phys. Chem.*, 74, 1910, (139).

[4] Dorsey, *Easton, Pa., J. Ind. Eng. Chem.*, 11, 1919, (234).

[5] McBain, *London, Phil. Mag.*, 18, 1909, (916).

REAL HYPERSURFACES CONTAINED IN ABELIAN VARIETIES

By S. Lefschetz

Department of Mathematics, University of Kansas

Communicated by E. H. Moore, April 29, 1919

1. In a recent note of these Proceedings (April, 1919), I showed that an abelian variety of genus p and rank one, V_p, is birationally transformable into a real one if and only if it possesses 2 p independent linear cycles $\gamma_1, \gamma_2, \ldots \gamma_{2p}$, with respect to which p integrals of the first kind have a period matrix of type $\Omega = \| \omega_{k,1}, \ldots \omega_{k,p}; i\omega_{k,p+1}, \ldots i\omega_{k,2p} \|$; $(k = 1, 2, \ldots p)$, the (ω)'s being real. I propose now to investigate the number ρ' of algebraically distinct real hypersurfaces which V_p, if real, may have. This number $\rho' \leqq \rho$, Picard number of V_p, may also be defined as the maximum number of real hypersurfaces which cannot be logarithmic singularities of a simple integral of the third kind.

2. In a general way V_p be an abelian variety of rank one, real or not, with the independent linear cycles $\gamma_1, \gamma_2, \ldots \gamma_{2p}$. By associating γ_μ with γ_ν we obtain a superficial cycle (μ, ν) and any other depends upon those of this type. In particular denoting by (A^{p-1}) the two dimensional cycle formed by A^{p-1}, curve of intersection of $p - 1$ algebraic hypersurfaces of the same continuous system as a given one A, we have

$$m\,(A^{p-1}) \sim \sum_1^{2p}{}^{\mu,\nu}\; m_{\mu,\nu}\,(\mu, \nu),\ (m_{\mu,\nu} \text{ integer} = -\,m_{\nu,\mu}). \tag{1}$$

It may be shown that if no integral of the first kind is constant on A the alternate form

$$\Sigma\, m_{\mu,\nu}\, x_\mu\, y_\nu \tag{2}$$

is a principal form of Ω as defined by Scorza (Palermo Rendic., 1916), and conversely to a principal form (2) corresponds an algebraic hypersurface A. Moreover to algebraically distinct hyersurfaces correspond linearly independent principal forms from which follows at once $\rho = 1 + k$, where k is Scorza's index of singularity for Ω.

3. Let us now assume V_p real. A real hypersurface A of V_p is transformed into itself by T, transformation of the variety which permutes its pairs of conjugate points and this property is characteristic for A. It may be shown that there are real curves A^{p-1},—let the one of No. 2 be one of them, and α its real line (locus of its real points). A small oriented circuit tangent to α in (A^{p-1}) is transformed by T into one of opposite orientation, for in the neighborhood of α, T behaves like an ordinary plane symmetry. It follows that T transforms the superficial cycle (A^{p-1}) into its opposite. Taking into account the fact that this cycle is a two sided manifold and also

the effect of T upon the linear cycles γ_μ of No. 1, we find at once that all the m's not of the type $m'_{\mu,p+\nu}$, $(\mu, \nu \leqq p)$ are equal to zero, hence ρ' *is equal to the number of independent forms of type.*

$$\sum_1^p{}^{\mu,\nu}\, m'_{\mu,p+\nu}\,(x_\mu y_{p+\nu} - x_{p+\nu} y_\mu) \tag{3}$$

which belong to Ω.

If V_p is pure $\rho' \leqq p$, for otherwise Ω would possess a degenerate form (3). This is to be contrasted with Scorza's result $1 + k \leqq 2p - 1$, or $\rho \leqq 2p - 1$ if Ω is pure.

4. Assuming $\rho' = 2$ let L, L', be the matrices formed by the determinants of two forms (3). They are both of type

$$\left\| \begin{array}{c|c} 0 & \Delta \\ \hline \Delta' & 0 \end{array} \right\|$$

where each square represents a matrix with p rows and columns, the matrices in the main diagonal having only zeroes for terms. As $L^{-1}\, L'$ is of the form

$$\left\| \begin{array}{c|c} \Delta & 0 \\ \hline 0 & \Delta' \end{array} \right\| = \|\, a_{\mu\nu} \,\|,\ (\mu, \nu = 1, 2, \ldots 2p;\ a_{\mu, p+\nu} = a_{p+\mu, \nu} = 0),$$

V_p has a complex multiplication defined by

$$\sum_1^p{}^{k}\, \lambda_{jk}\omega_{k\mu} = \sum_1^p{}^{\nu}\, a_{\mu\nu}\omega_{j\nu};\quad \sum_1^p{}^{j}\, \lambda_{j,k}\,\omega_{k,p+\mu} = \sum_1^p{}^{\nu}\cdot a_{p+\mu,p+\nu}\,\omega_{j,p+\nu}$$
$$(j, \mu = 1, 2, \ldots p),$$

the (λ)'s being necessarily real as they can be replaced by their conjugates. Finally the characteristic equation of this complex multiplication

$$\|\, a_{\mu\nu} - \epsilon_{\mu\nu}\, x \,\| = 0,\ (\epsilon_{\mu\nu} = 0,\ \mu\ \nu;\ \epsilon_{\mu\mu} = 1)$$

is necessarily reducible and a perfect square if V_p is pure.

5. Let us examine the case of a real hyperelliptic surface of rank one. The number ρ' has then the value 1 or 2, if the surface is pure not elliptic. A fundamental period matrix corresponding to linear cycles forming a minimum base may be reduced to the form

$$\left\| \begin{array}{l} 1,\ 0,\ \dfrac{m}{2} + ia,\ \dfrac{n}{2} + ib \\[2ex] 0,\ 1/\delta,\ \dfrac{n}{2} + ib,\ \dfrac{r}{2} + ic \end{array} \right\|,$$

where m, n, r, δ, are positive integers and $ac - b^2 > 0$. If $\gamma'_1, \gamma'_2, \ldots \gamma'_{2p}$, are the corresponding linear cycles those of No. 1 are given by

$$\gamma_1 = \gamma'_1,\ \gamma_2 = \gamma'_2,\ \gamma_3 = 2\gamma'_3 - m\gamma'_1 - n\delta\gamma'_2,\ \gamma_4 = 2\gamma'_4 - n\gamma'_1 - r\delta\gamma'_2,$$

and in general $\rho = \rho' = 1$, unless there is a singular relation as defined by G. Humbert. If the surface is not elliptic this relation can only be of type

$$\lambda a + \mu b + \gamma \delta c = 0, \ (\lambda, \mu, \nu, \text{ integers}) \tag{3}$$

and there can only be one such relation. In this case $\rho = \rho' = 2$, and the condition of existence becomes now, assuming as we may, $\nu > 0$,

$$\lambda a^2 + \mu ab - \nu \delta b^2 > 0$$

which assures us of the effective existence of the surface. If there are two singular relations such as (5) the surface is elliptic and $\rho = \rho' = 3$.

In addition to (5) there may be in the non-elliptic case as well as in the other a singular relation independent of (5) and reducible to the form

$$\lambda (b^2 - ac) + \mu = 0, \ (\lambda, \mu, \text{ positive integers})$$

and then $\rho - \rho' = 1$, both cases being realizable. Thus there are six distinct types of real hyperelliptic surfaces for which ρ, ρ' have the values: (1, 1), (1, 2), (2, 2), (2, 3), (3, 3), (3, 4), the last three corresponding to elliptic cases.

NATIONAL RESEARCH COUNCIL

EXTRACTS FROM THE MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

At the National Research Council Building, April 15, 1919, at 9.30 A.M.

Present: Messrs. Bancroft, Clevenger, Cross, Dunn, Flinn, Hale, Howe, Hussey, Johnston, Leuschner, Merriam, Millikan, Noyes, Walcott, Washburn and Yerkes. Mr. Hale in the chair.

The minutes of the meeting of the Executive Board, March 11 and of the meetings of the Interim Committee, March 18, 25, April 1, and April 8, were approved. The actions taken by the Interim Committee are included in the record of these minutes.

At these various meetings reports were presented by the Chairman of the Council and by the Chairmen of the Divisions of Science and Technology on the progress of the nominations from scientific and technical societies for representatives to serve on the Divisions of Science and Technology of the Council, on the organization meetings of various Divisions and on the election of officers, executive committees, and sub-committees of the Divisions. In accordance with action taken by the Executive Board, each divisional organization will be printed in the PROCEEDINGS after its completion and subsequent approval by the Executive Board, except that sub-committees of Divisions will be reported in the Minutes of the meeting at which they are approved. The membership and officers of the Divisions of Physical Sciences, of Engineering, of Chemistry and Chemical Technology, of Geology and Geography, and of Biology and Agriculture are announced in the present number.

Moved: That the first meeting of each of the permanent Divisions, at which the Division is organized and elects its officers and executive committee, be recognized as the annual meeting of the Division for this year. (*Adopted.*)

Moved: That all appointments made hereafter shall be considered to terminate on the last day of June of the appropriate year. (*Adopted.*)

Moved: That the question as to when the present membership of the Council ceases be referred to the Committee on Orzanization with the request that they formulate and circulate resolutions regarding this matter before the next meeting of the Interim Committee. (*Adopted.*)

Moved: That the Secretary be requested to bring to the attention of the Divisions the provisions of Section 6, Article V, for fixing by lot the term of office of the new members of the Divisions. (*Adopted.*)

Moved: That the Division of Physical Sciences be authorized to appoint a special committee to consider the advisability of enlarging the functions of the International Bureau of Weights and Measures and to prepare suitable recommendations in this respect.

Moved: That an additional allotment of $1,000 be made to the general maintenance fund of the Engineering Division; and that the transportation expenses of the members of the Division attending the first meeting, be met by the Council. (*Adopted.*)

Moved: That an appropriation of $500 be made to the Executive Board to cover the expense of preparation of an annotated analytical bibliography, according to the scheme proposed by Mr. Fulcher, of the literature dealing with the effects of over-strain and blue heat on steel. (*Adopted.*)

Moved: That in view of Mr. Hale's desire to be relieved of his duties as Chairman of the Council at an early date Mr. J. C. Merriam be elected Acting Chairman as from March 15. (*Adopted.*)

Moved: That there be constituted an Interim Committee of the Executive Board which shall include in its membership the Chairman, Secretary, and Treasurer of the Council and the Chairman (or in his absence the Vice-Chairman) of each of the Divisions of the Council; that the Chairman of the Council shall be Chairman of the Interim Committee; and that this Interim Committee shall act for the Executive Board between meetings of the latter. (*Adopted.*)

Moved: That reports of the conferences of members of the Council with research representatives of Middle West and Pacific Coast institutions, held at Chicago and at Berkeley, California, on February 1st and March 8th, respectively, be incorporated in the minutes of the Council. (*Adopted.*)

Moved: That an address entitled 'Industrial Research' by Dr. Frank B. Jewett, which was presented before the Royal Canadian Institute, Toronto, February 8, 1919, be published as a bulletin of the National Research Council. (*Adopted.*)

The Chairman of the Council presented a communication from the Secretary of the General Education Board, informing the Council that the Board had made an appropriation of $25,000 for the purpose of a study to be conducted under the auspices of the National Research Council by Mr. Robert M. Yerkes and Mr. Lewis M. Terman for the purpose of developing intelligence tests for the mental rating of school children, payments to be made to the National Academy of Sciences.

Moved: That the National Research Council express its appreciation of the action of the General Education Board in appropriating a sum of $25,000 for the purpose of developing intelligence tests for the mental rating of school children, and extend a vote of thanks for this gift. (*Adopted.*)

Moved: That the sum of $450, or so much thereof as may be necessary, be appropriated from the funds available for the Engineering Division to cover the transportation expenses of certain members of the Committee on the Use of Tellurium and Selenium attending a meeting of the Committee to be held in Buffalo. (*Adopted.*)

Mr. Washburn presented a brief report of the initial meeting of the new Division of Chemistry and Chemical Technology held in Washington, March 21 and 22.

Moved: That the American Ceramic Society be invited to form with the Division a joint Committee on Ceramic Research. (*Adopted.*)

Moved: That the Committee on Synthetic Drugs be continued; and that the Committee on Explosives Investigations be continued temporarily. (*Adopted.*)

Moved: That the Council authorize the sending of a delegate to a meeting called by the Société de Chimie Industrielle to be held in Paris April 14, at which certain international chemical questions will be discussed and that the selection of this delegate be left to the Chairman and Chairman-elect of the Division, and the Chairman of the National Research Council. (*Adopted.*)

Moved: That Mr. Leuschner be elected Acting Secretary of the National Research Council from April 16 to June 30, 1919. (*Adopted.*)

Moved: That the Chairman appoint a Committee to consider all matters relating to organization of administration. (*Adopted.*)

Appointed: Messrs. Cross (Chairman), Clevenger, Hussey, Leuschner, Yerkes.

Moved: That the present Budget Committee be discharged and that the Chairman appoint a committee to consider the budget for the ensuing year. (*Adopted.*)

Appointed: Messrs. Leuschner (Chairman), Cross and Clevenger.

Moved: That the transportation expenses of the members of the Division of Biology and Agriculture attending the first meeting of the Division be met by the Council. (*Adopted.*)

Moved: That the President of the National Academy of Sciences be requested to appoint as members of the Research Council the representatives of societies and others nominated to membership in the Divisions of Engineering, Chemistry, and Physical Sciences, for such terms as have been or may be determined by lot in the Divisional meetings. (*Adopted.*)

The Chairman of the Division of Engineering recommended the approval of the following Committees:

Nominating Committee.—C. A. Adams (Chairman), E. P. Hyde, and Ambrose Swasey.

Executive Committee.—C. A. Adams, D. S. Jacobus, E. G. Spilsbury, and the Chairman and Vice-Chairman of the Division.

Publication Committee.—A. M. Greene (Chairman), E. P. Hyde, and E. G. Spilsbury.

Finance Committee.—W. R. Walker, (Chairman), Gano Dunn, C. F. Rand, and A. A. Stevenson.

Heat Treatment of Carbon Steel.—Henry N. Howe (Chairman), R. M. Boylston, and Albert Sauveur.

Pulverizing.—G. H. Clevenger (Chairman).

Committee on Insulation.—F. B. Jewett (Chairman), C. A. Adams, Leroy Clark, Wallace S. Clark, W. A. Delmar, F. M. Farmer, A. E. Kennerly, F. W. Peek, H. J. Shanklin, C. E Skinner, J. B. Whitehead.

Moved: That the Committees as recommended by the Engineering Division be authorized and that the appointment of the members as given above be approved. (*Adopted.*)

Mr. Clevenger presented a request of the American Welding Society that the National Research Council designate a representative for appointment as Director in the American Welding Society.

Moved: That in accordance with a request received from the American Welding Society the Division of Engineering be authorized to appoint a member of the Division as director in the American Bureau of Welding with full powers. (*Adopted.*)

The Acting Chairman, Mr. Merriam, presented a communication from Mr. W. A. Averill of the Central Bureau of Planning and Statistics requesting that a weekly report be presented on Friday of each week on the work of the National Research Council.

Moved: That in accordance with a request received from the Central Bureau of Planning and Statistics weekly reports of activities of the Research Council be transmitted to the Central Bureau of Planning and Statistics. (*Adopted.*)

The Acting Chairman, Mr. Merriam, announced the appointment on April 4, of the following committees and requested that these appointments be approved:

Committee on Organization *of the Division of Industrial Relations.*—Henry M. Howe, (Chairman), W. D. Bancroft, G. H. Clevenger, and E. F. Nichols.

Committee on Organization *of the Government Division.*—R. M. Yerkes (Chairman), W. D. Bancroft, and E. F. Nichols.

Committee on Program of National Research Council *for Meeting of National Academy of Sciences.*—A. O. Leuschner (Chairman), Whitman Cross, Henry M. Howe and R. M. Yerkes. (*Approved.*)

Mr. Howe, as Chairman, submitted a report of the Committee on Organization of the Division of Industrial Relations.

Moved: That the report of the Committee on Organization of the Division of Industrial Relations be adopted in the following amended form:

Organization of the Division of Industrial Relations

It is believed that the work of this Division, particularly its first work should be largely so directed as to stimulate industrial administrators to broaden their research activities, and to persuade the smaller industries to combine for the creation and maintenance of efficient research laboratories for the common good.

The chief contribution which the Research Council has to offer the industries is the organized assistance of its Divisions of Science and Technology which are directly available for suggesting problems of industrial research and coöperating in their solution. Hence, these Divisions will serve to stimulate the industries to carry out larger and more effective schemes of coöperative research, suggested by the Division of Industrial Relations.

The Division will require a Chairman and a Vice-Chairman. The Chairman should preferably be in a position to devote his whole time to the work and should possess the highest qualifications. He must be a man who can command the attention and respectful hearing of large industries. He should have the necessary directing and executive ability combined with the power to weld together the various schemes of coöperative research.

The Division of Industrial Relations should serve as the medium through which coöperation is secured for the Divisions of Science and Technology. The actual work of organizing and directing industrial researches will be the specific work of the Divisions of Science and Technology.

The Committee therefore infers that the need of activity on the part of this Division is not so pressing now as is the case with the Divisions of Science and Technology.

The Committee recommends that the Division be constituted as follows:

Ex Officio Members.—The Chairmen of the Divisions of Chemistry and Chemical Technology, Engineering, and Physical Sciences.

Representatives of Government Bureaus.—A representative each of the Bureaus of Chemistry, Mines, and Standards.

Members at Large.—At least eight representatives intimately connected with industrial research or highly experienced in industrial administration. (*Adopted.*)

Moved: That the present Committee on Industrial Research be requested to report nominations for membership on the Division of Industrial Relations in accordance with the approved constitution of the Division. (*Adopted.*)

Moved: That consideration be given to changing the functions of the present Advisory Committee of the Division of General Relations so that it may serve as an Advisory Committee to the Executive Board. (*Adopted.*)

Mr. Merriam, as Chairman of the Section on Relations with Educational Institutions and State Committees, presented a report on the organization of the Division of Educational Relations.

Moved: That in accordance with the report submitted by the Section on Relations with Educational Institutions and State Committees, the Division of Educational Relations be organized as follows:

Organization of the Division of Educational Relations

It is recommended that the Research Committees in educational institutions be asked to group themselves according to six regions; that the committees in each region be requested to form an organization consisting of one or more representatives from each committee, that this organization meet for conference on the problems of Research Committees at least once each year and elect a chairman for the group; and that the Chairman of each of the six regional organizations be nominated to represent the Research Committees in the Division of Educational Relations of the National Research Council.

The following organization of the six regions is recommended:

1. *Pacific Coast Region.*—Washington, Oregon, California, Idaho, Nevada, Utah, Arizona.
2. *West Central Region.*—North Dakota, South Dakota, Nebraska, Kansas, Montana, Colorado, Wyoming.
3. *East Central Region.*—Minnesota, Iowa, Wisconsin, Michigan, Illinois, Ohio, Indiana, Kentucky.
4. *South Central Region.*—New Mexico, Texas, Oklahoma, Missouri, Arkansas, Louisiana.
5. *South Atlantic Region.*—Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida, Alabama, Tennessee, Mississippi,
6. *North Atlantic Region.*—Maine, Vermont, New Hampshire, Massachusetts, Rhode Island, Connecticut, New York, Pennsylvania, New Jersey, Delaware, Maryland, District of Columbia.

It is recommended that the membership of the Division of Educational Relations be constituted as follows:

Regional representatives of research committees in educational institutions	6
Association of American Universities	1
National Association of State Universities	1
Association of American Agricultural Colleges and Experiment Stations	1
Association of American Colleges	1
American Association of University Professors	1
United States Bureau of Education	1
Divisional Nominations, not to exceed	8
	20

It is recommended that the divisional nominations should represent investigators or educators of note who have given special consideration to the problem of research in educational institutions. (*Adopted.*)

Mr. Merriam, as Chairman of the Section on Relations with Educational Institutions and State Committees, presented a report on the organization of the Division of States Relations.

Moved: That in accordance with the report submitted by the Section on Relations with Educational Institutions and State Committees, the Division of States Relations be organized as follows:

Organization of the Division of States Relations

It is recommended that the State Research Committees be asked to group themselves according to six regions; that the committees in each region be requested to form an organization consisting of one or more representatives from each committee; that this organization meet for conference on the problem of state scientific organization at least once each year and elect a chairman for the group; and that the chairman of each of the six regional organizations be nominated to represent the States Committees in the Division of States Relations of the National Research Council.

The following organization of the six regions is recommended:

1. *Pacific Coast Region.*—Washington, Oregon, California, Idaho, Nevada, Utah, Arizona.
2. *West Central Region.*—North Dakota, South Dakota, Nebraska, Kansas, Montana, Colorado, Wyoming.
3. *East Central Region.*—Minnesota, Iowa, Wisconsin, Michigan, Illinois, Ohio, Indiana, Kentucky.
4. *South Central Region.*—New Mexico, Texas, Oklahoma, Missouri, Arkansas, Louisiana.
5. *South Atlantic Region.*—Virginia, West Virginia, North Carolina, South Carolina, Georgia, Florida, Alabama, Tennessee, Mississippi.
6. *North Atlantic Region.*—Maine, Vermont, New Hampshire, Massachusetts, Rhode Island, Connecticut, New York, Pennsylvania, New Jersey, Maryland, Delaware, District of Columbia.

It is recommended that the membership of the Division of States Relations be constituted as follows:

Regional representatives of State Research Committees	6
Representatives of the Divisions of Science and Technology, National Research Council	7
Representatives of the Division of Educational Relations, Division of Industrial Relations, and of the Research Information Service	3
Divisional Nominations—not to exceed	8
	24

(*Adopted.*)

Moved: That the Section on Relations with Educational Institutions and State Committees give further consideration to the apportionment of States in the regional divisions with reference to the organization of the Division of States Relations. (*Adopted.*)

Mr. Yerkes, on behalf of the Committee on Organization of the Government Division submitted the following report:

After consideration of the relations of the proposed Government Division to the other Divisions of General Relations and the possible functions of the proposed Government Division, the Committee decided to recommend that organization of the Government Division be postponed until the other Divisions of General Relations have been organized.

Moved: That in accordance with the recommendation of the Committee on Organization of the Government Division, the organization of the Government Division be postponed until the other Divisions of General Relations shall have been organized. (*Adopted.*)

Mr. Yerkes, Chairman of the Research Information Service, presented the following report on the organization of the Research Information Service and moved that it be approved:

Organization of the Research Information Service

It is recommended that the membership of the Committee include: (1) Representatives of the Divisions of the National Research Council, (2) Representatives of the Government Departments, and (3) members at large, as follows:

Representatives from the Council.—The Chairman of each Division.

Representatives from Government Departments.—One each for Agriculture, War, Navy, Commerce, Labor, Interior, Justice, State, Post Office, and Treasury.

Members at large.—The number not to exceed ten, representing institutions or other organizations of special importance for the Division.

The Committee would consist of thirty-three members, assuming the National Research Council representation to be thirteen, representation of Government Departments ten, and memberhip at large ten.

It is further recommended that appointments shall be so arranged that not more than one-third of the Committee membership shall retire in any one year.

The Committee would be expected to elect a Vice-chairman and to organize a small Executive Committee.

The following tentative plan of work for the Research Information Service is submitted:

1. The proper cataloguing, analysis, and filing of informational reports from various sources. (Force required two.)

2. Research personnel information. Preparation of catalogue to serve as research personnel directory. Coöperation with Editor of *American Men of Science* recommended. (Force one or two.)

3. Collection of information and investigation of ways of furthering scientific bibliographies, abstracts, and handbooks, especially analytical abstracts and handbooks. This involves survey of the present situation in coöperation with Committee on Aids to Scholarship of the Institute of International Relations. (Force two.)

4. Organization of scientific and technical research information. Establishing of contacts with informational sources. (Force one or two.) (*Adopted.*)

Moved: That the Interim Committee place itself on record as understanding that the Chairman of the National Research Council is ex officio a member of all committees. (*Adopted.*)

Moved: That Miss Ruth Cobb be appointed to assist with the work of the Research Information Service at a salary of $1800 a year, the appointment to take effect not later than May 1, 1919. (*Adopted.*)

The Chairman presented the resignation of Mr. S. J. Farnsworth, Engineering Associate to the Scientific Attaché at London, to take effect April 1.

Moved: That the resignation of Mr. S. J. Farnsworth as Engineering Associate to the Scientific Attaché at London, to take effect April 1, be accepted with regret, and that the Chairman express to him the appreciation of the Council for the valuable services which he has rendered. (*Adopted.*)

Moved: That Mr. Henry M. Howe be appointed Scientific Attaché to the American Embassy at Paris, with appointment to date from April 16, 1919. (*Adopted.*)

Moved: That Mr. Clevenger be appointed Acting Chairman of the Division of Engineering during the absence of Mr. Howe, the Chairman. (*Adopted.*)

Mr. Howe, Chairman of the Division of Engineering, recommended that the Division of Engineering be authorized to form a small committee to study the relation of Neumann bands in iron and steel to the rate of rupture as caused by explosions or other stresses. It was stated that this work could be

carried on with little or no expense on account of the proposed coöperation of the Explosives Section of the Bureau of Mines and other agencies.

Moved: That the Division of Engineering be authorized to form a committee for the study of the relation of the Neumann bands in iron and steel to the rate of rupture as caused by explosions or other stresses. (*Adopted.*)

Moved: That the Acting Chairman appoint a nominating committee to nominate a Chairman, a Vice Chairman, and not more than ten members at large of the Executive Board of the National Research Council to be elected at a special meeting of the Executive Board on Wednesday, April 30, at 4 p.m. (*Adopted.*)

Appointed: Messrs. Hale (Chairman), Noyes, and Millikan.

Resolutions were proposed by the Treasurer and by the Acting Secretary of the Council to interpret the following former actions taken by the Executive Board:

Joint meeting of Board with Council of National Academy, February 11, 1919:

That the scale of annual salaries of the National Research Council be as follows: Chairman of the National Research Council, $10,000; Chairmen of Divisions of Science and Technology, $6,000, with an additional allowance of $1,000 for traveling expenses if they are sent abroad by the Council; Scientific Attachés, $6,000, until the Council is in a position to pay more.

Joint Meeting of Board with Council of National Academy, March 11, 1919:

That the new Divisions be considered to be effective when the divisional nominations have been approved by the Executive Board, and that present officers of Divisions shall continue to serve until their successors shall have been elected.

Interim Committee meeting, March 18, 1919:

That the first meeting of each of the permanent Divisions, at which the Division is organized and elects its officers and executive committee, be recognized as the annual meeting of the Division for this year.

Moved: That the resolution of February 11 be amended by the addition of the words "It is understood that this scale of salaries is to go into effect under the new organization on July 1, 1919, provided that officers elected under the new organization prior to that time shall be entitled to the foregoing salaries from the date on which they enter upon their duties on full time subject to special action of the Executive Board." (*Adopted.*)

Moved: That the resolution of March 11 be amended by the insertion of the words "under present salary arrangement" between the words "serve" and "until," and by the addition of the clause "and if they are elected to act for newly elected officers until the latter assume their duties, that they shall be entitled to the salaries provided for under the new organization subject to special action of the Executive Board." (*Adopted.*)

Moved: That consideration of individual cases that may come under these amended resolutions be referred to the Interim Committee with power. (*Adopted.*)

Mr. Cross presented a report of progress of the Committee on Organization of Administration with the following recommendation:

Moved: That William M. Davies and T. C. Mooney be appointed draftsmen at a salary of $125 a month from the date of their discharge from the army until further notice up to June 30, 1919. (*Adopted.*)

Mr. Leuschner, as Chairman of the Committee on Program of the National Research Council for the annual meeting of the National Academy of Sciences submitted the following recommendation:

Since all matters pertaining to the organization and activities of the National Research Council, which require action by the Academy, will be reported by the Council of the Academy at the regular business sessions of the Academy, the committee recommends that at the open meeting of the Academy on Wednesday afternoon, April 30, Dr. Hale present an address on the past work and future plans of the National Research Council and that this address be followed by brief statements of the work of individual divisions by their chairmen and by especially invited representatives of research activities related to the war. (*Approved.*)

The report of the Treasurer for the month ending March 31, 1919, was presented and placed on record.

Mr. Cross reported for the Budget Committee that an arrangement had been made by which disbursements for the London and Paris offices of the Research Information Service would be made through the Council of National Defense in Washington, after discontinuance of its disbursing office in London, about June 1.

Mr. Hale reported that in accordance with the resolution adopted March 11, 1919, at the joint meeting of the Executive Board with the Council of the National Academy of Sciences, authorizing a project of which the main purpose is to promote fundamental research in Physics and Chemistry in educational institutions primarily through the maintenance of a system of National Research Fellowships, a communication had been addressed by the President of the National Academy of Sciences and by the Chairman of the National Research Council to Dr. George E. Vincent, President of the Rockefeller Foundation, requesting an appropriation of $500,000, to be extended through a period of five years to enable the Council to carry out a project for promoting fundamental research in Physics and Chemistry in educational institutions in the United States through the maintenance of a system of National Research Fellowships in Physics and Chemistry and through such supplementary features as may promote the broad purpose of the project and increase its efficiency and that in reply to this request the following communication had been received from Mr. Edwin R. Embree, Secretary of the Rockefeller Foundation:

THE ROCKEFELLER FOUNDATION
61 BROADWAY, NEW YORK

April 10, 1919.

My dear Dr. Hale:

I have the honor to inform you that a meeting of the Executive Committee of the Rockefeller Foundation held April 9, 1919, the following resolutions were adopted:

RESOLVED that the sum of Fifty thousand dollars ($50,000) be, and it is hereby, appropriated, of which so much as may be necessary shall be paid to the NATIONAL RESEARCH COUNCIL during the year 1919 for the maintenance of a system of NATIONAL RESEARCH FELLOWSHIPS IN PHYSICS AND CHEMISTRY under the direction of the Research Board of that Council and in general conformity with the plans outlined in letter of application.

RESOLVED that the Rockefeller Foundation pledge itself to appropriate to the NATIONAL RESEARCH COUNCIL for the maintenance of a system of NATIONAL RESEARCH FELLOWSHIPS IN PHYSICS AND CHEMISTRY such additional sums for use in succeeding years as shall make available for expenditure during the period from May 1, 1919, to June 30, 1925, a total sum not to exceed Five hundred thousand dollars ($500,000), it being understood that the appropriation for any one year shall be for not more than One hundred thousand dollars ($100,000) plus any unexpended balances from appropriations for previous years.

Very truly yours,

EDWIN R. EMBREE,
Secretary.

Moved: That a committee be appointed to express to the Rockefeller Foundation the appreciation of the National Research Council for the Foundation's willingness to cooperate with the Council in the promotion of research, and of its munificence in supporting by the sum of Five Hundred Thousand Dollars the maintenance of National Research Fellowships in Physics and Chemistry.

Appointed: Messrs. Hale (Chairman), Noyes, and Millikan.

An announcement in regard to these Fellowships was issued by the National Research Council and is printed with these Minutes, page 313.

Mr. Yerkes, Chairman of the Research Information Service, presented a report of the Committee on Organization of the Government Division which was approved in the following amended form:

(1) The membership of the Government Division shall consist of the heads of such bureaus, civil and military, of Departments of the United States Government as shall be determined by the Executive Board of the National Research Council and by the Council of the National Academy of Sciences in accordance with Article V, Section I, of the Organization of the National Research Council.

(2) The Chairman of the Government Division shall be nominated by the Executive Board of the National Research Council and the President of the National Academy of Sciences shall be requested to present the name so nominated to the President of the United States for designation as Chairman of the Government Division.

(3) The Chairman of the Research Information Service shall be a member of the Government Division and shall serve as its Secretary.

The Executive Board concurred in the opinion of the committee that the Government Division has a very important function in bringing together at

intervals the heads of scientific bureaus of the Government and in promoting effective coöperation between them and also between their several bureaus and the National Research Council.

Mr. Hale presented a report on the organization of the Division of Foreign Relations and stated that in so far as has been determined at this time the representation on the Division would be as follows:

Representing the State Department. Hon. William Phillips, Assistant Secretary of State.
Representing the National Academy of Sciences.—Foreign Secretary, George E. Hale.
Representing the American Philosophical Society.—Henry Fairfield Osborn.
Representing the American Academy of Arts and Sciences.—A. A. Noyes.

Mr. Noyes reported that the Division of Chemistry and Chemical Technology had considered the report to the Executive Committee of the International Research Council from the Committee on International Coöperation in Chemistry appointed at the Paris Conference and recommended that it be concurred in by the Council with certain changes proposed by the Division.

Moved: That the report of the Division of Chemistry and Chemical Technology with reference to the proposed International Chemical Council be referred to the Division of Foreign Relations with the understanding that Messrs. Bancroft, Noyes, and Washburn be invited to meet with the Division for consideration of the report.

In accordance with these resolutions the following recommendations in regard to the formulation of an International Chemical Council were approved.

1. That an International Chemical Council be constituted; and that, if possible, arrangements be made for transferring the funds originally given to the International Association of Chemical Societies to this International Chemical Council.

2. That the object of the International Chemical Council be to initiate and promote international cooperation in chemistry; for example, by arranging:

(a) for international cooperation in the preparation and publication of chemical literature.

(b) for the appointment of international commissions to deal with special chemical questions of standardization (such as atomic weights, nomenclature, etc.)

(c) for international cooperation in the prosecution of special research projects.

(d) for the calling of international chemical conferences for various purposes; and also for the organization of an International Chemical Congress with meetings at stated intervals, and including all of the scientific and technological branches of chemistry.

3. That the International Chemical Council be constituted of delegates representing the leading chemical societies and other chemical research organizations of the several allied and neutral countries, these delegates to be selects as described in Paragraph 5.

4. That the International Chemical Council be affiliated with the International Research Council; and that the National Research Council of each country or its National Academy when no Research Council has been created, act as the intermediary in communications between the International Chemical Council and the chemical organizations of that country, and arrange for the proper representation of those organizations in accordance with Paragraph 5.

5. That the delegates from each country shall in general be chosen by the major chemical societies in that country, but that the number and distribution of such delegates and their voting strength within the delegation be determined initially by the National Research Council of that country, with the understanding that in countries where a National Research

Council shall not have been organized the National Academy itself shall fulfill this function until the National Research Council is organized.

6. That upon all questions voted upon by the International Chemical Council the number of votes cast by the various countries shall be determined by their population as follows:

Countries of less than	5	million inhabitants have 1 vote.
Countries between	5 and 10	million inhabitants have 2 votes.
Countries between	10 and 15	million inhabitants have 3 votes.
Countries between	15 and 20	million inhabitants have 4 votes.
Countries over	20	million inhabitants have 5 votes.

The inhabitants of colonies and possessions are included in the population of the country to which they belong, according to the indications of its Government. Each self-governing Dominion has the same number of votes as an independent country according to the above scale.

7. That the International Chemical Council, as soon as it shall be organized, shall elect an Executive Committee of seven members, which shall exercise such functions as may be assigned to it by the Council. The Executive Committee shall appoint an Executive Secretary, who shall have charge of correspondence and of the central office of the Council.

8. That until the International Chemical Council shall be organized and its Executive Committee appointed, the Committee on International Cooperation in Chemistry appointed by the Paris Conference shall act as a provisional Executive Committee for purposes of organization. Its membership shall, however, be increased by the addition of four members representing industrial chemistry, to be appointed respectively by the Royal Society of London, the Academie des Sciences de France, the Academia dei Lincei, and the National Research Council of the United States. This Committee shall elect a chairman and a secretary; but the latter need not be a member of the Committee.

Mr. Leuschner reported that the Committee on the Enlargement of the Functions of the International Bureau of Weights and Measures, appointment of which had been previously authorized by the Interim Committee, was constituted as follows: S. W. Stratton, Chairman, Comfort A. Adams, Joseph S. Ames, Gano Dunn, Henry M. Howe, Edward B. Hyde, Albert A. Michelson, Ernest F. Nichols, Edward B. Rosa, and the Chairman of the Division of Physical Sciences—ex officio and that this Committee had submitted the following report:

WHEREAS, practically all scientific investigations and much technological work required uniformity in fundamental standards of measurement, values of physical constants, and methods of measurement; and

WHEREAS, there is at present no provision for bringing about this uniformity other than that of the Treaty establishing the International Bureau of Weights and Measures,—charged with matters pertaining to length and mass only.

IT IS RECOMMENDED,—that the Treaty providing for the establishing and maintenance of the International Bureau of Weights and Measures be amended to provide for the following functions:

1. To serve as a depository for such standards as require the preservation of prototypes at a central place, and to make comparisons between these prototypes and the national prototypes of countries subscribing to the Treaty.

2. To establish international values of constants by the correlation of data produced at the various national and other scientific laboratories, such as the mechanical equivalent of heat, melting points, boiling points, gravitation constant, fundamental electrical units, velocity of light, etc.

3. To undertake investigations concerning standards, constants, or methods of measurement, which from their nature must be undertaken jointly by the nations subscribing to the Treaty.

4. To correlate and utilize, as far as possible, the work of the various national and other laboratories pertaining to standards, or methods of measurement, and to promulgate the results.

5. To provide for an increase in the membership of the Committee charged with the direction of the work of the International Bureau in order to secure a group of experts representative of the wider range of subjects.

Moved: That the Executive Board of the National Research Council express approval of the principle contained in the suggestion of the Executive Committee of the Division of Physical Sciences, that the scope of the International Bureau of Weights and Measures be enlarged, and that steps be taken, in consultation with national and international bodies interested, to determine the constants to be included, with a view to enlarging the function of the Bureau in harmony with the existing work and jurisdiction of other international bodies. (*Adopted.*)

On behalf of the Committee on Enlargement of the Functions of the International Bureau of Weights and Measures, Mr. Leuschner recommended that Messrs. Comfort A Adams, Joseph S. Ames, and A. A. Michelson be appointed as a special committee to report on a wave length standard for a unit of length. (*Approved.*)

Mr. Leuschner, Acting Chairman of the Division of Physical Sciences, submitted the following recommendations:

1. That Messrs. William Bowie, F. R. Moulton and C. F. Marvin be appointed a Committee on Variation of Latitude of the American Section of the International Geophysical Union, to confer with a similar committee of the American Section of the International Astronomical Union and to make joint recommendations with this Committee in regard to the future organization of researches on the variation of latitude.

2. That Messrs. L. A. Bauer, William Bowie, Whitman Cross, H. F. Reid, C. F. Marvin, A. O. Leuschner, and R. S. Woodward (Chairman), be appointed a committee to prepare recommendations regarding international cooperation in geophysical subjects for consideration by the American Section of the International Geophysical Union, with the understanding that this Committee has power to increase its membership.

3. That Mr. William Bowie be appointed Acting Chairman of the American Section of the International Geophysical Union.

4. That the organization meeting of the American Section of the Geophysical Union be held in Washington in conjunction with the June meeting of the American Section of the International Astronomical Union.

Moved: That the foregoing recommendations be adopted and that the nominations be approved, and that the further organization of the American Section of the International Geophysical Union be left with its Acting Chairman with power. (*Adopted.*)

On motion of Mr. Howe, the Committee on Organization of the Division of Industrial Relations, was discharged.

Mr. Howe presented a communication from Mr. Alfred D. Flinn, Secretary of Engineering Foundation, transmitting a report of the Foundation's Committee on Relations with the National Research Council. Mr. Dunn directed attention to certain changes in the report concerning the membership of the Division agreed upon by the Engineering Division at its meeting in New York on April 12. The report as amended is as follows:

Council shall not have been organized the National Academy itself shall fulfill this function until the National Research Council is organized.

6. That upon all questions voted upon by the International Chemical Council the number of votes cast by the various countries shall be determined by their population as follows:

Countries of less than 5 million inhabitants have 1 vote.
Countries between 5 and 10 million inhabitants have 2 votes.
Countries between 10 and 15 million inhabitants have 3 votes.
Countries between 15 and 20 million inhabitants have 4 votes.
Countries over 20 million inhabitants have 5 votes.

The inhabitants of colonies and possessions are included in the population of the country to which they belong, according to the indications of its Government. Each self-governing Dominion has the same number of votes as an independent country according to the above scale.

7. That the International Chemical Council, as soon as it shall be organized, shall elect an Executive Committee of seven members, which shall exercise such functions as may be assigned to it by the Council. The Executive Committee shall appoint an Executive Secretary, who shall have charge of correspondence and of the central office of the Council.

8. That until the International Chemical Council shall be organized and its Executive
Committee appointed, the Committee on International Cooperation in Chemistry appoin
by the Paris Conference shall act as a provisional Executive Committee for purposes of or
ization. Its membership shall, however, be increased by the addition of four members
senting industrial chemistry, to be appointed respectively by the Royal Society of
the Academie des Sciences de France, the Academia dei Lincei, and the National
Council of the United States. This Committee shall elect a chairman and a sec
the latter need not be a member of the Committee.

Mr. Leuschner reported that the Committee on the Enlarg
Functions of the International Bureau of Weights and Mea-
ment of which had been previously authorized by the Int
was constituted as follows: S. W. Stratton, Chairman, C
Joseph S. Ames, Gano Dunn, Henry M. Howe, Edward
Michelson, Ernest F. Nichols, Edward B. Rosa, and
Division of Physical Sciences—ex officio and that this
mitted the following report:

WHEREAS, practically all scientific investigations and muc
uniformity in fundamental standards of measurement, va
methods of measurement; and
WHEREAS, there is at present no provision for bringi
that of the Treaty establishing the International Burea
with matters pertaining to length and mass only.
IT IS RECOMMENDED,—that th
of the International Bureau
ing functions:
1. To serve as
at a central
prototypes
2. To e
the variou
heat, me
velocity

3. [illegible] the services of [illegible] Physics and [illegible].

4. [illegible] Medical Sciences, [illegible] of the Division [illegible].

5. [illegible] psychological rec- [illegible] tion.

(*Approved.*)

[illegible] the National Research [illegible] regret and the following

[illegible] tive services, characterized by [illegible] anization and in the past years [illegible] Ellery Hale the Council in accept- [illegible] ented hereby creates and bestows [illegible] of the National Research Council [illegible] of three to prepare and inscribe in [illegible] the services rendered and more fully [illegible] arch Council.

[illegible] (Chairman), Noyes, and Walcott.

[illegible] Brockett, *Assistant Secretary*.

[illegible] HIPS IN PHYSICS AND CHEMISTRY

[illegible] Rockefeller Foundation

[illegible] nal Research Council has been entrusted by [illegible] th the expenditure of an appropriation of [illegible] years for promoting fundamental research in [illegible] ly in educational institutions of the United States. [illegible] the plan is the initiation and maintenance of a [illegible] ch Fellowships, which are to be awarded by the [illegible] il to persons who have demonstrated a high order of [illegible] e purpose of enabling them to conduct investigations [illegible] ons which make adequate provision for effective prose- [illegible] physics or chemistry. The plan will include such sup- [illegible] as may promote its broad purpose and increase its [illegible]

[illegible] *ew.* Among the important results which are expected to [illegible] execution of the plan may be mentioned:

[illegible] of a scientific career to a larger number of able investigators [illegible] thorough training in research, thus meeting an urgent need of [illegible] d industries.

ENGINEERING FOUNDATION

Report of Committee on Relations with National Research Council

W. F. M. Goss, Chairman, Charles F. Rand, Silas H. Woodard, Frank B. Jewett Appointed at Meeting of Board, February 13, 1919.

April 10, 1919.

To Engineering Foundation Board:

Your Committee on Relations with National Research Council, after several meetings and conferences, recommends the approval by the Foundation Board of the following proposals:

1. Engineering Foundation, recognizing the desirability of maintaining close affiliation with National Research Council, proposes to collaborate with the Council "for the furtherance of research in science and engineering or for the advancement in any other manner of the profession of engineering and the good of mankind."

2. To contribute to the above end, as part of the policy of Engineering Foundation, office space in Engineering Societies Building has been engaged at the expense of Engineering Foundation, in addition to its own requirements, to serve as the New York office of the Engineering Division of the National Research Council, beginning May 1, 1919, and the Foundation, having brought its office to an adjacent room, in addition proffers to the Council and its Engineering Division, without charge, such secretarial services as the Foundation may from time to time determine.

3. National Research Council has proposed that its Engineering Division, comprising in all not less than 24 nor more than 29 members (of whom at least 7 and not more than 12 shall be members at large), be so organized as to include at least 5 members of Engineering Foundation, and (including these 5) 17 members of the Founder Societies. Engineering Foundation accepts this proposal as well calculated to meet the mutual requirements of the Foundation and the National Research Council.

4. Engineering Foundation proposes to collaborate with National Research Council in the activities of its Engineering Division and to make such appropriations of funds to aid specific undertakings of the Division as the Foundation may from time to time determine.

5. It is understood that all publications relating to research work in which the Foundation shall have participated, will be issued under the joint names of the Engineering Foundation and the National Research Council.

Respectfully,
W. F. M. Goss.
Chairman.

Moved: That the policy recommended by the Engineering Foundation's Committee on Relations with the National Research Council in its report of April 10, 1919, is acceptable to the National Research Council and will receive its approval if adopted by the Foundation, and that in anticipation of the adoption of the report by the Engineering Foundation the National Research Council express its appreciation of the Foundation's cooperation with the Council in the promotion of engineering research. (*Adopted.*)

Mr. Howe presented a recommendation of the Engineering Division that the number of members at large be increased from seven to twelve.

(*Approved.*)

Moved: That the Engineering Division move its offices to New York at as early a date as possible after May 1, 1919. (*Adopted.*)

Moved: That the material belonging to the National Research Council, with the exception of letter files and drawings now located in the shops of the Carnegie Institute of Technology at Pittsburgh, be turned over to that Institute with the request that they act as custodian of it until called for by the National Research Council. (*Adopted.*)

On behalf of the Division of Engineering, Mr. Howe offered the services of the Division to the National Research Fellowship Board in Physics and Chemistry.

On behalf of the Executive Committee of the Division of Medical Sciences, Mr. Hussey recommended that a grant of $800 from the funds of the Division be made to support the work of making an analysis of the psychological records of medical officers taken during the period of mobilization.

(*Approved.*)

Mr. Hale presented his resignation as Chairman of the National Research Council to take effect April 30, 1919.

The resignation of the Chairman was accepted with regret and the following resolution was unanimously adopted:

RESOLVED: That as a mark of appreciation of the constructive services, characterized by accomplishment as well as vision, rendered in the early organization and in the past years chairmanship of the National Research Council by George Ellery Hale the Council in accepting the resignation of the Chairmanship he has just presented hereby creates and bestows upon him in perpetuity the title of Honorary Chairman of the National Research Council and requests the presiding officer to appoint a committee of three to prepare and inscribe in the records a suitable minute more fully setting forth the services rendered and more fully expressing the sense of obligation of the National Research Council.

The Chairman appointed Messrs. Dunn (Chairman), Noyes, and Walcott.

The Meeting adjourned at 1 p.m.

PAUL BROCKETT, *Assistant Secretary*.

NATIONAL RESEARCH FELLOWSHIPS IN PHYSICS AND CHEMISTRY

SUPPORTED BY THE ROCKEFELLER FOUNDATION

General Statement.—The National Research Council has been entrusted by the Rockefeller Foundation with the expenditure of an appropriation of $500,000 within a period of five years for promoting fundamental research in physics and chemistry primarily in educational institutions of the United States.

The primary feature of the plan is the initiation and maintenance of a system of National Research Fellowships, which are to be awarded by the National Research Council to persons who have demonstrated a high order of ability in research, for the purpose of enabling them to conduct investigations at educational institutions which make adequate provision for effective prosecution of research in physics or chemistry. The plan will include such supplementary features as may promote its broad purpose and increase its efficiency.

Purposes in View. Among the important results which are expected to follow from the execution of the plan may be mentioned:

(1) Opening of a scientific career to a larger number of able investigators and their more thorough training in research, thus meeting an urgent need of our universities and industries.

(2) Increase of knowledge relating to the fundamental principles of physics and chemistry, upon which the progress of all the sciences and the development of industry depend.

(3) Creation of more favorable conditions for research in the educational institutions of this country.

Administration.—The plan will be administered by the Research Fellowship Board of the National Research Council. This Board consists of six members appointed for terms of five years, and of the chairmen ex officiis of the Division of Physical Science and the Division of Chemistry and Chemical Technology of the National Research Council. The members of the Board are

Henry A. Bumstead, Professor of Physics, Yale University.
Simon Flexner, Director of Laboratories, Rockefeller Institution for Medical Research.
George E. Hale, Director of Mount Wilson Observatory.
Elmer P. Kohler, Professor of Chemistry, Harvard University.
Robert A. Millikan, Professor of Physics, University of Chicago.
Arthur A. Noyes, Director of the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology.
Wilder D. Bancroft, Professor of Physical Chemistry, Cornell University.
Chairman of the Division of Chemistry and Chemical Technology.
——— — ————, Chairman of the Division of Physical Sciences.

Coöperation of Educational Institutions.—National Research Fellows will be permitted to conduct their investigations at institutions that will coöperate in meeting their needs. These needs differ widely from those of students seeking only instruction. Able investigators, actively engaged in productive research, are needed to inspire and guide the work of the Fellows. Research laboratories, adequately manned with assistants and mechanicians, and amply supplied with instruments, machine tools, and other facilities, are indispensable; and funds to provide supplies and to satisfy the constantly recurrent demands of research must be available. Above all, there must exist the stimulating atmosphere found only in institutions that have brought together a group of men devoted to the advancement of science through pursuit of research.

The Research Fellowship Board expects to make arrangements by which educational institutions will associate the Research Fellows with their graduate departments and offer the most favorable conditions for the prosecution of their researches.

The applicant will indicate one or more institutions at which, in his opinion, his research work can be conducted to the best advantage.

Fellowship Appointments.—The appointments of National Research Fellows will be made only after careful consideration of the scientific attainments of all candidates, not only of those who apply on their own initiative, but also of those who are brought to the attention of the Fellowship Board by professors in educational institutions and by other investigators throughout the country. In making the appointments much weight will also be given to the

judgment shown by the applicant in selecting and planning his proposed research.

The Research Fellowships will for the most part be awarded to American citizens who have had training equivalent to that represented by the Doctor's degree. The salary will ordinarily be $1,500 for the first year. The Research Fellowship Board will not, however, be bound by rigid rules of procedure. Thus it may offer larger salaries to those of exceptional attainment or wider experience, and may give appointment to competent investigators who have had training other than that represented by the Doctor's degree. The Research Fellows will be appointed for one year; but they will be eligible for successive reappointments, ordinarily with increase of salary.

Fellowship Regulations.—Research Fellows are expected to devote their entire time to research, except that during the college year they may at their option give not more than one-fifth of their time (outside preparation included) to teaching of educational value to themselves, or to attendance on advanced courses of study. They may associate graduate students with their researches. They shall not engage in work for remuneration during the term of their appointment. Fellows who have not received the Doctor's degree may, with the approval of the institution, offer their research work in partial fulfillment of the requirements for that degree.

Fellows are expected to submit to the Board shortly before the first of April of each year a detailed report on the progress of their researches. They must also present an account of their researches in form for publication before withdrawing from the Fellowship; and final salary payments will be deferred until this condition is fulfilled. It is understood that all results of investigation by the Fellows shall be made available to the public without restriction.

Fellowship appointments are subject to the condition that after they are accepted by the applicant, they will not be vacated within the year without consent of the Research Fellowship Board.

Fellowship Applications.—It is expected that fifteen to twenty Research Fellowships will be available during the coming year, and that the number will be increased in subsequent years. Applications for these Fellowships should be made on the form provided for the purpose, and should be sent to the Secretary of the Research Fellowship Board, National Research Council, 1023 Sixteenth Street, Washington, D. C., to whom all other correspondence should also be addressed. Applications will be received up to September 1, 1919 for Fellowships available during the next academic year; but a limited number of appointments will be made on the basis of the applications received. before April 20, 1919.

Washington, D. C., March 29, 1919.

ORGANIZATION OF DIVISIONS OF SCIENCE AND TECHNOLOGY

Division of Physical Sciences

C. E. Mendenhall, *Chairman.*
A. O. Leuschner, *Acting Chairman.*

Executive Committee

C. E. Mendenhall, *Chairman.*
William Bowie.
A. O. Leuschner, *Acting Chairman.*
R. A. Millikan.
H. N. Russell.
E. B. Wilson.

Representatives of Societies

	Term of Office Years
American Astronomical Society	
W. W. Campbell	2
H. N. Russell	1
Joel Stebbins	3
American Physical Society	
H. A. Bumstead	1
William Duane	2
Irving Langmuir	2
Ernest Merritt	3
R. A. Millikan	1
E. B. Wilson	3
American Mathematical Society	
E. W. Brown	1
L. E. Dickson	3
H. S. White	2

Members at Large

J. S. Ames	2
L. A. Bauer	3
William Bowie	2
Henry Crew	1
C. F. Marvin	1
Max Mason	3
M. I. Pupin	1
S. W. Stratton	2
A. Trowbridge	3

Division of Engineering

Henry M. Howe, *Chairman.*
Galen H. Clevenger, *Vice-Chairman.*

Executive Committee

Henry M. Howe, *Chairman.*
Comfort A. Adams.
Galen H. Clevenger, *Vice-Chairman.*
D. S. Jacobus.
E. G. Spilsbury.

Representatives of Societies

	Term of Office Years
American Society of Mechanical Engineers	
Arthur M. Greene	3
W. F. M. Goss	1
D. S. Jacobus	2
American Institute of Electrical Engineers	
Comfort A. Adams	2
F. B. Jewett	3
W. R. Whitney	1
American Institute of Mining Engineers	
Hennen Jennings	2
Philip N. Moore	1
Joseph W. Richards	3
American Society of Civil Engineers	
Anson Marston	3
H. H. Porter	1
George S. Webster	2
American Society for Testing Materials	
A. A. Stevenson	1
American Society of Illuminating Engineers	
Edward P. Hyde	2
Western Society of Engineers	
Arthur N. Talbot	3
Society of Automotive Engineers	
Charles F. Kettering	1

Members at Large

Henry M. Howe, *Chairman*	1
Galen H. Clevenger, *Vice-Chairman*	2
Edward Dean Adams	3
John J. Carty	2
Gano Dunn	2
Van H. Manning	2
Charles F. Rand	1
E. G. Spilsbury	1
Bradley Stoughton	3
S. W. Stratton	3
Ambrose Swasey	1
William H. Walker	3

Division of Chemistry and Chemical Technology

E. W. Washburn, *Acting Chairman.*
W. D. Bancroft, *Chairman.*
Julius Stieglitz, *Vice Chairman.*

Executive Committee

E. W. Washburn, *Acting Chairman.*
W. D. Bancroft, *Chairman-elect.*
Julius Stieglitz, *Vice Chairman.*
A. B. Lamb.
A. A. Noyes.
C. L. Alsberg.

Representatives of Societies

	Term of Office Years
American Chemical Society.	
C. L. Alsberg	1
W. D. Bancroft, *Chairman-elect*	1
C. G. Derick	1
J. M. Francis	3
E. C. Franklin	2
W. F. Hillebrand	3
John Johnston	2
Julius Stieglitz, *Vice Chairman*	3
J. E. Teeple	2
American Electrochemical Society	
Colin G. Fink	3
American Institute of Chemical Engineers	
Hugh K. Moore	1
American Ceramic Society	
Albert V. Bleininger	2

Members at Large

C. H. Herty	3
G. A. Hulett	3
A. B. Lamb	2
A. A. Noyes	1
C. L. Parsons	2
E. W. Washburn	1

Division of Geology and Geography

E. B. Mathews, *Vice Chairman.*

Executive Committee

E. B. Mathews, *Vice Chairman.*
Isaiah Bowman.
A. H. Brooks.
J. M. Clarke.
N. M. Fenneman.
David White.

Representatives of Societies

	Term of Office Years
Association of American Geographers	
W. M. Davis	2
N. M. Fenneman	3
J. Russell Smith	1
American Geographical Society	
Isaiah Bowman	2
Geological Society of America	
J. M. Clarke	2
Whitman Cross	3
R. A. Daly	2
H. E. Gregory	1
A. C. Lawson	3
C. K. Leith	1

Paleontological Society
T. Wayland Vaughan 1
National Geographic Society
Gilbert Grosvenor 3

Members at Large

Ralph Arnold 2
Eliot Blackwelder 3
A. H. Brooks 2
A. L. Day 3
Ellsworth Hungtington 2
Douglas Johnson 1
E. B. Mathews, *Vice Chairman* 3
R. A. F. Penrose, Jr. 1
David White 1

Division of Biology and Agriculture

C. E. McClung, *Chairman.*
L. R. Jones, *Vice Chairman.*

Executive Committee.

C. E. McClung, *Chairman.*
L. R. Jones, *Vice Chairman.*
I. W. Bailey.
F. R. Lillie.
G. R. Lyman.
H. F. Moore.
A. F. Woods.

Representatives of Societies

Term of Office Years

American Society of Agronomy
Charles V. Piper 3
American Society of Bacteriologists
Samuel C. Prescott 1
Botanical Society of America
William Crocker 2
A. S. Hitchcock 1
L. R. Jones 3
Ecological Society of America
W. M. Wheeler 2
American Society of Economic Entomologists
P. J. Parrott 3
Society of American Foresters
Barrington Moore 3
American Genetics Association
G. N. Collins 1
American Society for Horticultural Science
U. P. Hedrick 1
American Phytopathological Society
George R. Lyman 2

Society of American Zoologists	
M. F. Guyer	2
F. R. Lillie	1
G. H. Parker	3
Members at Large	
I. W. Bailey	1
B. E. Livingston	3
C. E. McClung, *Chairman*	1
C. F. Marbut	3
A. G. Mayor	2
H. F. Moore	2
J. R. Murlin	3
W. H. Osgood	2
A. F. Woods	1

EDITORIAL NOTE

With the next issue (August, 1919) the PROCEEDINGS will return to the larger type which was initially adopted but which was reduced a year and a half ago as a War economy. The return would have been made sooner but for the large amount of matter already set up and delayed in publication by a series of unavoidable accidents.

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 AUGUST 15, 1919 Number 8

SOME APPLICATIONS OF THE VARIATION OF HYDROGEN OVERVOLTAGE WITH THE PRESSURE

By D. A. MacInnes and A. W. Contieri

Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology

Communicated by A. A. Noyes, June 2, 1919

In a previous communication[1] it was shown that the hydrogen overvoltage at a given electrode can be expressed, at low current densities, with a considerable degree of approximation, by the equation:

$$2\,F\,E = \frac{3RT}{pr}\,\gamma$$

in which E, r, p, and γ are, respectively, the overvoltage, the radius of the hydrogen bubbles, the pressure, and the surface tension. The value of r was found experimentally to be independent of the pressure, and a plot of the variation of the overvoltage with the pressure was found to follow, very nearly at least, the hyperbola required by the above expression.

It next became interesting to investigate some chemical processes in which hydrogen overvoltage is intimately involved in order to see whether changes in these processes, produced by variations in the external pressure, are also in the direction predicted by the theory. Three such processes are: (1) the corrosion of metals in acid solutions, (2) reduction in acid solutions by metals, and (3) the electrodeposition of metals.

1. Metal Corrosion.—When a metal above hydrogen in the electromotive series is placed in a solution of an electrolyte there is a tendency for the metal to ionize; i.e., to split into ions and electrons; for instance, with iron, the following reaction tends to take place

$$Fe = Fe^{++} + 2e, \tag{1}$$

(e = electron). However, no chemical action will ensue unless another reaction, involving the absorption of the electrons liberated in Reaction 1, can also occur. In the absence of oxidizing agents the only possible reaction is

$$2H^+ + 2e = H_2 \tag{2}$$

Any factor which tends to decrease the velocity of Reaction 2, i.e., which increases the overvoltage, will decrease the corrosion represented by Reaction 1. Watts and Whipple[3] have found that, contrary to statements in the chemical literature, a decrease of the external pressure will produce a decrease of corrosion of metals in acids. These workers, however, attribute the decreased corrosion to the absence of air in the solutions which were under reduced pressure. As it seemed probable to us that the effect is, largely at least, due to an increase in the overvoltage with decreased pressure, we repeated their experiments, taking care to exclude oxygen from the acid. Table 1, which contains

TABLE 1

METAL	LOSS IN WEIGHT	
	At 1 atmospheric pressure	At $\frac{1}{15}$ atmospheric pressure
	gram	*gram*
Zn	0.0927	0.0594
Fe	0.0797	0.0469
Cd	0.0018	0.0018

typical results for Zn, Fe, and Cd, shows that the effect of decreased pressure on the corrosion of the first two of these metals is in the direction predicted. No bubbles of hydrogen were observed to leave the surface of the cadmium, so no change of overvoltage, and therefore no change of the corrosion, with the pressure is to be expected.

2. *Reduction, in Acid Solutions, by Metals.*—On placing a strip of iron into a slightly acidified solution of $FeCl_3$ the metal enters solution according to Reaction 1. The two ionic reactions that compete, so to speak, for the liberated electrons are

$$Fe^{+++} + e = Fe^{+++} \tag{3}$$

and Reaction 2. Here we can predict that a decrease of pressure will, by increasing the overvoltage, reduce the velocity of Reaction 2, with the result that Reaction 3 will be favored. The experimental results of a series of measurements are plotted in figure 1, in which abscissas represent pressures and ordinates reduction in a given time. Due to

irregularities in the surface of the iron the points are scattered, but the trend is clearly the same as that of the overvoltage-pressure curves given in the previous paper. This appears to be a promising method of increasing the speed and efficiency of a large class of reductions.

3. The Electrodeposition of Metals.—If hydrogen were evolved at a cathode at its reversible potential under all conditions no metal higher in the electromotive series than hydrogen could be deposited electrolytically from an aqueous solution. Due to hydrogen overvoltage, however, it is possible to obtain deposits of these metals as high in the

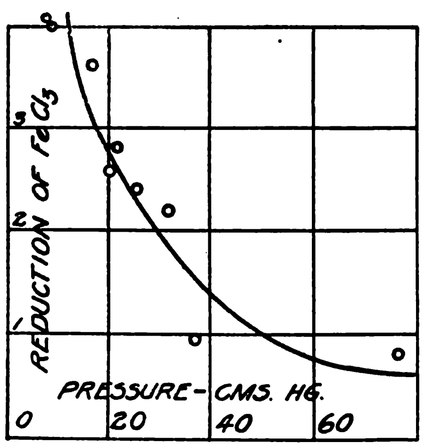


FIG. 1

series as zinc. The current efficiency of such a deposition depends, mainly, upon the relative magnitudes of the potential between the solution and the metal, and the hydrogen overvoltage of the metal; the efficiency increasing, of course, with an increase of the overvoltage. The figures in table 2 indicate that the increase of hydrogen overvoltage produced by a decrease of pressure is accompanied, in the case of the deposition of zinc, by an increase in the efficiency of the deposition. These depositions were all made from the same solution, and at the same current density and temperature.

TABLE 2

Pressure, *cms. Hg*	76	13	12	6
Efficiency, *per cent*	75	81	92	97.5

[1] These Proceedings, May, 1919, also *J. Amer. Chem. Soc.*, 41, 1919, (194).

[2] *Trans. Amer. Electrochem. Soc.*, 32, 1917, (257).

THE EFFECTS OF TRANSPLANTING LIMBS UPON THE FORMATION OF NERVE PLEXUSES AND THE DEVELOPMENT OF PERIPHERAL NEURONES

By S. R. Detwiler

Osborn Zoological Laboratory and Anatomical Laboratory, School of Medicine, Yale University

Communicated by R. G. Harrison, June 6, 1919

Previous experiments on the transplantation of limbs (Braus,[1] Banchi,[2] Gemelli[3] and Harrison[4]) have shown, with the exception of those of Banchi, that limbs placed in an abnormal (heterotopic) position will acquire a system of peripheral nerves derived from that part of the central nervous system of the host which corresponds to the position of the implanted limb rudiment. That such nerves may be partially functional has been shown both by slight spontaneous movements of the limbs and by movements in response to electrical stimulation.

The majority of the experiments to which reference has just been made were carried out upon anuran embryos at a period when the peripheral nerve paths were in part or wholly laid down. Accordingly, the limb rudiments were placed in the direct pathway of one or more developing spinal nerves which merely continued their growth into the rudiment so placed. In all of these experiments the transplanted limb was developed from an additional rudiment, taken from another embryo, the normal limb rudiments of the host being left intact.

Experiments carried out by the author upon the urodele, Amblystoma punctatum, in which the anterior limb rudiment was transplanted to an abnormal position at a period prior to initial outgrowth of the spinal nerves, strongly suggest that the limb exerts a guiding influence upon the segmental nerve supply and determines the path taken by the spinal nerves effecting its innervation. The positive reaction towards this influence appears to be greater in the nerves developing from the normal limb level of the cord than in those developing from more posterior segments of the cord.

Positive evidence has also been found to show that the functional activity of the transplanted limb has initiated a hyperplasia of the peripheral afferent neurones, the spinal ganglia connected with the heterotopic limb being considerably larger than their counterparts which do not supply an homologous organ. Evidence that the ganglionic hypertrophy has been brought about by an actual hyperplasia

of the elements has been obtained by making a count of the ganglion cells.

Although diligent search has been made, no evidence has as yet been obtained to show that the functional activity of the transplanted limb has effected a hyperplasia of the peripheral somatic motor neurones, there being no constant differences in size between the motor horn areas supplying the heterotopic limb and the opposite side where the limb is not present.

The experiments were carried out upon embryos in the so-called tail bud stage (stage 29). In the majority of the experiments the anterior limb rudiment was excised and re-implanted in the same embryo at distances ranging from one to seven segments posterior to the normal position (autoplastic transplantation). Other experiments consisted of transplanting an additional anterior limb rudiment to an embryo in which the normal limb rudiments were left intact (homoplastic transplantations). The larvae were preserved at intervals from thirty to eighty days after the operation, the majority of cases being under observation for about sixty days.

The results of the transplantations are presented in table 1. The figures in table 1 A illustrate the significant fact, that, as the limbs are implanted farther and farther away from the normal situation, there occurs a corresponding decrease in their ability to function until a position is reached (six segments posterior to the normal, series AS6) in which all exhibit very imperfect movements, there being no cases with perfect adaptive function.

Additional anterior limbs transplanted successively three, four and five segments posterior to the normal intact limb of the host (table 1 B), while exhibiting slight twitching movements in cases, never attain the completeness of function reached by autoplastic limbs implanted in the same relative position.

The gradual decrease in the function of the autoplastic limbs as they are implanted more and more remote from the normal limb region seems to be directly correlated with the segmental nerve contribution (table 2).

The normal anterior limb is supplied by a plexus composed of the third, fourth and fifth spinal nerves (fig. 1). In the tail bud stage of embryonic development the anterior limb rudiment constitutes a slightly thickened region of somatopleural mesoderm lying just ventral to the pronephros and extending from the anterior border of the third somite to the posterior border of the fifth.

TABLE 1

A. Showing the results of transplanting the right anterior limb rudiment a number of segments (AS1, AS2, AS3, etc.) posterior to its normal position.

B. Showing the results of transplanting an additional anterior limb rudiment to an embryo in which the normal limb rudiments have been left intact. HS3, HS4 and HS5 indicate the successive number of segments the additional limb rudiment was implanted posterior to the normal right anterior limb of the host.

	SERIES	OPERATIONS	POSITIVE EXPERIMENTS	NORMAL LIMBS		REDUPLICATIONS		FUNCTIONAL LIMBS		NO FUNCTION		FUNCTION IMPAIRED		FUNCTION PERFECT	
				Cases	Per cent	Cases	Per cent	Cases	Per cent	Cases	Per cent	Cases	Per cent	Cases	Per cent
A	AS1	25	20	19	95	1	5	20	100	0	0	1	5	19	95
A	AS2	21	18	17	94.5	1	5.5	17	100	0	0	3	16.7	15	83.3
A	AS3	24	17	13	76.5	4	23.5	15	88.2	2	11.8	4	23.5	11	64.7
A	AS4	34	29	18	67.0	11	33	23	79.3	6	20.7	11	37.9	12	41.4
A	AS5	35	30	11	36.7	19	63.3	20	66.7	10	33.3	16	53.3	4	13.4
A	AS6	35	29	22	75.9	7	24.1	15	51.7	14	48.3	15	51.7	0	0
B	HS3	10	9	2	22.2	7	77.8	9	100	0	0	6	66.7	3	33.3
B	HS4	30	21	8	38.1	13	61.9	11	52.4	10	47.6	11	52.4	0	0
B	HS5	30	25	11	44.0	14	56.0	13	52.0	12	48.0	13	52.0	0	0

TABLE 2

Showing the segmental nerve contribution to the anterior limb when implanted successively a number of segments (1 to 7) posterior to its normal position.

SERIES	CASE	POSITION OF LIMB NUMBER OF SEGMENTS POSTERIOR TO NORMAL	SEGMENTAL NERVE CONTRIBUTION								
			3	4	5	6	7	8	9	10	11
Normal..............	1	0	3	4	5						
AS1..................	12	1	3	4	5						
AS2..................	5	2	3	4	5						
AS2..................	12	2	3	4	5						
AS3..................	9	3		4	5	6					
AS3..................	18	3		4	5	6					
AS4..................	12	4		4	5	6	7				
AS4..................	27	4				6	7	8			
AS4..................	26	4			5	6	7				
AS4..................	30	4			5	6	7				
AS5..................	25	5			5	6	7	8	9		
AS5..................	27	5				6	7	8	9		
AS5..................	23	5					7	8	9		
AS5..................	30	5				6	7	8	9		
AS5..................	26	5					7	8	9		
AS6..................	29	6						9	9	10	
AS6..................	5	6							9	10	
AS7..................	5	7								10	11

The outgrowing third, fourth and fifth spinal nerves in Amblystoma, effecting connection with the limb rudiment at a period when it occupies its maximum extent (anterior border of the third somite to the posterior border of the fifth), become converged into a plexus as a result of concentration of the rudiment into the definitive limb bud which centers under the fourth myotome. Such convergence having taken place, a typical normal plexus as illustrated in figure 1 is produced.

However, in the examination of the segmental nerve supply to the anterior limb when transplanted posteriorly through distances ranging from one to seven segments, it is found that limbs so placed receive the

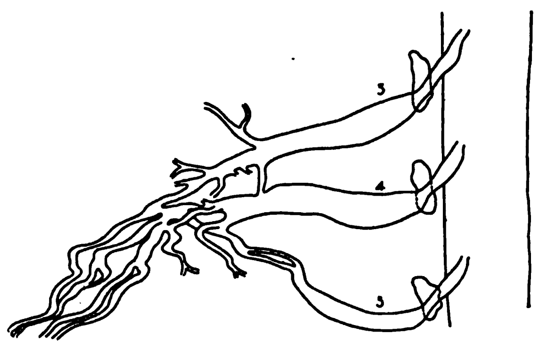


FIG. 1. GRAPHIC RECONSTRUCTION OF THE NORMAL LEFT BRACHIAL PLEXUS OF CASE AS4[26], PRESERVED SIXTY-EIGHT DAYS AFTER THE OPERATION. × 20

bulk of their nerve components from spinal segments situated anterior to the implanted limb rudiment rather than from segments corresponding to the new position of the limb (table 2 and fig. 2).

As has been observed previously, a very significant feature of table 2 illustrates the tendency of the transplanted limbs to be innervated by the nerves developing from the normal limb level of the cord, this tendency being most marked in the fourth and fifth nerves both of which, in the cases studied, were found to elongate in a postero-lateral direction a greater distance to effect innervation than did the third nerve or those developing from segments of the cord posterior to the normal limb level.

Anterior limbs when implanted the distance of one or two segments posterior to the normal position receive the normal segmental nerve contribution (table 2, series AS1 and AS2). The function of such

limbs is practically as complete as that of the limbs in their normal position. When transplanted posteriorly the distance of three segments, the third spinal nerve no longer contributes to the innervation of the limb and the function of limbs in this position is less complete (table 2, series AS3).

When transplanted posteriorly the distance of four segments, all cases studied, with one exception, show that the fourth spinal nerve no longer contributes to the innervation of the limb and that limbs in this position exhibit still greater imperfection of function (table 2, series AS4).

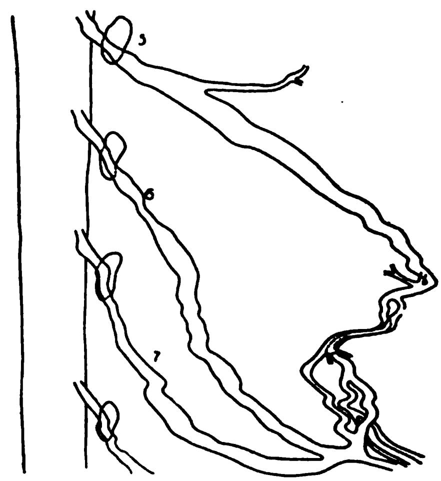


FIG. 2. GRAPHIC RECONSTRUCTION OF THE RIGHT BRACHIAL PLEXUS OF CASE AS4[26], SHOWING THE SEGMENTAL NERVE SUPPLY TO THE RIGHT ANTERIOR LIMB TRANSPLANTED THE DISTANCE OF 4 SEGMENTS POSTERIOR TO THE NORMAL POSITION. × 20

In the series AS5 in which the limbs were implanted the distance of five segments posterior to the normal position, there occurred only one case of those studied (case AS5[25], table 2) in which the limb received innervation from the fifth spinal nerve, the remainder being innervated by nerves posterior to the normal brachial nerves. With this general failure of the limbs in this position to receive innervation from the normal brachial nerves, it is found that still greater restrictions are placed upon their movements, there being only four cases in thirty which functioned normally (table 1 A).

In the series AS6 in which the limbs are so far removed that they receive no contribution from the normal limb nerves nor from the

sixth and seventh nerves, there are no cases which function normally, forty eight per cent being totally incapable of movements and fifty one per cent showing only greatly impaired movements (table 1 A and table 2).

Inasmuch as there are practically no structural deficiencies within the transplanted limb itself, the gradual decrease in the function of the limbs, as they are implanted more and more remote from the normal region, suggests the following possible factors as conditioning their degree of function, (a) structural deficiencies in the shoulder girdle, (b) deficiencies in the shoulder musculature, (c) the failure of certain of the shoulder muscles to receive innervation and (d) the absence of proper central neurone connections.

The shoulder girdle being a mosaic (Detwiler[6]), there is considerable variability in the degree of its development in the transplanted position, yet it is found that its development, in general, is no less complete in cases where the limb has been removed a considerable distance from the normal region than in those where the limb has been removed only a short distance. The difference, therefore, in the degree of function of the limbs could hardly be a result of this factor alone. Secondly, a study of the cross section anatomy of the shoulder region of the transplanted limbs shows that shoulder muscle differentiation does not become less complete as the limbs are implanted more and more posteriorly. No cases have been found in which there was complete absence of any of the muscles which typically develop in the heterotopic position.

Thirdly, peripheral efferent innervation to the shoulder muscles, although somewhat less complete quantitatively in the more posterior positions than in cases where the transplanted limb receives segmental nerves from all or a part of the normal limb level of the cord, is no less developed qualitatively, practically all of the individual muscles receiving some nerve fibers. Certainly the degree of defective peripheral innervation could hardly account entirely for the very imperfect movements exhibited by limbs in the series AS5 and AS6 (table 1 A).

The remaining factor viz: defective connections within the central nervous system appears to be the only one which will adequately account for the marked deficiency of function in limbs transplanted so far posteriorly as to be beyond the point where they receive peripheral innervation from the normal limb level of the cord.

Although in normal larvae of this age, the most obvious motor responses to various types of peripheral stimulation consist of total swimming reactions, under certain controlled conditions motor responses

may be almost entirely limited to co-ordinated movements of the limbs. Such responses may be carried out perfectly by the transplanted limbs when their peripheral innervation is derived from the normal limb level of the cord, but the ability of the transplanted limbs to exhibit movements, co-ordinated with the opposite intact limb, decreases markedly when their peripheral innervation is derived from segments well beyond the normal limb level (series AS6).

As has been shown by Herrick[6] we have developed here a central nervous architecture by means of which peripheral sensory stimuli pass through more or less localized ascending sensory tracts from the cord to the medulla (tractus spino-bulbaris), to the midbrain (tractus spino-tectalis) and to the thalmus (tractus spino-thalamicus). These stimuli may become finally discharged into the somatic motor centers of the spinal cord by means of descending tracts such as the tractus thalamo-bulbaris, tractus tecto-bulbaris, the fasciculus longitudinalis medialis and the tractus bulbo-spinalis. According to Herrick the cell bodies of the tractus bulbo-spinalis lie in the general motor tegementum of the medulla and their axones are directed ventrally into the ventral funiculi of the same and the opposite side. It is highly probable that a certain number of these fibers normally develop only as far as the third, fourth and fifth segments of the cord for specific discharge into the normal appendicular somatic motor centers. The fact that transplanted limbs, receiving peripheral innervation from these levels, exhibit movements which are co-ordinated with the opposite intact limb, strongly suggest such a condition. The behavior of limbs innervated mainly from the sixth, seventh and eighth segments of the cord (series AS5) suggests that these descending neurones, which normally end in the limb level, may be induced to continue their growth an additional segment or two to meet the functional demands imposed upon them by the transplanted limb. Their incapacity for further functional regulation is suggested by the loss of co-ordinated function and the greatly impaired movements that are exhibited by limbs of the series AS6, probably none of which receive peripheral innervation from segments of the cord anterior to the eighth.

The increase in the number of cases with total loss of function as the limbs are implanted more and more posteriorly (table 1 A) would also suggest that there occurs a corresponding increased deficiency in the connections of the purely intraspinal correlation neurones.

Additional limbs transplanted respectively three, four and five segments posterior to the normal intact limb of the host (table 1 B), never

attain the completeness of function attained by limbs in the same relative positions with the normal limb extirpated. Although such limbs may be well supplied with peripheral nerves, derived from segments of the cord posterior to the normal limb level, their greatly impaired movements appear to be a consequence of their inadequate supply of central efferent neurones, which run apparently only as far as the normal limb level where they discharge into the somatic motor centers of the normal intact limb.

The generally restricted and non-adaptive movements which these limbs do exhibit upon stimulation are probably effected through more or less imperfectly connected intraspinal, intersegmental correlation neurones of the levels from which peripheral innervation is derived.

A more complete account of the experiments reported in this paper will appear in later publications.

[1] Braus, H., *Morph. Jahrb.*, **35**, 1906.
[2] Banchi, A., *Anat. Anz.*, **28**, 1906.
[3] Gemelli, F. A., *Rev. Pathologia Nervosa Mentale*, **11**, 1906.
[4] Harrison, R. G., *J. Exp. Zool.*, **4**, 1907.
[5] Detwiler, S. R., *Ibid.*, **4**, 1918.
[6] Herrick, C. J., *J. Comp. Neur.*, **24**, 1914.

THE INTERFEROMETRY OF RAPID VIBRATIONS[1] CHIEFLY IN RELATION TO TELEPHONE CURRENTS

By C. Barus

Department of Physics, Brown University

Communicated June 13, 1919

1. *Introductory.*—The preceding apparatus[2] with telescopic or microscopic enlargement of the telephonic vibrations, behaved on the whole so satisfactorily, that it seemed worth while to try a similar design on the interferometer. I was inclined to doubt the feasibility of the plan; but it appeared on trial that the high tension wires actually keep the auxiliary mirrors of the interferometer practically quiet; so that in the absence of alternating current it is not difficult to find the fringes. Tense wires are out of step with the usual laboratory tremors. The system needs no special damping.

The displacement of achromatic fringes due to the induced secondary current is normal to their direction. The objective of the vibration telescope is to oscillate in the direction of the fringes and to

be coupled with the primary current. A full account of the changes of phase and amplitude in any transformer system may then be obtained from the fringe ellipses usually seen in the interferometer.

2. *Apparatus. Wide Bifilar.*—This is in large measure a modification of the apparatus described heretofore[3] except that special attachments have been added for sharply reaching the resonance tension of the wire. The latter is shown at $d\ e\ e'\ d'$ in figure 1 (front elevation), being the thinnest steel music wire, about 0.023 cm. in diameter. Its ends are wound around the stiff screws b, b', provided with locknuts, and rotating in horizontal short strong rods a, a', attached to stout standards (not shown) fixed to the bed plate A, B of the interferometer. The wire dd' passed around the grooved pulleys w, w', and above the grooved pulley x, carried in a fork and screw stem y. The latter may be raised or lowered by the nut u, which rests upon the massive carriage BB, supported by the slides AA' of the apparatus. Provision must be made (slotted sheath and pins, not shown) to prevent y from turning on its axis. Tension is roughly given to the wire at the screws b, b', and the fine adjustment is thereafter made at the nut u. This worked very satisfactorily.

The vibrator proper cc' is attached at the middle of the wires d, d', and carries the parallel auxiliary mirrors m, m', of the quadratic interferometer. A thin steel umbrella rib seemed well adapted to fulfill the requirements of cc' though a light soft iron tube would have been preferable.

The telephones T and T' are adjustable on special standards, attached to the bed plate (carriage BB) and placed horizontally, one in front and the other toward the rear of the vibrator cc'. It is desirable that one be adjustable on a micrometer screw and spring, so that the distance of the poles of both from cc' may be regulated.

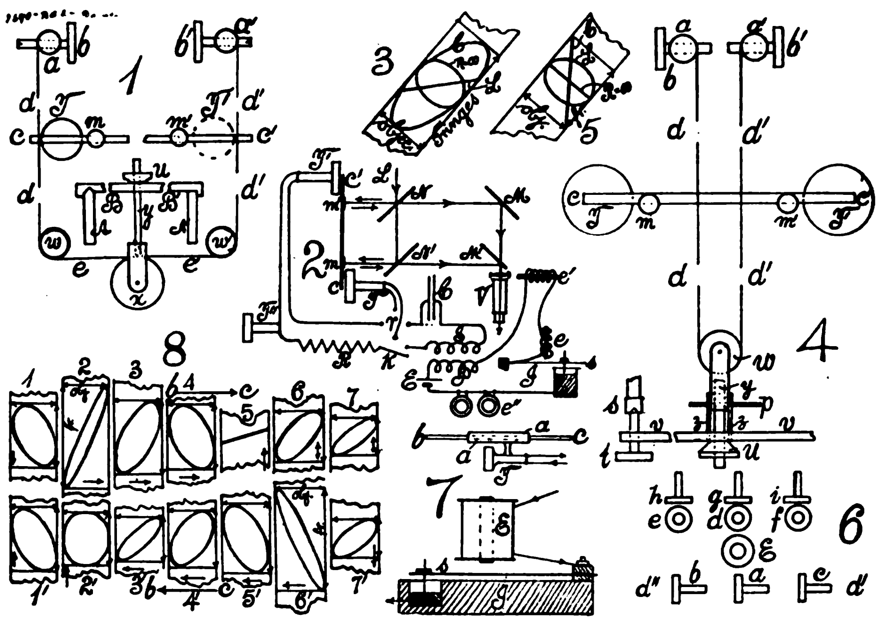
The achromatic fringes in the fine slit image of the telescope field of the interferometer must be observed with a vibration telescope, and it was found desirable to control the latter by a special electromagnet. Figure 2 is a diagram of the parts of the apparatus as a whole, $M\ M'\ N\ N'$ being the mirrors of the interferometer (M' on a normal micrometer screw), m, m' the auxiliary mirrors on the vibrator cc'; T', T, are the telephones (one provided with a switch r), V the vibration telescope, I the mercury interruptor. T'' is an auxiliary telephone for the ear.

The primary consists of the linear coil P described in the preceding paper, the storage cells E (usually four), and the two small electromagnets e, e', for controlling the interruptor and the objective of the

vibration telescope, V. The secondary S was the coil "B" of the preceding paper, wound on glass. This was in circuit with a rheostat R (up to 40000 ohms), the telephones T, T', T'', and the key K. The condenser C in the secondary is available when needed, and the inductive resistance e'' in the primary.

The whole of the parts shown in figure 1 could be slid fore and aft on the carriage BB and AA', to accommodate the interferometer.

3. *Observations with the Slit Image.*—It will be seen that if an ordinary telescope is used at V, figure 2, the sharp slit image must widen to a band here, as in the preceding apparatus; but the design is much



less sensitive because the rays are parallel throughout. The slit image nevertheless suffices admirably for finding the resonance tension. For this purpose the screw b or b', figure 1, is first manipulated till the image begins to widen. The fine adjustment is then made at u, y, x, till the maximum band-width is reached. One easily recognizes in this way three harmonics, the fundamental at lowest tension and small band-widths; the octave at larger tension with maximum band-width; and the next overtone at still larger tension and diminished band-width. Above this I did not go as the stress on the wire would have been excessive. The reason for the prominence of the octave here, is not

obvious. It does not occur in the later work. While the fundamental showed a band-width of 5 scale-parts, the same for the octave was 42 cm. wide, and for the next harmonic 26 cm. wide.

The remarks made presuppose that the telephones are acting in concert, on opposite sides of the vibrator. We may refer to this as an arrangement in series. When the telephones are acting in opposition, the band-width, s, decreases to one or a few scale-parts, depending on the symmetry of adjustment, etc., which if perfectly made should throw the differential s out entirely.

4. *Observations with the Interferometer.*—As has been stated the fringes are easily found because the rapid motion of the vibrator implies considerable damping. The slit image is thus quite stationary and the fringes clear and strong. On starting the inductor, the fringes at once vanish and after breaking the circuit slowly reappear, unless the vibration telescope is used. When the period of the latter differed from that of the induction (to be very weak), the even band of fringes changed to wave lines travelling in opposed directions and of continually increasing amplitude. Eventually the crests or troughs only are seen and these but on one side (as some micrometer adjustment for one or the other will be necessary to keep them in the field), again travelling in pairs, in opposite directions, through each other. On breaking the circuit these pulses, slowly coalesce into the wave bands and finally into the even band.

The case here presented is that of a relatively slowly vibrating telescopic objective, at V. If the frequency of the latter can be counted, the frequency of the alternator may be deduced from the number of moving crests in the field. Thus when the frequency of the objective was $n' = 5$, there were four crests in motion, implying a frequency of $n = 20$ for the interruptor of the coil. In this respect the case of different periods is advantageous.

The net double amplitude of the waves measured by the ocular micrometer was about 4 scale-parts and $i = 4 \times 10^{-6}$ amperes came to a scale-part, in case of the present small fringes.

The next step in advance consisted in adding an electromagnet, e; figure 2, at the objective of the vibration telescope V, in series with the electromagnet e of the interruptor, I. The two springs at V and I, moreover, were adjusted to about the same period. The electromagnet e' could be rotated on a vertical axis, so that by moving it closer to or further from the steel spring of V, any degree of band-widths was obtainable in the telescope. Magnificent octave fringes were obtained in this way. They moved merely on opening and closing the circuit.

5. *Decreased Bifilar Distances.*—The easy accomplishment of the above experiments, where the distance between the bifilar threads was about 30 cm. and their length 60 cm., encouraged me to reduce the distances between the threads until the system was virtually torsional. In such a case the displacement at the ends of the vibrating beam (cc', figure 4) is no longer limited to that of the bifilar wires. Figure 4 is a front elevation of the new apparatus; the steel wires dd' being at a distance of about 6 cm., their length 60 cm., the distance between the auxiliary mirrors m, m', 10 cm., and between the magnets of the telephones T, T', 16 cm. The vibrator cc' was (as above) a steel umbrella rib. The ends of the wires dd' were again wound about the stiff screws b, b' held in posts a, a' rigidly attached to the bed plate of the interferometer (not shown). The wires were stretched below by the pulley w and screw y, controlled by the nut u pressing against the rail v, also rigidly attached to the bed plate of the interferometer. Provision must be made (slotted sheath z and pin p) to prevent y from turning on its axis. A variety of braces are to be introduced to obviate synchronous vibrations of parts of the apparatus.

The adjustment for resonance is here more difficult than in the preceding case, and y must be a fine or a differential screw and the nut u work smoothly. Resonance should first be established by aid of the slit image in a telescope with fixed objective and with small resistance (200 ohms) in circuit. After this a large resistance (10,000 ohms and more) may be inserted and the work continued with the interference fringes when further adjustment is possible.

In a later design of the apparatus, the bar v, rigidly fixed at one end, was elastically controlled at the other end by the fine threaded screw t, pushing against the rigid socket s. This is an excellent fine adjustment and when used does not disturb the fringes.

6. *Observations with the New Apparatus.*—The first trials were made with the white slit image. Using the coil with a virtual electromotive force of about 0.7 volts and a small resistance (200 ohms), the resonance conditions were easily reached in the well-braced apparatus. The resistance was then increased to 10,000 ohms and the corresponding band-width found. When obtained the band-widths s, persists. The sensitiveness observed if s is 18 scale parts, was $0.7/18 \times 10^{-4} = 4 \times 10^{-6}$ amperes per scale-part (0.01 cm.) of the slit image. Per $\Delta N = 10^{-4}$ cm. of the micrometer of the interferometer, where $b\theta = \Delta N \cos i$ and $\theta = s/2f$ ($f = 23$ cm. being the focal length of the telescope, $b = 10$ cm. the breadth of the ray parallelogram, $i = 45°$), the current would be estimated as 1.3×10^{-7} ampere.

The coil No. 10, with about 90 turns giving 0.001 volts per turn was substituted for B. With the ocular micrometer I obtained 10^{-7} ampere per scale-part. This is less than what was estimated; but a shortage is here inevitable. The fringes used were fairly large, but completely under control.

Unlike the preceding case, the fringes now obtained were of the elliptic type, so that there is unison between interruptor and vibrator. The ellipses were often magnificent. There was little difficulty in measuring their breadth normal to the direction of fringes when quiet, as this is the fringe displacement. On making or breaking the circuit the ellipses oscillate in the well known way, and it may take a part of a minute or more before they subside into the bands (circuit broken). The ellipses remained in the field with resistances up to 10,000 ohms, below which the ΔN micrometer had to be adjusted to bring either axial extremity into view. Notwithstanding the feeble current, the telephone was still quite audible; so that the sensitiveness of the ear has not been exceeded. When the degree of resonance between the telephonic vibrator and the vibrating objective is exceptionally perfect, marked ellipses may appear in the absence of current, in spite of the fact that the telescope has an independent mounting. This very annoying phenomenon is hard to eliminate.

Another smaller coil with but ten turns giving 0.001 volt per turn was installed with the object of ultimately approaching a condition of silence in an audible telephone. The results were striking throughout but can not be given here.

7. *Capacity and Selfinduction in the Secondary.*—The phase differences thus far observed are attributable to the selfinduction of the secondary. It is interesting therefore to test whether the lead due to capacity can be equally well observed. The circuit, figure 2, was therefore provided with a condensor C containing up to one microfarad, in steps of tenths. An auxiliary telephone T'' was also inserted as a detector. The results were successful at once, as shown in figures 3 and 5. In figure 3, $R = \infty$ is the symmetrical ellipse, obtained on open circuit. This changed rapidly into the oblique ellipse C when 0.5 microfarads were inserted, and the latter into the bands L (with a range of 30 scale-parts) when the circuit was closed with about 3000 ohms. In another adjustment (fig. 5) of primary, $R = \infty$ gave normal bands (i.e., the fringes do not vibrate); the capacity 0.5 microfarads now gave the oblique bands C and the selfinduction ($R = 5000$ ohms) the nearly symmetrical ellipse L. Space prevents the insertion of other relevant results.

8. *Selfinduction in the Primary.*—The present experiments contain an element of uncertainty owing to the mechanical (or possibly magnetic) coupling of the vibrator cc', on the interferometer, figure 2, and the objective of the vibration telescope V. I have not thus far been able to eliminate this. Information was sought by inserting additional self-induction into the primary. The two small electromagnets, figure 2, e'', (about an inch long), which could be used either separately or in series, were available for this purpose.

The different elements of harmonic motion involved in the experiment may be analyzed as follows. The whole is fundamentally subject to the vibration period of the spring at the interruptor of the primary, which gives the impressed electromotive force

$$l = l_0 \sin \omega t \tag{1}$$

The current induced in the primary controls the objective of the vibration telescope, which thus moves with a lag α subject to

$$i = i_0 \sin (\omega t - \alpha) \tag{2}$$

and this may be modified by the resistance and inductance in the primary.

The objective is as stated either mechanically or magnetically coupled with the vibrator on the interferometer in a way yet to be ascertained; for each has an independent mounting. Hence the vibrator displacements s are subject to an equation with a lag or lead

$$s = s_0 \sin (\omega t - \beta) \tag{3}$$

in the absence of current in the secondary ($R = \infty$).

Finally the secondary, if carrying current has its own lag or lead γ depending on the R, L, C there inserted, and is thus subject to an equation

$$s = s_0 \sin (\omega t - \gamma) \tag{4}$$

where γ is essentially associated with β, as seen in the preceding paragraphs.

If we suppose the coupling implied in equation (3) to be uniform, the lag in equation (2) may be made obvious.

In these experiments the vibration figures were very large and very definite in the successions of their changes with L, however frequently repeated; but unfortunately the induction in the secondary is soon quenched.

9. *Direct Telephonic Induction.*—The influence of the oscillating magnetic field on the telephone is much more pervasive than one is apt to

suppose. The effect moreover is particularly marked if the telephone is open, i.e. with no connection between the clamps. The stray vibrating field produced by a small electromagnet (say $\frac{1}{4}$ inch iron, 2 inches long) is thus quite audible even beyond 50 cm. from the electromagnet. The degree of response depends moreover on the orientation (fig. 6) of the telephone relative to the electromagnet E. If we take the three cardinal positions of the plane of the coil or the diaphragm, the vertical positions e, d, f, and the fore and aft horizontal positions h, g, i, have their maximum response in the plane of symmetry $g\ d\ E$. The right and left horizontal positions d'', b, a, c, d', give minimum response (telephone silent) in this plane (E, a), with maxima at symmetrical positions, b and c. A convenient reversal of magnetic field is thus obtained.

Although all telephones show the phenomenon pretty well, since it is more distinct on open circuit (which implies a current oscillating from clamp to clamp) it would be well worth while to wind a telephonic bobbin provided with a capacity for the particular purpose of catching the stray magnetic field, such as is here encountered. Without proceeding to this extent, I used the telephone as a secondary as shown in figure 7, where E is the electromagnet of the interruptor I, s being the vibrating break circuit spring. The telephone depending adjustably from the sleeve a may be slid right and left or rotated into any horizontal position relative to E, and the current obtained measured by the vibrator.

In the endeavor to minimize the mechanical coupling, the telescope (separately mounted) was placed at about a meter from the vibrator. In this case the phase difference of the vibrations of fringes and objective was annulled, but the bands in the absence of current were nevertheless somewhat oblique to the direction of the vibration of the objective, showing that the fringes still vibrated.

With this exception the behavior of the telephone inductor was admirable. In passing from the positions b to c by sliding the telephone, the ellipses regularly passed through the oblique bands, indicating that these successive ellipses, even if of nearly equal size, were opposite in their phase rotation. This was the case when the secondary was closed with 5000 ohms and the inevitable inductance; also when a capacity was placed in the secondary and finally on passing from an inductance to a capacity in the secondary. The effect produced by changes of capacity of 0.5 microfarad was marked. The alternate-current effect, moreover, was still apparent when the circuit was closed with 25,000 ohms and the telephone practically silent.

The most direct criterion as to changes of phase is the rotation of ellipses as indicated in figure 8. I shall give a few examples of what is observed in the telephone displacement in question.

In the absence of current the fringe bands were nearly horizontal parallel lines. The secondary was closed with 5000 ohms and the inductance of the three telephones. From the position *b* (ellipse 1, figure 8 quiescent) the inductor telephone was quickly displaced to position *c*. The enormously eccentric, finally linear ellipse, 2, follows, which then rotates and contracts counterclockwise through the figures 3 and 4 into the sharp bands (usually but not always) no. 5. These duplications then separate on further rotation into the final quiescent form, 7. The arrows indicate the drift of one of the four points of tangency. On returning from *c*, by quickly sliding the telephone inductor into the position *b*, the figures roll clockwise from 7′ to 1′. Number 7′ passes at once through the highly eccentric ellipse 6′, though in other slower adjustments intermediate sharp duplicates like 5 may be detected between 6′ and 7′. The stretched ellipses, which follow immediately after the change of aspect of the telephone bobbin to the magnetic lines, are noteworthy. They result from the sudden reversal of the magnetic field in spite of the vibration. Ellipses cross over or change sign of rotation at 2 and 6′, but not near 3′ or between 5, the latter being oscillations. The corresponding cases for capacity, were similar on the whole, though less pronounced. Moreover the first and final forms were not quite in opposed phases.

10. Narrow bifilar.—After obtaining the favorable results just described, it seemed obvious that the sensitiveness could be further increased by diminishing the distance between the bifilar wires. Accordingly, with the same inductor, figure 4, the above wires (diameter 0.023 cm.) were adjusted at but 1.5 cm. apart by decreasing the diameter of the lower pulley. A few other modifications were added. The results however were disappointing throughout.

A final observation may be added. The auxiliary audible telephone responded with about equal loudness when the telephone circuit was closed with 400 ohms and when a condenser of 1 microfarad capacity was inserted. But the vibrator reacted in the former case (resistance and selfinduction) with a deflection of 23 scale-parts, whereas in the latter (capacity) the response was at most 2 scale-parts; and this required a slightly different tension of wire. The metallically closed circuit therefore affected the vibrator at least 12 times more strongly than the oscillation due to the capacity of 1 microfarad. Small capaci-

ties like 0.1 microfarad fail to influence the vibrator though to the ear the sound is quite loud. The capacity should have to be increased to 10 or 20 microfarads for equal effects on telephone and vibrator. In another experiment the closed circuit gave 20 scale-parts. The insertion of 4 microfarads decreased this to 4 scale-parts, which is again a demand of about 20 microfarads for an equality of behavior. On the other hand, while the telephone responds for a phenomenally small capacity, it soon ceases to increase in loudness (for 1, 2, 3, 4, mf., or resistances), whereas the deflections of the vibrator increase regularly.

[1] Advance note from a Report to the Carnegie Institution of Washington, D. C.

[2] These PROCEEDINGS, 5, 211–217, (1919).

[3] These PROCEEDINGS, 4, 328–333, (1918). Carneg. Publ. No. 249, 3, 1919, chap. v.

ON THE PRESSURE VARIATION OF SPECIFIC HEAT OF LIQUIDS

BY C. BARUS

DEPARTMENT OF PHYSICS, BROWN UNIVERSITY

Communicated June 13, 1919

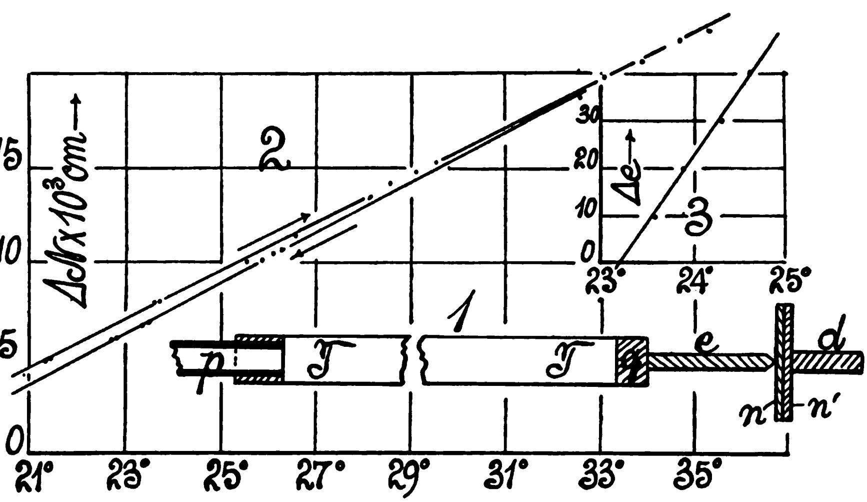
1. *Introductory.*—The measurement of the specific heat of a liquid in its relation to pressure is surrounded by so many difficulties, that any method which gives a fair promise of success deserves to be carefully scrutinized. During the course of my recent work on interferometry, I have had this in view, and the plan which the present paper proposes is particularly interesting as it seems to be quite selfcontained.

2. *Equations.*—If θ, p, ρ, c, denote the absolute temperature, the pressure, the density, and the specific heat at constant pressure, respectively, of the liquid, and if $\alpha' = (dv/v)/d\theta$ is its coefficient of volume expansion, the relation of these quantities may be expressed by the well known thermodynamic equation

$$d\theta/dp = \alpha'\theta/J\rho c \tag{1}$$

where J is the mechanical equivalent of heat, and the transformation is along an adiabatic. The main difficulty involved would therefore be the measurement of the temperature increment; for dp could be read off on a Bourdon gauge after a partial stroke of the lever of my screw compressor, with facility. It is my purpose to find $d\theta$ by the displacement interferometer. To fix the ideas; suppose the liquid in

question is introduced into a long steel tube TT, figure 1, and that the tubulure p conveys the increments of pressure dp. The end p is rigidly fixed. The other end q of the tube is free to move. By aid of the stylus, e, the elongation is registered on the plate n of a contact lever read by interferometry, the lever being identical in construction with the apparatus described in my paper on magnetic elongation.[1] Thus the interferometer will indicate the elongations due both to the pressure increment and to the corresponding temperature increment of the suddenly compressed liquid, and it becomes a question to what degree the two may be adequately separated. If Δl, Δp, $\Delta\theta$ are corresponding increments of the length, l, of the tube and the



pressure and temperature of its liquid content, we may write successively, if $\Delta l = \Delta l' + \Delta l''$,

$$\Delta l'/l = (r_1^2/3k(r_2^2 - r_1^2))\,\Delta p = \beta\Delta p, \text{ say,} \tag{2}$$

$$\Delta l''/l = \alpha\Delta\theta \tag{3}$$

where α is the coefficient of expansion, k the bulk modulus, r_1 and r_2 the internal and external radius of the steel tube of length l. Hence

$$\alpha\Delta\theta = \Delta l/l - \beta\Delta p \tag{4}$$

and equation (1) becomes

$$d\theta/dp = \frac{\Delta l/(l\Delta p) - \beta}{\alpha} = \frac{\alpha'\theta}{J\rho c} \tag{5}$$

or

$$c = (\alpha'\theta/J\rho)\,(\alpha/(\Delta l/(l\Delta p) - \beta) \tag{6}$$

Hence c may be found from observations of Δl and Δp, provided α' and ρ, α and β are sufficiently known.

3. Measurement of the Pressure Coefficient β.—For this purpose the tube TT, figure 1, is placed in a water jacket of constant temperature, and β found by internal pressure, directly. Experiments of this kind were contributed with some detail in an earlier paper.[2] The method then used consisted in finding β from the displacement of the spectrum ellipses under known conditions; but the present method of the contact lever and achromatic fringes may be considered preferable, particularly if the tube contains water, for which the thermal discrepancy is small.

Moreover, since $\Delta\theta$ is primarily aimed at, β should be made as small as possible. This may be done by selecting relatively thick walled tubes of small external diameter. A few data are here desirable. Using an ocular micrometer plate 1 cm. long with scale parts of 0.01 cm. each and fringes of moderate size (one or two scale parts in width) we may write as in the preceding paper (l. c.)

$$\Delta l/l = 3 \times 10^{-7}\,\Delta e \tag{7}$$

where Δe is the displacement of the achromatic fringe on the ocular scale corresponding to the elongation $\Delta l/l$.

Hence for steel tubes ($k = 1.8 \times 10^{12}$) the following data apply

TUBES	$2r_1$	$2r_2$	$\beta \times 10^7$	$\Delta e'/\Delta p$
	cm.	cm.		
I	1.0	0.9	8.0	2.7
II	1.0	0.8	3.3	1.1
III	0.7	0.6	5.2	1.7
IV	0.7	0.5	1.9	0.63

β being reckoned per atmosphere.

4. Measurement of α.—For this purpose the tube TT is to be clean and empty, the nozzle p removed and a long-stemmed thermometer passed from end to end of the tube, through the end p. Externally the tube is surrounded by a coil of wire for electric heating and appropriately jacketed. Measurements made in this way with a brass tube are given in figures 2 and 3 and they would have been quite satisfactory if the tube had been properly protected from loss by radiation. (ΔN is read off on the displacement micrometer at an interferometer mirror; Δe in the ocular scale).

If for the steel tube, $\alpha = 10^{-6} \times 12$, equations (3) and (6) then give us

$$\Delta\theta = 0.025\,\Delta e$$

or about 40 scale parts of the ocular micrometer per degree of temperature.

5. Liquids.—It remains to select suitable liquids for experiment. For water at 27°, $d\theta/dp = 0.0019$; for ethylic alcohol at about 18°, $d\theta/dp = 0.017$; for ether at 18°, $d\theta/dp = 0.028$, pressures being measured in atmospheres. Thus for the four tubes specified in §3, the respective displacements in the ocular micrometer would be (per atmosphere)

I	$\Delta e' = 2.7$	water:	$\Delta e'' = 0.08$
II	1.1	alcohol	$\Delta e'' = 0.68$
III	1.7	ether	$\Delta e'' = 1.14$
IV	0.63		

The case of water may be dismissed for here the thermal displacement per atmosphere, $\Delta e''$, is a small fraction of the elastic displacement in the ocular micrometer. But alcohol and ether show satisfactory conditions. Thus a sudden half turn of the lever of the screw compressor producing 100 atmospheres would displace the fringes, in case of tube III and ether, 173 scale parts elastically and 114 scale parts thermally, together 287 scale parts. Stops of 30 atmospheres would be advisable. Tube IV with 63 and 114 scale parts respectively is even more advantageous.

It remains to estimate the diminution of $\Delta\theta$ owing to the partition of heat between the liquid and the tube. If $\Delta\theta'$ is the increment of the combined system of liquid and tube the ratio will be

$$x = \frac{\Delta\theta'}{\Delta\theta} = 1/\left(1 + \frac{r_2^2 - r_1^2}{r_1^2}\frac{c'\rho'}{c\rho}\right)$$

if c' and ρ' are the specific heat and density of the solid. These ratios x for the tubes and liquids in §3 and the corrected $\Delta e''$ are easily tabulated. Tubes of the type I are unsatisfactory. In case of tubes of the types II or III the thermal displacement would be but about 5% of the elastic displacement in case of water; but in case of alcohol 25 to 35%, and of ether 35 to 55%. In tubes of the type IV the advantages of thicker walls and small external diameter have again decreased. The problem of selecting the best tube admits of general solution.

If we combine equations (4) and (8) and put

$$A = \Delta l/l; \quad B = \Delta p/3k; \quad C = \rho'c'/\rho c; \quad y = r_2^2 - r_1^2/r_1^2$$

the result is

$$\alpha\Delta\theta' = \frac{A - B/y}{1 + Cy} \tag{9}$$

Here y is the ratio of solid and liquid sections and we inquire what value of y will make $\Delta\theta'$ a maximum provided A, B, C are constant. If the thermal and elastic elongations are to be equal $A = 2B$. Differentiating (9) and reducing:

$$1/y = C(\sqrt{1 + A/BC} - 1) \tag{10}$$

and since y must be positive the radical is positive. Now if $A = 2B$, for example, the ratio of diameters $2r_1$ to $2r_2$ would in all cases have to exceed 0.65. If $A = 3B$, the case of water remains nearly the same, but for ether and alcohol the diameter ratio approaches 0.9.

[1] These PROCEEDINGS, 5, 1919, (267–272).
[2] Carnegie Publ. No. 249, 1917, pp. 84–94.

STUDIES OF MAGNITUDES IN STAR CLUSTERS, IX. THE DISTANCES AND DISTRIBUTION OF SEVENTY OPEN CLUSTERS

By HARLOW SHAPLEY

MOUNT WILSON OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Communicated by G. E. Hale, June 14, 1919

The question of whether globular clusters are really or only apparently absent from the mid-galactic segment makes the study of the distances of open clusters particularly important. These objects are relatively near to the galactic circle, and many appear to be at such great distances along the plane as to support the hypothesis that obstructing matter is insufficient to occlude globular clusters in mid-galactic regions.[1] On the other hand there is evidence that globular clusters actually may not be absent from low galactic latitude,[2] and the following discussion of open clusters and other relevant factors suggests that the question must be considered an unsettled one for the time being.

Although the question of the reality of the region of avoidance affects but little the general conclusions reached in earlier papers regarding globular clusters, spiral nebulae, and the Galaxy, two modifications should be made to previously suggested interpretations,[3] in case we demonstrate the existence of much obstructing matter along the galactic

plane beyond the confines of the local cluster: First, the distance of the center of the system and the total extent of the Galaxy may be considerably greater than inferred from the visible globular clusters; second, the transition from globular to open clusters, if there be such an evolution,[4] is not necessarily rapid and inevitable when the globular clusters enter thickly populated galactic regions.[5] It should be noted that the diameter-parallax correlations[6] are some insurance that a hypothetical *partial* obstruction has exaggerated neither the distances of globular clusters nor the galactic dimensions. If dark clouds obscure distant globular clusters they are remarkably thorough (except in one or two possible cases[7]), the obscuration at any point appearing to be absolute or non-existent. According to the magnitude results now available for faint galactic stars, the obstructing clouds also are of such character that they do not affect star colors appreciably.

Several years ago the correlation of the apparent diameter to the brightness of stars in open clusters was pointed out and the possibility noted of obtaining relative parallaxes from measures of magnitude or diameter.[8] My values of the relative parallaxes of these open groups, however, as well as the provisional absolute values, have remained unpublished in the hope that the accumulation of data on magnitudes and spectra would permit the determination of more definite absolute parallaxes. The open clusters contain few, if any, Cepheid variables, and appear almost without exception to be quite beyond the reach of direct measures of distance. Parallaxes of an accuracy comparable with that for globular clusters seem unattainable; but, notwithstanding some uncertainty in diameters due to looseness of structure and to intermixture with the Milky Way stars among which the open clusters usually lie, it appears quite possible to use their angular dimensions to determine a system of relative distances and to use the apparent magnitudes of the red giants or the B-type stars in as many clusters as possible to establish the scale and zero point for absolute distances.

My observational work on open clusters comprises: (1) Thirty spectrograms of some 200 faint stars in various northern groups, made with a slitless spectrograph of small dispersion at the 80-foot focus of the 60-inch reflector; (2) more than a hundred direct photographs at the primary focus of the 60-inch; (3) the determination of magnitudes and colors of about two thousand stars; (4) the measurement of the diameters and form of all known open clusters on Franklin-Adams charts, Harvard photographs, or Mount Wilson plates. This work has been supplemented by similar data from other sources, chiefly from the

observations at Harvard on the spectra in a few bright clusters[9] and from the work of Adams and van Maanen on the double cluster in Perseus.[10]

The accompanying table contains the parallaxes and space coördinates of the 70 open groups which are sufficiently rich, condensed, symmetrical, and distinct from the background to make practicable the use of diameter as a criterion of distance. Melotte's well-known catalogue[11] of clusters contains more than twice as many open groups as are listed here, but a large number of his clusters are so ill-defined and poor in numbers that they appear to be little else than chance groupings of Milky Way stars. The omission of these irregular and scattered aggregations does not operate selectively in matters of distance or distribution.

The first column of the table contains the number of the cluster in Dreyer's *New General Catalogue* or its indices. A few that are not listed by Dreyer retain their numbers in Melotte's catalogue. Parallaxes of the fourteen clusters marked with an asterisk have been derived directly from measured apparent magnitudes combined with absolute magnitudes estimated on the basis of observed colors, or spectra, or both. The adopted mean absolute magnitudes of early type stars depend on the studies of Kapteyn,[12] Plummer,[13] and Charlier,[14] and on my own results for the luminosities of blue stars in globular clusters. Apparently without exception the brightest stars in open clusters (as in globular clusters) are giants in luminosity.

By using the measured diameters and adopted distances of these specially studied clusters to determine a parallax-diameter curve, the parallaxes of the other open clusters have been estimated on the basis of diameters alone; in most cases, however, the relative distances have been roughly checked with the aid of the magnitudes of the brightest stars as estimated by Bailey[15] or as derived from Mount Wilson photographs. The diameters are the means of measures by Bailey, Melotte, Shapley, and Miss Davis. In keeping with the probable accuracy, most of the parallaxes have been rounded off to a single significant figure after computing the linear galactic coördinates, which are given in the fourth and fifth columns. The very largest parallaxes are uncertain because of the lack of precision in the corresponding end of the parallax-diameter curve; the very smallest are uncertain because of the large effect of small errors in measurement of diameter; most of the diameters, however, fall within limits for which the curve is well-defined and for which the measures of diameter by the four observers are

in good agreement. It is expected that the measures and computations underlying this work and a more complete discussion will appear in a *Mount Wilson Contribution.*

POSITIONS IN SPACE OF SEVENTY OPEN CLUSTERS

N. G. C.	GALACTIC LONGITUDE	PARALLAX	DISTANCE (UNIT 100 PARSECS		N. G. C.	GALACTIC LONGITUDE	PARALLAX	DISTANCE (UNIT 100 PARSECS	
			Along plane	From plane				Along plane	From plane
457	93°	0.00016	62	− 5.3	2682*	183°	0.00025	34	+22.0
663	97	0.0002	50	− 1.2	I. C. 2488	245	0.0002:	43	− 2.7
869*	102	0.0008	12	− 0.9	3114	250	0.001:	10	− 0.5
884*	103	0.0008	12	− 0.8	3532†	257	0.002:	4	+ 0.1
1039	111	0.0006	17	− 5.0	I. C. 2714	259	0.0002	59	− 1.2
1245	114	0.0002	44	− 7.1	Mel 105	260	0.00007	140	− 4.7
Pleiades*	134	0.015	0.6	− 0.3	Mel 108	262	0.0001	90	+10.5
1528	119	0.0004	24	0.0	4349	267	0.00025	40	+ 1.0
1807	154	0.0002	42	− 9.7	5281	277	0.00007	140	− 0.4
1912*	139	0.0003	32	+ 0.6	5617	282	0.0002	50	+ 0.2
1960*	142	0.0002	48	+ 1.1	5823	289	0.00015	67	+ 3.3
2099*	145	0.0004	26	+ 1.5	5999	293	0.0001	100	− 3.5
2168*	154	0.0006:	18	+ 0.8	6005	293	0.00006	160	− 9.2
2266	154	0.00009	110	+20.0	6067	297	0.0002	45	− 1.7
2287	199	0.0006:	16	− 2.6	6124	308	0.0005	21	+ 2.2
2323*	189	0.0002	50	− 0.4	6134	302	0.00014	71	− 0.1
2324	181	0.0001	91	+ 6.4	6192	308	0.0001	83	+ 3.2
2355	170	0.0001	98	+22.0	6222	309	0.00007	140	− 0.4
2360	197	0.00014	71	− 0.8	6242	313	0.00014	71	+ 2.8
2421	204	0.0001	77	+ 1.2	6259	310	0.0002	53	− 1.5
2420	165	0.0001	94	+34.0	I. C. 4651	308	0.0002	50	− 7.0
2423	199	0.0003	34	+ 2.7	6405*	324	0.001:	8	− 0.1
Mel 71	197	0.0001	77	+ 7.8	6451	327	0.00009	110	− 3.9
2439	214	0.0001	91	− 5.7	6475*	323	0.003:	4	− 0.3
2437	200	0.0005	21	+ 1.8	6494	338	0.0005	21	+ 0.8
2447	208	0.0002	43	+ 1.0	6520	330	0.00009	110	− 6.2
2477	221	0.0005	20	− 1.8	6603	340	0.00008	120	− 3.8
2489	214	0.0001	100	+ 0.3	Mel 204	341	0.0006:	16	− 1.5
2506	198	0.0001	98	+19.0	6645	343	0.0002	56	− 4.2
2516	241	0.002:	6	− 1.5	6705*	355	0.00014	71	− 4.2
2539	201	0.0003	31	+ 6.8	6709	10	0.0002	56	+ 3.8
2548	195	0.0007:	14	+ 4.2	6834	33	0.00008	120	+ 0.9
2567	217	0.0001	83	+ 5.5	6838*	24	0.00007	140	−14.0
2627	219	0.0001	90	+12.0	7654	80	0.00016	62	− 0.2
2632*	172	0.003:	25	+ 1.6	7789	83	0.0003	37	− 3.7

† The bright southern cluster N.G.C. 3532 is conspicuously elliptical.

The well-known concentration of open clusters in the Milky Way is shown by the smallness of the tabular distances from the plane. This condition allows a good representation of the distribution in space

simply through plotting (as in figure 1) the galactic longitude and the distance projected on the galactic plane. No striking lack of symmetry appears in this diagram, except the almost total absence of bright open clusters in the first 90° of galactic longitude.

The mean distance of the 70 open clusters along the plane is 5900 parsecs, all individual values (except for the Pleiades) lying between 400 and 16,000 parsecs. By taking the dip of the central line of the Milky Way[16] as 1°, and the distance of the sun above the plane as 60 parsecs,[17] the distance of the stars and star clouds that enter Newcomb's

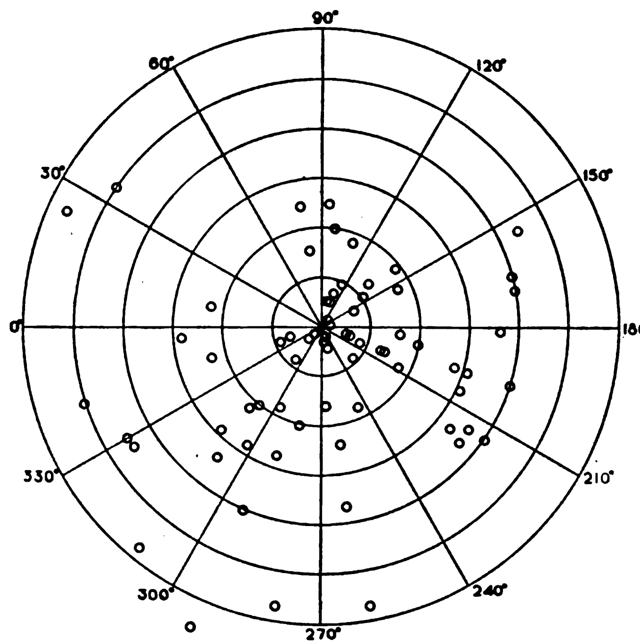


FIG. 1. DISTRIBUTION OF OPEN CLUSTERS IN THE GALACTIC PLANE
The direction angles are galactic longitudes; the annuli are 25,000 parsecs in width

(visual) determination of the position of the galactic circle is of the order of $60/\sin 1° = 3500$ parsecs. Although the uncertainty of this value is large, it seems reasonable to infer that the open clusters are intermingled with the non-cluster stars of the galactic stratum.

The diagram in figure 2 illustrates, for all the open clusters, globular clusters, and Cepheid variables falling between galactic longitudes 290° and 360°, the distances projected on the galactic plane plotted as abscissae against the distances from the plane as ordinates. This region of the sky, containing the great Milky Way clouds of Ophiuchus,

Sagittarius, and Scorpio, is symmetrical about the point that is indicated by the distribution of globular clusters as the center of the galactic system. It is mainly the absence of globular clusters from low galactic latitudes throughout this interval of 70° in longitude that gives rise to the phenomenon of a region of avoidance. The diagram shows that the distribution of stellar material is probably fairly continuous along the galactic plane; from the local cluster the Cepheid variables (and various other types of highly luminous galactic stars) extend to the nearer star clouds and open clusters, and the latter are recorded among the more distant star clouds along the plane nearly as far as the center of the system of globular clusters.

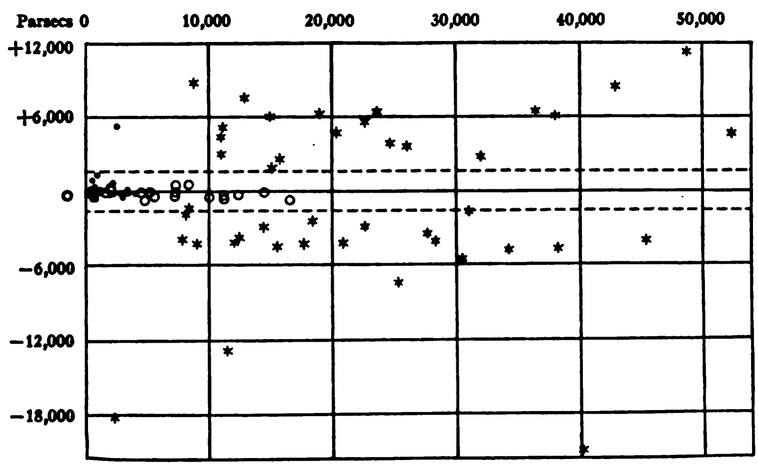


FIG. 2. DISTRIBUTION OF THE GLOBULAR CLUSTERS (ASTERISKS), OPEN CLUSTERS (OPEN CIRCLES), AND CEPHEID VARIABLES (DOTS) THAT FALL BETWEEN GALACTIC LONGITUDE 290° AND 360°

Ordinates are distances from the galactic plane; abscissae are distances along the plane

If, as appears very probable, the system of globular clusters outlines the galactic system, why do we not find large numbers of open clusters in the vicinity of and beyond the center, between the two halves of the assemblage of globular clusters? Nearer the sun there are some 70 open groups within the mid-galactic segment—the segment which appears to be their natural and only domain. Is the distant central region that is devoid of globular clusters also in part avoided either actually or apparently by open clusters? The observed scarcity of open clusters in this direction leads us to question the reality of the avoidance; it may be that patches of obscuring material conceal both open and globular clusters, as well as many of the more distant stars,

perhaps thus playing a large part in the conspicuous rifts in the Milky Way.

Barnard's dark markings, recently catalogued,[18] do not furnish direct evidence of this obscuration, for, singularly enough, more than half of his objects fall outside the region of avoidance, if we exclude one small region near Messier 11 for which 30 separate positions are listed. It appears that most of the markings may be affiliated with the local cluster, and at no great distance from the sun. Thus in the Taurus-Orion region, two-thirds of the dark markings have negative galactic latitudes, lying on the average more than 10° south of the galactic circle; in the opposite region the latitudes are largely positive, the dark markings in Ophiuchus and Scorpio lying intermingled with the seemingly unaffected globular clusters. Along the middle line of the region of avoidance relatively few markings are recorded.

Indirectly, however, in Barnard's nebulae we have an argument favoring the hypothesis that globular clusters are concealed in mid-galactic regions, for, if a considerable amount of obscuration is associated with the relatively small local cluster, it suggests that such material may also be common in other stellar regions. Although the star counts in typical open and globular clusters fail to reveal as yet the presence of such obscuration, the distribution of stars in the Magellanic Clouds suggests the possibility of its presence there.

Another point of considerable weight against a real absence of globular clusters from the region of avoidance is the difficulty and improbability of such a dynamical condition. The distribution of globular clusters in space shows their very close relationship to the Galaxy; the average velocity and probable mass both appear to be very great; the possibility, therefore, of repelling a globular cluster from the stellar stratum, or completely transforming it during a single passage, seems remote. From a gravitational standpoint we should naturally expect the frequency of clusters to be greatest at or near the galactic plane, and that many oscillations must occur before the hypothetical assimilation and transformation is completed for an average globular cluster.

Of the several arguments favoring the reality of the empty zone, we recall that the most important are the completeness of the observed absence at all distances from the sun, and the various suggestions of immediate genetic relationship between the external globular clusters and the open clusters within, including the observation that the globular clusters nearest the plane appear to be the most open.

[1] Shapley, Harlow, *Mt. Wilson Contr.*, No. 152, 1917 (1–28), pp. 22, 23 and footnotes; No. 157, 1918 (1–26), p. 10; No. 161, 1918 (1–35), sections I, II, and III.

[2] *Ibid.*, No. 157, 1918 (1–26), p. 12; No. 161, 1918 (1–35), section IV.

[3] *Ibid.*, No. 157, 1918 (1–26); No. 161, 1918 (1–35), sections VII and VIII.

[4] *Ibid.*, No. 161, 1918 (1–35), section VIII; No. 157, 1918 (1–26), pp. 12–14.

[5] But whether the absence of clusters is real or only apparent, we must remember that the assignment of a definite thickness of three or four thousand parsecs to the galactic segment is mostly a matter of convenience and approximation; the intention is merely to suggest that practically every known object except spiral nebulae and globular clusters is within a thousand parsecs or so of the central plane of a greatly extended, indefinitely bounded stellar stratum.

[6] Shapley, Harlow, *Mt. Wilson Contr.*, No. 152, 1917 (1–28), fig. 1; No. 161, 1918 (1–35), section V.

[7] *Ibid.*, No. 152, 1917 (1–28), p. 22, footnote 2.

[8] *Ibid.*, No. 115, 1915 (1–21), p. 11 and fig. 1.

[9] Pickering, E. C., *Ann. Obs. Harvard Coll., Cambridge, Mass.*, **26**, 1891 (260–286).

[10] Adams, W. S., and van Maanen, A., *Astr. J. Albany, N. Y.*, **27**, 1913, p. 187.

[11] Melotte, P. J., *Mem. R. Astr. Soc., London*, **60**, 1915 (175–186).

[12] Kapteyn, J. C., *Mt. Wilson Contr.*, Nos. 82 and 147, *Astrophys. J., Chicago, Ill.*, **40**, 1914 (43–126), **47**, 1918 (104–133, 146–178, 255–282).

[13] Plummer, H. C., *Mon. Not. R. Astr. Soc., London*, 73, 1913 (174–191).

[14] Charlier, C. V. L., *Meddelanden Lunds Astr. Obs., Lund*, Series 2, No. 14, 1916 (1–108).

[15] Bailey, S. I., *Ann. Obs. Harvard Coll., Cambridge, Mass.*, **60**, 1908, No. VIII.

[16] Newcomb, S., *Pub. Carnegie Inst., Washington, D. C.*, No. 10, 1904 (1–32).

[17] Shapley, Harlow, *Mt. Wilson Contr.*, No. 157, 1918 (1–26), p. 23.

[18] Barnard, E. E., *Astrophys. J., Chicago, Ill.*, **49**, 1919 (1–23).

A COMPARISON OF CERTAIN ELECTRICAL PROPERTIES OF ORDINARY AND URANIUM LEAD

By P. W. Bridgman

Jefferson Physical Laboratory, Harvard University

Communicated, June 17, 1919

A comparison of the physical properties of chemical isotopes is of significance because of the light it may throw on the corresponding mechanisms. Comparisons of the properties of ordinary and uranium lead have hitherto been made with respect to the atomic volume,[1] thermoelectric quality,[2] and emission spectra.[3] No differences have been detected, except possibly a very slight shift in one of the spectrum lines. It is not to be expected that large differences exist with regard to other physical properties, but nevertheless a verification by direct experiment is not without interest.

Through the kindness of Prof. T. W. Richards there was made available for me 20 grams of lead of radio-active origin on which he has already published chemical data,[4] and also a similar quantity of puri-

fied ordinary lead. The radio-active lead was from Australian carnotite, and showed an atomic weight of 206.34, which is therefore 0.41% lower than that of ordinary lead. The theoretical and experimental value for the atomic weight of the pure end product of the disintegration of uranium is 206.08, so that this sample was probably composed of 76% pure isotope and 24% ordinary lead. The radio-active lead contained not over 5 parts in 100,000 of impurity, mostly silver, and the ordinary lead was a trifle less pure, showing also a trace of copper. For the experiments both these samples were formed into wire 0.035 cm. in diameter by cold extrusion through a steel die. The samples were cast into ingots ready for extrusion by Professor Richards, who melted them in hydrogen and then continued the fusion for ten minutes in vacuum.

The measurements recorded here are comparisons of the pressure coefficient of electrical resistance, temperature coefficient of resistance, and specific resistance. The comparison of pressure coefficient of electrical resistance was made with more accuracy than the other measurements because a specially adapted apparatus designed for another purpose was available.

In order to compare the pressure coefficients, approximately equal lengths of the two varieties of lead were wound non-inductively on either end of a bone core, which was placed in the pressure chamber. The two terminals of each wire were soldered to independent leads which were brought through the walls of the pressure chamber through an insulating plug of a design essentially like that previously described,[5] except that there were three, instead of one, insulated stems through the plug. The two wires were made the two extension coils of a Carey Foster bridge. In this way the difference of the pressure effects on the two coils could be measured. The absolute value of the pressure effect on ordinary lead had been previously determined with sufficient accuracy.[6] The apparatus for producing pressure was the same as that previously described.[5]

Readings were made to 12,000 kgm./cm.^{2} at 1000 kgm. intervals, with increasing and decreasing pressure, and at two temperatures, 25° and 85°. An independent set of readings was made to determine the effect of pressure on the resistance of the leads, which turned out to be almost negligible. At 25° the decreases of resistance of the two kinds of lead under 12,000 kgm. were the same within 0.02% of the total decrease, and at 85° within 0.03%. Assuming that the possible error in reading the slider settings of the Carey Foster bridge was 0.1 mm. and that the errors conspired in the most unfavorable way, the possible

instrumental error in the readings would have been 0.04%. The agreement is therefore as close as could be expected. At pressures lower than 12,000 the agreement of the readings was also always within possible instrumental errors.

The temperature coefficients were compared by comparing the resistances at atmospheric pressure at 25° and 85°, while still wound on the same bone core as was used for the pressure measurements. Over this range the temperature coefficients differed by 0.06% of themselves. This is slightly greater than the possible instrumental error, but is less than possible discrepancies due to differences of handling which I have previously found in different lengths from the same sample of ordinary lead.

The specific resistances were compared by comparing the actual resistances of the samples already measured. Correction for inequalities of dimensions was made by measuring the length of each sample and its weight, and from these computing the average cross section assuming that the densities were directly proportional to the atomic weights.[1] In order to measure the length, the wire had to be cut from the core, unwound, and straightened. Because of the mechanical softness of lead, this is a difficult operation without error, so that the comparison of specific resistance is less accurate than of pressure or temperature coefficient. Two independent comparisons of specific resistance were made. The discrepancies of these two comparisons differed in sign; the average of the two comparisons showed an agreement of specific resistance of 0.06%.

Conclusion.—These measurements establish that any difference between the pressure coefficient of resistance, temperature coefficient and specific resistance is at least many fold less than the difference of the atomic weights. It seems indicated with a high degree of probability that the same conclusion will be found also to apply to the compressibility and thermal expansion. The results fortify the point of view embodied in recent theories of electrical resistance that the processes involved in electrical conduction take place in the outer part of the atomic structure.

[1] Richards, T. W., and Wadsworth, C., 3rd, *J. Amer. Chem. Soc., Easton, Pa.*, **38**, 1916, (221–227 and 1658–1660).

[2] Richards, T. W., *Year Book, Carnegie Inst., Washington*, **16**, 1917, (299–300).

[3] Aronberg, L., *Astroph. J., Chicago*, **47**, 1918, (96–101).

[4] Richards, T. W., and Wadsworth, C., 3rd, *J. Amer. Chem. Soc.*, **38**, 1916, (2613–2622).

[5] Bridgman, P. W., *Proc. Amer. Acad., Boston*, **49**, 1914, (627–643).

[6] Bridgman, P. W., *Ibid.*, **52**, 1917, (573–646).

NATIONAL RESEARCH COUNCIL

EXTRACTS FROM MINUTES OF MEETINGS OF THE EXECUTIVE BOARD, JOINT MEETING OF THE EXECUTIVE BOARD OF THE NATIONAL RESEARCH COUNCIL WITH THE COUNCIL OF THE NATIONAL ACADEMY OF SCIENCES

IN THE UNITED STATES NATIONAL MUSEUM

APRIL 30, 1919, AT 4.55 P.M.

Present: Messrs. Bancroft, Clevenger, Cross, Dunn, Hale,* Leuschner, McClung, Manning, Merriam, Millikan, Noyes,* Ransoms,* Stratton, Walcott,* Woodward, Woods.

Mr. Walcott in the chair during the transaction of the business of the Council of the Academy.

Mr. Merriam in the chair during the transaction of the business of the National Research Council.

The minutes of the meetings of the Interim Committee held April 22, and April 28, 1919, are included in this record.

Mr. Cross, Chairman of the Committee on Organization of Administration, presented a report of progress, recommending that there should be adopted a classification of the entire administrative force of the Council, both scientific and clerical, to serve as a basis for determining responsibility, the method of supervision, and the salary scale, and asked for authorization in behalf of the Committee to take the necessary steps for the establishment of an equitable bonus system. The proposed classification was approved and it was

Moved: That the Interim Committee approve the general plan of a standard salary scale for non-scientific employees of the Research Council with bonuses to meet the increased cost of living and authorize the Committee on Organization of Administration to work out an equitable basis for such bonuses. (*Adopted.*)

At his own request, Mr. Cross was relieved of the chairmanship of the Committee on Organization of Administration, and Mr. Leuschner, Acting Secretary of the Council, was appointed to succeed him as chairman of the Committee.

On recommendation of Mr. Hale, Chairman of the Division of Foreign Relations, the following tentative organization of the Executive Committee of the Division was authorized: Foreign Secretary of the National Academy of Sciences, Chairman; Chairman of National Research Council, Chairman of the Research Information Service, Dr. H. A. Bumstead.

Mr. Washburn, Acting Chairman of the Division of Chemistry and Chemical Technology, requested information regarding the plans of the Government

* Members of the Council of the National Academy of Sciences.

with reference to the publication of results of scientific investigations bearing on the war and regarding the Research Council's connection with any plans the Government might have in this respect.

Moved: That the Research Information Service secure information on the plans of the Government for the publication of results of scientific investigations bearing on the war. (*Adopted.*)

Mr. Clevenger, Acting Chairman of the Division of Engineering, reported the following organization of the committee authorized April 8, to study the relation of Neumann bands in iron and steel to the rate of rupture caused by explosions or other stresses: Professor Charles E. Munroe, Chairman; Mr. S. B. Howell, Mr. F. B. Foley, Professor Albert Sauveur, Professor William D. Campbell, Lieut.-Colonel C. G. Storm, Lieut.-Commander O. M. Hustevedt.

(*Approved.*)

On behalf of the Division of Chemistry and Chemical Technology, Mr. Washburn, Acting Chairman, presented a report on the Publication of Compendia of Chemical Literature, etc. This report was originally prepared by a Committee of the American Chemical Society and later considered and concurred in by the Division.

Moved: That copies of the Report on Publication of Compendia of Chemical Literature, etc., be sent to the members of the Executive Board prior to the next meeting, and that a committee consisting of two members of the Division of Chemistry and Chemical Technology, two members of the Division of Physical Sciences, one member from the Bureau of Standards, and one member from the Smithsonian Institution, be appointed to report on the proposed project at the next meeting of the Executive Board. (*Adopted.*)

The Committee was constituted as follows: Representing the Division of Chemistry and Chemical Technology, Messrs. W. D. Bancroft and E. W. Washburn; representing the Division of Physical Sciences, Messrs. H. A. Bumstead and R. A. Millikan; representing the Bureau of Standards, Mr. S. W. Stratton; representing the Smithsonian Institution, Mr. F. E. Fowle.

Mr. Merriam presented for later consideration the following recommendation of the Committee on Psychology of the Council:

It is recommended by the Committee on Psychology of the National Research Council that the Division of Anthropology and Psychology be constituted of equal numbers of psychologists and anthropologists. It is further recommended that the number for each science be nine. Of the nine members representing psychology the American Psychological Association would nominate six (two each succeeding year), and the National Research Council the remainder (one each year).

Mr. Clevenger, Acting Chairman of the Division of Engineering, reported the election by the Executive Committee of Mr. C. F. Rand as Chairman of the Finance Committee of the Division, vice William R. Walker who is unable to serve.

Mr. Clevenger brought up the question of furnishing the New York Office of the Division of Engineering. After discussion

Moved: That the Division of Engineering be authorized to purchase furnishings for its New York office in the United Engineering Building at 29 West 39th Street, and that a sum of not more than $1400, or as much thereof as may be necessary, be set aside for this purpose from the unappropriated funds at the disposal of the Council. (*Adopted.*)

Mr. Leuschner, Acting Secretary of the National Research Council, presented a preliminary report on the membership of the Executive Board under the new organization of the National Research Council, and indicated that the new Board could not be completely organized at the present meeting.

Mr. Hale, for the committee appointed April 15, to nominate a Chairman and Vice Chairman and not more than ten members at large of the Executive Board, reported that the committee was not prepared to nominate a Chairman at this time, and presented the following names for election as members at large of the Executive Board: Messrs. Dunn, Millikan, Noyes, and Welch.

Moved: That the Acting Secretary of the Council cast the ballot for the election of Messrs. Dunn, Millikan, Noyes, and Welch as members at large of the Executive Board of the National Research Council, and that the President of the National Academy be requested to appoint them members of the National Research Council. (*Adopted.*)

Mr. Leuschner, the Acting Secretary, thereupon reported that he had cast the ballot for the election of Messrs. Dunn, Millikan, Noyes, and Welch as members at large of the Executive Board of the National Research Council, and the Chairman declared these nominees so elected.

In view of the incomplete organization of the new Board it was

Moved: That the former ex-officio members of the Executive Board of the National Research Council continue to serve on the Board until their successors shall have been elected, and that all members at large of the Executive Board continue to serve until the election of members at large shall have been completed and the new members shall have been appointed to the Council. (*Adopted.*)

Mr. Hale, Chairman of the Nominating Committee, stated that two members were leaving Washington and

Moved: That the present Nominating Committee be discharged and the Acting Chairman be requested to appoint a new Committee on Nominations. (*Adopted.*)

At the suggestion of the Acting Chairman, Mr. Merriam it was

Moved: That the Acting Chairman of the National Research Council be authorized to appoint a committee of three to serve on the part of the National Research Council on a joint committee with representatives of the Chemical Foundation, to study and report on the relation of the National Research Council to the Chemical Foundation in regard to the question of chemical research in educational institutions. (*Adopted.*)

The Acting Chairman stated that Mr. A. A. Hammerschlag, director of the Carnegie Institute of Technology, of Pittsburgh, has offered the use of the shops of the institution and the services of its staff to the National Research Council, in connection with research problems which the Council might undertake.

Moved: That a vote of thanks be extended to Dr. A. A. Hammerschlag, director of the Carnegie Institute of Technology, Pittsburgh, by the National Research Council, for courtesies extended in the shops of the Institution to the National Research Council in connection with the work on war problems, and for the offer of further assistance. (*Adopted.*)

Mr. Merriam, in behalf of the Section on Relations with Educational Institutions and State Scientific Committees, under the old organization, recommended that the appointment of chairmen for the Divisions of Educational Relations and of States Relations be deferred, that for the present a Secretary for transacting the business of both be appointed, and that Dr. Albert Barrows, of the University of California, be appointed to this position.

On motion, these recommendations were unanimously approved.

On nomination of the Section on Relations with Educational Institutions and State Scientific Committees it was

Moved: That the President of the National Academy of Sciences be requested to appoint Messrs. S. P. Capen, A. Flexner, H. E. Gregory, A. O. Leuschner, J. C. Merriam, H. S. Pritchett and R. M. Yerkes members of the National Research Council on the Division of Educational Relations. (*Adopted.*)

Moved: That the Division of State Relations include one representative from six organizations concerned particularly with research problems of the state, as follows: Association of American State Geologists, five others to be named. (*Adopted.*)

Mr. Cross, Treasurer of the Council, gave notice at the meeting of the Interim Committee of April 22, 1919, that he had tendered his resignation as Treasurer of the National Academy of Sciences to take effect May 1, 1919.

Mr. Hale presented the following resolutions adopted by the National Academy of Sciences:

That the organization of the National Research Council as presented from the Council of the National Academy of Sciences be adopted as a whole, and that power of amendment be given to the Council of the National Academy of Sciences.

That the Council of the National Academy of Sciences be authorized to represent the Academy in all dealings with the National Research Council.

That the Council of the National Academy of Sciences be authorized (1) to secure funds to meet the needs of the Academy and of the National Research Council (2) to prepare a budget apportioning such funds (3) to select and purchase a site for a building, using any available Academy funds in case a grant for the selection of a suitable building can be secured.

That Section 2 of Article 4 of the Organization of the National Research Council be amended to read:

"Section 2. The affairs of the National Research Council shall be administered by an Executive Board, of which the officers of the Council, the President and Home Secretary of the National Academy of Sciences, the President of the American Association for the

Advancement of Science, the Chairmen and Vice-Chairmen of the Divisions of Science and Technology, and the Chairmen of the Divisions of General Relations shall be ex officio members. The Council of the National Academy of Sciences and the Executive Board of the National Research Council, meeting in joint session, may nominate additional members, not to exceed twelve in number, who, if not already members of the National Research Council, shall be appointed thereto, in accordance with Article V, section 4."

That the Council of the National Academy of Sciences be authorized to proceed with arrangements for the publication of a popular science journal, in co-operation with the American Association for the Advancement of Science, provided arrangements can be made for properly financing the journal and for safeguarding the scientific quality of its contents.

That the following new rules of the National Academy of Sciences be approved:

The Treasurer is authorized to act ex officio as Treasurer of the National Research Council.

It is the opinion of the Council of the National Academy of Sciences that the provision in the Organization of the National Research Council making the Treasurer of the National Academy of Sciences ex officio the Treasurer of the National Research Council should go into effect May 1, 1919, or as soon thereafter as practicable.

This opinion shall be transmitted to the Executive Board of the National Research Council with the suggestion that it should approve the action specified.

On such approval the Chairman of the National Research Council should request the President of the Carnegie Institution of Washington to take such steps as may be necessary to place the account of the National Research Council at the Riggs National Bank in the custody of the Treasurer of the National Academy of Sciences, and to cause further payments from the Carnegie Corporation of New York in support of the National Research Council to be paid directly to the Academy.

Moved: That the Executive Board of the National Research Council approve the action of the National Academy of Sciences suggesting that the provision in the Organization of the National Research Council, in making the Treasurer of the National Academy of Sciences ex officio Treasurer of the National Research Council, should go into effect May 1, 1919, or as soon thereafter as practicable, and that the Chairman of the National Research Council is directed to request the President of the Carnegie Institution of Washington to take such steps as may be necessary to place the account of the National Research Council at the Riggs National Bank in the custody of the Treasurer of the National Academy of Sciences and to cause further payments from the Treasurer of the Carnegie Corporation of New York in support of the National Research Council to be paid directly to the National Academy of Sciences. *(Adopted.)*

That the Treasurer shall have assistance of a salaried and bonded officer, the bursar, who shall be chosen by the Finance Committee and be directly responsible to the Treasurer.

That a Budget Committee on the expenses of the National Academy of Sciences and the National Research Council, to consist of the President of the Academy, the Chairman of the National Research Council, and the Treasurer of the Academy and the Research Council, be appointed, the President of the Academy to act as Chairman.

Concerning notification of present members of the National Research Council that their term of service terminated with the new organization.

Moved: That in ordinary course the Secretary be authorized in his discretion to communicate with the members of the National Research Council under the old organization, stating that the new organization has taken effect and thanking them for their services, which were so patriotically given during the war period. *(Adopted.)*

Mr. Hale stated that it was expected that the National Research Council might change its quarters and that in the new quarters there would be room for exhibits, which are desirable in connection with the work of the National Research Council, and he

Moved: That an exhibit by the Western Electric Company in this connection be authorized. *(Adopted.)*

Mr. Noyes submitted the following draft of recommendations of the Division of Chemistry and Chemical Technology with respect to the formation of an International Chemical Council:

RECOMMENDATIONS OF THE DIVISION OF CHEMISTRY AND CHEMICAL TECHNOLOGY WITH RESPECT TO THE FORMATION OF AN INTERNATIONAL CHEMICAL COUNCIL*

1. That an International Chemical Council be constituted; and that, if possible, arrangements be made for transferring the funds originally given to the International Association of Chemical Societies to this International Chemical Council.

2. That the object of the International Chemical Council be to initiate and promote international coöperation in chemistry; for example, by arranging:

(a) For international coöperation in the preparation and publication of chemical literature.

(b) For the appointment of international commissions to deal with special chemical questions of standardization (such as atomic weights, nomenclature, etc.)

(c) For international coöperation in the prosecution of special research projects.

(d) For the calling of international chemical conferences for various purposes; and also for the organization of an International Chemical Congress with meetings at stated intervals, and including all of the scientific and technological branches of chemistry.

3. That the International Chemical Council be constituted of delegates representing the leading chemical societies and other chemical research organizations of the several allied and neutral countries, these delegates to be selected as described in Paragraph 5.

4. That the International Chemical Council be affiliated with the International Research Council; and that the National Research Council of each country, or its National Academy when no Research Council has been created, act as the intermediary in communications between the International Chemical Council and the chemical organizations of that country, and arrange for the proper representation of those organizations in accordance with Paragraph 5.

5. That the delegates from each country shall in general be chosen by the major chemical societies in that country, but that the number and distribution of such delegates and their voting strength within the delegation be determined initially by the National Research Council of that country, with the understanding that in countries where a National Research Council shall not have been organized the National Academy itself shall fulfill this function until the National Research Council is organized.

6. That upon all questions voted upon by the International Chemical Council the number of votes cast by the various countries shall be determined by their population as follows:

Countries of less than 5 million inhabitants have 1 vote.
Countries between 5 and 10 million inhabitants have 2 votes.
Countries between 10 and 15 million inhabitants have 3 votes.
Countries between 15 and 20 million inhabitants have 4 votes.
Countries over 20 million Inhabitants have 5 votes.

The inhabitants of colonies and possessions are included in the population of the country to which they belong, according to the indications of its Government. Each self-governing Dominion has the same number of votes as an independent country according to the above scale.

7. That the International Chemical Council, as soon as it shall be organized, shall elect an Executive Committee of seven members, which shall exercise such functions as may be assigned to it by the Council. The Executive Committee shall appoint an Executive Secretary, who shall have charge of correspondence and of the central office of the Council.

---

* The designation "Council" will probably be changed to "Union" to correspond to similar international organizations in astronomy, geophysics, etc.

8. That until the International Chemical Council shall be organized and its Executive Committee appointed, the Committee on International Coöperation in Chemistry appointed by the Paris Conference shall act as a provisional Executive Committee for purposes of organization. Its membership shall, however, be increased by the addition of four members representing industrial chemistry, to be appointed respectively by the Royal Society of London, the Academie des Sciences de France, the Academia dei Lincei, and the National Research Council of the United States. This Committee shall elect a chairman and a secretary; but the latter need not be a member of the Committee.

SPECIAL MEETING OF THE EXECUTIVE BOARD

AT THE NATIONAL RESEARCH COUNCIL BUILDING

MAY 8, 1919, AT 9.00 A.M.

Present: Messrs. Bancroft, Clevenger, Cross, Leuschner, Manning, Merriam, Ransome, Stratton, Walcott, Washburn, and Yerkes.

Mr. Merriam, Chairman of the Council in the Chair.

The question of moving the National Research Council to more suitable quarters was discussed.

Moved: That a Committee of three in addition to the Acting Chairman of the Council be appointed, with power to act in the carrying out of plans and making arrangements to move the offices of the National Research Council to more suitable quarters. (*Adopted.*)

Appointed: Mr. Walcott, Chairman, Messrs. Cross, Dunn, Merriam.

In accordance with a request received from the American Council on Education, the Acting Chairman, Mr. Merriam, recommended that the representation of the National Research Council on the American Council on Education be increased from one to three members.

Moved: That Messrs. A. O. Leuschner and R. M. Yerkes be added to the representation of the National Research Council on the American Council on Education. (*Adopted.*)

The Chairman presented a communication from Mr. Wm. M. Davis, Chairman of a Committee of the National Academy of Sciences on Pacific Exploration, recommending that a general committee on Pacific Exploration be organized by the National Research Council. In this connection Mr. Mathews, Acting Chairman of the Division of Geology and Geography, reported that the subject of Pacific Exploration had been fully discussed in the Division of Geology and Geography, and submitted a recommendation for the appointment of an appropriate divisional committee.

Moved: That the appointment of a sub-committee on the Exploration of the Pacific by the Division of Geology and Geography be approved; that each of the other Divisions of the Council interested be requested to appoint similar sub-committees; that the chairmen of these sub-committees be members of a general Committee on the Exploration of the Pacific to be appointed by the Acting Chairman; and that Mr. Davis be informed of this action. (*Adopted.*)

On the recommendation of Mr. Yerkes, Acting Chairman of the Committee of Psychology, it was

Moved: That the Board authorize the expenditure of not to exceed $50.00 for the preparation of a record of war service of American psychologists. (*Adopted.*)

On further recommendation of Mr. Yerkes, presented in the absence of the Acting Chairman of the Division of Medicine, it was

Moved: That the expenditure of a sum not to exceed $300 to further a special study of the intelligence ratings of medical officers in relation to education and professional success, be authorized, it being understood that the sum of $300 is additional to the sum of $800 already available for this purpose. (*Adopted.*)

Mr. Washburn, Acting Chairman of the Division of Chemistry and Chemical Technology reported on the desirability of continuing the Committee on Explosives Investigations.

Moved: That the Committee on Explosives Investigations be continued, and that a conference of representatives of the Division of Chemistry and Chemical Technology, of the Army, of the Navy, and of the Bureau of Mines be held for the purpose of arranging for an annual contribution of $2000 each from the Army, the Navy, and the Bureau of Mines for the support of the work of the Committee on Explosives Investigations. (*Adopted.*)

The Chairman stated that this special meeting of the Executive Board was called for the purpose of electing a Chairman for the National Research Council for the ensuing year which action had not been possible at the meeting held on April 30. In the absence of Mr. Hale, Chairman of the former Nominating Committee, the Chairman presented the name of Mr. James R. Angell, of the University of Chicago, as the unanimous choice of the Nominating Committee for Chairman. Mr. Walcott

Moved: That the report of the Nominating Committee be approved and that the Acting Secretary cast the ballot for Mr. James R. Angell, of the University of Chicago, as Chairman of the National Research Council for the year beginning July 1, 1919.
(*Adopted.*)

The Secretary cast the ballot for Mr. James R. Angell as Chairman of the National Research Council for the year beginning July 1, 1919, and the Chairman announced that the election of Dr. Angell was unanimous.

Mr. Cross reported that his resignation as Treasurer of the National Academy of Sciences and ex-officio Treasurer of the National Research Council had been accepted at the Annual Meeting of the National Academy held April 30, 1919, and that he had turned over all accounts and the books of his office to his successor, Mr. F. L. Ransome. Mr. Cross requested to have his accounts as Treasurer of the National Research Council, including the funds appropriated by the Carnegie Corporation and the Rockefeller Foundation, audited by expert accountants.

Moved: That the report of the retiring treasurer be received and approved, that with the system of auditing of the accounts now in effect it would not be necessary to have the accounts audited at this time on the transfer of the funds, and that a committee of three be appointed to express in a suitable way the appreciation of the National Research Council of the work which Mr. Whitman Cross has so patriotically given to the National Research Council during the period of the war. (*Adopted.*)

Appointed: Mr. Woodward, Chairman, Messrs. Leuschner, Washburn.

The Secretary presented a communication from Mr. Clevenger, Acting Chairman of the Division of Engineering, in regard to the desirability of completing certain war work for the United States Bureau of Mines, and securing financial support and coöperation for the Pulverizing Committee of the Division of Engineering.

Moved: That the Acting Chairman of the Division of Engineering, Mr. Clevenger, be authorized to proceed to Colorado for the purpose of giving personal attention to matters growing out of certain war work for the Bureau of Mines, and for the purpose of securing the financial support and coöperation for the Pulverizing Committee of the Division of Engineering. (*Adopted.*)

Mr. Leuschner, Acting Chairman of the Division of Physical Sciences, reported the election of Mr. C. E. Mendenhall as Chairman of the Division, of Mr. A. O. Leuschner as Acting Chairman of the Division during Mr. Mendenhall's absence in London as Scientific Attaché, and of the following as members of the Executive Committee: William Bowie, R. A. Millikan, H. N. Russell, E. B. Wilson, the Chairman, ex officio, the Vice-Chairman, ex officio.

(*Approved.*)

The Acting Chairman of the Division of Geology and Geography, Mr. E. B. Mathews, presented a report of the meeting of the Division held on May 1 and 2, and announced the following elections:

Vice-Chairman and Acting Chairman, E. B. Mathews; Executive Committee, Isaiah Bowman, Alfred H. Brooks, John M. Clarke, N. M. Fenneman, David White.

Moved: That the appointments be confirmed and that the report be approved.
(*Adopted.*)

The Acting Chairman of the Division of Geology and Geography presented a tentative draft of a resolution for submission to the Secretary of War, the Chief of Staff, and the Military Committees of Congress, providing that in the reorganization of the army provision should be made for a number of reserve officers who are primarily scientific experts in physics, chemistry, geography, biology, geology, psychology, mathematics, engineering, medicine, and such other sciences as may be appropriate, and that they be relieved from certain training in order to devote their entire time to research work.

Moved: That the matter contained in the report be referred to a committee to be appointed by the Acting Chairman of the Council in accordance with a previous resolution of the Interim Committee. (*Adopted.*)

Appointed: Mr. Walcott, Chairman, Messrs. Mathews, Leuschner.

The Division of Geology and Geography suggested a survey of (a) the financial resources available for research in existing museums, universities, and allied institutions; (b) the boards or persons controlling the disposal of funds; (c) the discovery of private sources of support, together with best lines of approach.

Moved: That a survey of the kind be approved and referred to the Research Information Service. (*Adopted.*)

Mr. Washburn, Acting Chairman of the Division of Chemistry,

Moved: That a special committee on the Chemistry of Colloids be appointed to undertake the following specific projects:

(1) To arrange a series of lectures on different aspects of colloid chemistry by competent specialists and to make public the fact that such lectures have been arranged for and can be given at any institution or before a meeting of any interested body which will make the necessary financial arrangements with the committee.

(2) To secure the preparation and publication of a text book and laboratory manual on colloid chemistry suitable for use in University courses in this field, bothb ooks to be prepared under the auspices of the Committee and subject to the criticism of its full membership and any other experts which the Committee may desire to consult.

(3) To make a research census of investigations and investigators in the field of colloid chemistry. (*Adopted.*)

Moved: That the Committee on Colloid Chemistry be constituted with the following personnel:

Mr. H. N. Holmes, Oberlin College, Oberlin, Ohio, Chairman.
Mr. Jerome Alexander, 59th Street and 11th Avenue, New York, N. Y.
Mr. W. D. Bancroft, National Research Council, Washington, D. C.
Mr. G. H. A. Clowes, Eli Lilly Company, Indianapolis, Ind.
Mr. W. A. Patrick, Johns Hopkins University, Baltimore, Md.
Mr. J. A. Wilson, 203 Juneau Avenue, Milwaukee, Wis.

(*Adopted.*)

A letter was presented from Dr. Charles E. Munroe, Chairman of the Committee on Explosives Investigations, relating to the appointment of a committee to arrange for a series of coöperative investigations dealing with the specific heats of explosive materials and recommending that a committee of three members, to consist of the Chairman of the Division, the Chairman of the Explosives Investigations Committee, and one additional member (to be selected by them) of recognized standing as an expert in calorimetry, be appointed.

Appointed as third member: Dr. W. P. W. White.

(*Approved.*)

Moved: That the sum of $500 be allotted from unappropriated funds for the use of the Division of Biology and Agriculture. (*Adopted.*)

MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

At the National Research Council Building

May 13, 1919, at 9.30 a.m.

Present: Messrs. Bancroft, Dunn, Flinn, Hussey, Leuschner, Mathews, Merriam, Ransome, Washburn, Woods, Woodward, and Yerkes.

Mr. Merriam, Chairman of the Council, in the Chair.

The Chairman announced that President Walcott of the National Academy of Sciences had appointed the following Committee on Nominations of Members at Large of the Executive Board: Messrs. Dunn (Chairman), Merriam and Yerkes. This is a joint committee of the Council of the Academy and of the Executive Board of the National Research Council, appointment of which was provided for at the joint meeting of these bodies held April 30. This Committee will report to a joint meeting of the Council of the National Academy and of the Executive Board of the National Research Council.

The Chairman asked consideration of the number of Vice Chairmen to be elected under the new organization of the National Research Council, the number of Vice Chairmen under the old organization being three.

Moved: That there be three Vice Chairmen in the new organization of the National Research Council. (*Adopted.*)

Mr. Washburn, Acting Chairman of the Division of Chemistry and Chemical Technology, presented an amended report on the organization of the proposed International Chemical Union. The original report was presented at the meeting of the Executive Board on April 15 and was referred to the Division on International Relations. The amended report presented by Mr. Washburn, has been printed with the minutes of April 30.

Moved: That the plan of organization of the proposed International Chemical Union, as circulated with the minutes of April 30, be approved. (*Adopted.*)

Moved: That the whole question of delegates to international conferences to be held during the coming summer, be left to the Interim Committee with power and that the principle be adopted that in naming delegates alternates be named at the same time. (*Adopted.*)

Consideration of this report brought up the question of delegates to the various international conferences to be held in Europe during the coming summer.

Moved: That the question of delegates for the coming meeting of the International Chemical Union be referred to the Division of Chemistry for nominations and that the Division report to the Interim Committee. (*Adopted.*)

The Chairman submitted for consideration the question of the future of the Foreign Research Information Service. Mr. Yerkes, Chairman of the Research Information Service outlined the history of the Service during the war

and presented arguments in favor of its continuation. After thorough discussion in which Messrs. Dunn, Flinn, Leuschner, Merriam, Woods, Woodward, and Yerkes participated, the following motion was made.

Moved: That the Chairman of the Council and the Chairman of the Research Information Service present to the State Department a request that the Foreign Research Information Service be maintained, that the Scientific Attachés be continued at their present posts, that a committee be appointed with power to prepare and present suitable resolutions to the proper authorities in regard to the continuation of the Service, and that authority to present matters concerning international relations in regard to scientific research to the League of Nations at its meeting in Washington be left to the Committee with power.

(*Adopted.*)

Appointed: Messrs. Bancroft, Dunn, Merriam, Mathews, Yerkes (Chairman).

The Chairman submitted for consideration the question of the organization of the Government Division. After a general discussion of the subject in which practically all of the members of the Board participated, it was referred back to the Committee on Organization of the Government Division for future consideration.

The Chairman presented a communication from Mr. Johnston, Chairman of the Section on Industrial Research under the old organization of the Council, proposing that in accordance with action taken on April 8, regarding the organization of the Division of Industrial Relations under the new organization, the membership of the Division be constituted as follows:

The Chairman, Division of Physical Sciences, Charles E. Mendenhall (A. O. Leuschner, Acting Chairman).

The Chairman, Division of Engineering, Henry M. Howe (G. H. Clevenger, Acting Chairman).

The Chairman, Division of Chemistry and Chemical Technology, W. D. Bancroft (E. W. Washburn, Acting Chairman).

One representative of the Bureau of Chemistry, H. D. Gibbs.

One representative of the Bureau of Mines, F. G. Cottrell, (A. E. Wells, Acting).

One representative of the Bureau of Standards, G. K. Burgess.

Members at large of whom one-third shall serve for one year, one-third for two years, one-third for three years, the terms to be determined by lot: Leo H. Baekeland, John J. Carty, Gano Dunn, Alfred D. Flinn, Harrison E. Howe, John Johnston, C. E. K. Mees, Walter Rautenstrauch, C. P. Townsend, E. W. Washburn, and W. R. Whitney. Nomination of the twelfth member at large to be deferred.

Moved: That the foregoing nominations be approved with the recommendation to the President of the National Academy of Sciences that the nominees be appointed members of the National Research Council and assigned to the Division of Industrial Relations.

(*Adopted.*)

The Chairman stated that there was some question as to whether the Research Council should consider the salaries of Government employees having to do with research, in connection with the present movement of reclassification. It was the sense of the Executive Board that no action be taken.

Mr. Yerkes, Chairman of the Research Information Service, presented the following report:

In accordance with the action taken by the Interim Committee April 8, 1919 to the effect "that the Research Information Service secure information on the plans of the Government for the publication of results of scientific investigations bearing on the war," the Chairman of the Division of Research Information has made inquiries in the office of the Secretary of War and of the War Plans Division of the General Staff.

It appears that no general plan for the publication of scientific materials relating to military problems has been formulated. The several military departments or bureaus are proceeding with arrangements for the assembling and publication of their respective materials. It is desired by the Chief of the War Plans Division of the General Staff that plans for the preparation and publication of materials, as well as completed manuscripts, be submitted to him for consideration and approval before further action is taken.

Moved: That the report of Mr. Yerkes with reference to the plans of the Government for the publication of results of scientific investigations bearing on the war be approved and spread on the minutes. (*Adopted.*)

Mr. Dunn asked whether action was being taken in regard to retaining possession of the government furniture now in use by the Council. The Chairman stated that the matter was under consideration and that a report in regard to the same was expected in a short time.

The Secretary presented the report of the Treasurer, which was approved.

PAUL BROCKETT, *Assistant Secretary.*

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 SEPTEMBER 15, 1919 Number 9

RADIATION FROM A MOVING MAGNETON

By H. Bateman

Throop College of Technology, Pasadena

Communicated by R. A. Millikan, June 7, 1919

1. The rate of radiation of energy from a ring of electrons revolving in a circular orbit and from various other distributions of moving electric charges and magnetic poles has been calculated by G. A. Schott,[1] who finds that the rate of radiation of energy is almost invariably positive. This is certainly true in the case of a single electric pole describing a circular orbit as is indicated by the well known formulae of Larmor and Liénard for the rate of radiation. Thus electromagnetic theory in its present form lends no support to Bohr's idea of non-radiating orbits.

A steady distribution such as a Parson magneton which consists of a complete ring of electric charges following one another round the ring at a constant speed will evidently give no radiation when the ring is stationary as a whole, but as Schott remarks the ring may be expected to radiate energy when its centre has an acceleration.

Schott's results are so important that it is desirable that they should be confirmed by an independent method and an attempt has been made to devise a method by which the rate of radiation from a moving electric pole and magnetic doublet may be readily calculated. In two important cases we have confirmed Schott's surmise that the rate of radiation is positive.

2. Starting with the case of an *electromagnetic doublet*, i.e. an electric doublet and magnetic doublet which move together, we determine the electric force E and the magnetic force H from the equations

$$M \equiv H + iE = i \operatorname{rot} L$$

$$L = \frac{\partial G}{\partial t} + ic \operatorname{rot} G, \qquad G = \frac{1}{\nu} g(\tau)$$

$$4\pi g = q + ip - \frac{1}{c}(v \times p) + \frac{i}{c}(v \times q)$$

$$\nu = \xi'(\tau)[x - \xi(\tau)] + \eta'(\tau)[y - \eta(\tau)] + \zeta'(\tau)[z - \zeta(\tau)] - c^2(t - \tau)$$
$$\equiv r[(v \cdot s) - c]$$
$$r^2 = [x - \xi(\tau)]^2 + [y - \eta(\tau)]^2 + [z - \zeta(\tau)]^2 = c^2(t - \tau)^2. \qquad t \geq \tau.$$

In these equations v denotes the velocity at time τ of the moving point P whose coordinates at this instant are $\xi(\tau)$, $\eta(\tau)$, $\zeta(\tau)$; x, y, z are the coordinates of an arbitrary point Q; s is a unit vector in the direction of the line PQ; p and q are vectors representing the electric and magnetic moments at time τ; t is the time, and c the velocity of light.

From these equations we find that

$$E \times H = c^2 \frac{r^4}{\nu^4} s[g^* \cdot g_0^* - (s \cdot g^*)(s \cdot g_0^*) - i\{s \cdot (g^* \times g_0^*)\}]$$

where g^* is a certain complex vector and g^*_0 the conjugate complex vector. A similar expression may be obtained in the case of an electromagnetic pole and also in the case of the combination of an electromagnetic pole and an electromagnetic doublet. In the latter case the appropriate expression for g^* is

$$g^* = \frac{1}{\nu} b - 3\frac{r}{\nu^2}(s \cdot v')g' + \frac{3r^2}{\nu^3}(s \cdot v')^2 g - \frac{r}{\nu^2}(s \cdot v'')g$$

where

$$b = g'' + mv' + i\frac{m}{c}(v \times v'),$$

and

$$m = h + ie.$$

Here $4\pi e$ and $4\pi h$ are the electric and magnetic charges associated with the pole and primes denote differentiations with respect to τ.

Calculating the rate of radiation I across a very large sphere whose centre is at the moving point P we find that I may be represented by the real part of the following expression

$$\pi c^3 \left[\frac{8}{3}\frac{c^2 + 2v^2}{(c^2 - v^2)^4}(b \cdot b_0) - \frac{8}{(c^2 - v^2)^4}(v \cdot b)(v \cdot b_0) + \frac{16}{5}\frac{c^2 + v^2}{(c^2 - v^2)^5}\left\{12(v \cdot v')(b \cdot g_0')\right.\right.$$
$$\left. + 3v'^2(g' \cdot g_0') + 4(v \cdot v'')(b \cdot g_0) + 2v'^2(b \cdot g_0) + 2(v' \cdot v'')(g' \cdot g_0) + \frac{1}{3}v''^2(g \cdot g_0)\right\}$$
$$+ \frac{8}{5}\frac{3c^2 + 2v^2}{(c^2 - v^2)^6}\left\{24(v \cdot v')^2(g' \cdot g_0') + 16(v \cdot v')^2(b \cdot g_0) + \frac{144}{7}v'^2(v \cdot v')(g' \cdot g_0)\right.$$

$$+16(v\cdot v')(v\cdot v'')(g'\cdot g_0)+\frac{9}{7}v'^4(g\cdot g_0)+\frac{16}{7}(v\cdot v'')v'^2(g\cdot g_0)+\frac{32}{7}(v\cdot v')(v'\cdot v'')(g\cdot g_0)$$

$$+\frac{8}{3}(v\cdot v'')^2(g\cdot g_0)\Big\}+\frac{64}{7}\frac{2c^2+v^2}{(c^2-v^2)^7}\Big\{24(v\cdot v')^3(g'\cdot g_0)+9v'^2(v\cdot v')^2(g\cdot g_0)$$

$$+8(v\cdot v'')(v\cdot v')^2(g\cdot g_0)\Big\}-\frac{8}{5}\frac{1}{(c^2-v^2)^4}\Big\{6(v\cdot b)(v'\cdot g_0')+6(v\cdot g_0')(v'\cdot b)$$

$$+3(v'\cdot g')(v'\cdot g_0')+2(v'\cdot b)(v'\cdot g_0)+(v'\cdot v'')(g'\cdot g_0)+(v''\cdot g')(v'\cdot g_0)$$

$$+(v'\cdot g')(v''\cdot g_0)+\frac{1}{3}(v''\cdot g)(v''\cdot g_0)+2(v\cdot b)(v''\cdot g_0)+2(v\cdot g_0)(v''\cdot b)\Big\}$$

$$-\frac{32}{5}\frac{1}{(c^2-v^2)^5}\Big\{12(v\cdot v')(v\cdot g_0')(v\cdot b)+6(v\cdot v')(v\cdot g')(v'\cdot g_0')+6(v\cdot v')(v'\cdot g')(v\cdot g_0')$$

$$+3v'^2(v\cdot g')(v\cdot g_0')+2v'^2(v\cdot b)(v\cdot g_0)+4(v\cdot v')(v\cdot b)(v'\cdot g_0)+4(v\cdot v')(v'\cdot b)(v\cdot g_0)$$

$$+2(v'\cdot v'')(v\cdot g')(v\cdot g_0)+2(v\cdot v')(v\cdot g')(v''\cdot g_0)+2(v\cdot v'')(v\cdot g_0)(v'\cdot g')$$

$$+2(v\cdot v'')(v\cdot g')(v'\cdot g_0)+2(v\cdot v')(v\cdot g_0)(v''\cdot g')+\frac{4}{3}(v\cdot v'')(v\cdot g)(v''\cdot g_0)$$

$$+\frac{1}{3}v''^2(v\cdot g)(v\cdot g_0)+\frac{18}{7}v'^2(v\cdot g')(v'\cdot g_0)+\frac{18}{7}v'^2(v\cdot g_0)(v'\cdot g')+\frac{4}{7}(v\cdot v'')(v'\cdot g)(v'\cdot g_0)$$

$$+\frac{4}{7}(v'\cdot v'')(v'\cdot g_0)(v\cdot g)+\frac{4}{7}(v'\cdot v'')(v'\cdot g)(v\cdot g_0)+\frac{4}{7}(v\cdot v')(v'\cdot g)(v''\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')(v'\cdot g_0)(v''\cdot g)+4(v\cdot v'')(v\cdot b)(v\cdot g_0)+\frac{36}{7}(v\cdot v')(v'\cdot g')(v'\cdot g_0)$$

$$+\frac{4}{7}(v\cdot g)(v''\cdot g_0)v'^2+\frac{9}{14}v'^2(v'\cdot g)(v'\cdot g_0)\Big\}-64\frac{1}{(c^2-v^2)^6}\Big\{3(v\cdot v')^2(v\cdot g')(v\cdot g_0')$$

$$+2(v\cdot v')^2(v\cdot b)(v\cdot g_0)+2(v\cdot v')(v\cdot v'')(v\cdot g')(v\cdot g_0)+\frac{1}{3}(v\cdot v'')^2(v\cdot g)(v\cdot g_0)$$

$$+\frac{18}{7}(v\cdot v')^2(v\cdot g')(v'\cdot g_0)+\frac{18}{7}(v\cdot v')^2(v\cdot g_0)(v'\cdot g')+\frac{4}{7}(v\cdot v')(v\cdot v'')(v\cdot g)(v'\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')(v\cdot g)(v\cdot g_0)(v'\cdot v'')+\frac{4}{7}(v\cdot v')(v\cdot v'')(v\cdot g_0)(v'\cdot g)+\frac{18}{7}v'^2(v\cdot v')(v\cdot g')(v\cdot g_0)$$

$$+\frac{4}{7}(v\cdot v')^2(v\cdot g)(v''\cdot g_0)+\frac{2}{7}v'^2(v\cdot v'')(v\cdot g)(v\cdot g_0)+\frac{9}{56}v'^4(v\cdot g)(v\cdot g_0)$$

$$+\frac{9}{14}(v\cdot v')^2(v'\cdot g)(v'\cdot g_0)+\frac{9}{7}v'^2(v\cdot v')(v\cdot g)(v'\cdot g_0)+\frac{192}{7}\{24(v\cdot v')^3(v\cdot g')(v\cdot g_0)$$

$$+8(v\cdot v')^2(v\cdot v'')(v\cdot g)(v\cdot g_0)+9v'^2(v\cdot v')^2(v\cdot g)(v\cdot g_0)+12(v\cdot v')^3(v\cdot g)(v'\cdot g_0)\}$$

$$-576\frac{1}{(c^2-v^2)^8}(v\cdot v')^4(v\cdot g)(v\cdot g_0)-i\pi c^2\Big[\frac{4}{15}\frac{5c^2+v^2}{(c^2-v^2)^4}\{5[v\cdot(b\times b_0)]$$

$$+6[v'\cdot(b\times g_0')]+2[v''\cdot(b\times g_0)]\}+\frac{4}{5}\frac{7c^2+v^2}{(c^2-v^2)^5}\{4(v\cdot v'')[v\cdot(b\times g_0)]$$

$$
\begin{aligned}
&+ 12\,(v\cdot v')\,[v\cdot(b\times g_0')] + 6\,(v\cdot v')\,[v'\cdot(g'\times g_0')] + 3v'^2\,[v\cdot(g'\times g_0')] \\
&+ 4\,(v\cdot v')\,[v'\cdot(b\times g_0)] + 2v'^2\,[v\cdot(b\times g_0)] + 2\,(v\cdot v')\,[v''\cdot(g'\times g_0)] \\
&+ 2\,(v\cdot v'')\,[v'\cdot(g'\times g_0)] + 2\,(v'\cdot v'')\,[v\cdot(g'\times g_0)] + \frac{2}{3}\,(v\cdot v'')\,[v''\cdot(g\times g_0)] \\
&+ \frac{1}{3}\,v''^2\,[v\cdot(g\times g_0)] + \frac{18}{7}\,v'^2\,[v'\cdot(g'\times g_0)] + \frac{4}{7}\,(v'\cdot v'')\,[v'\cdot(g\times g_0)] \\
&+ \frac{2}{y}\,v'^2[v''\cdot(g\times g_0)] + \frac{16}{5}\,\frac{9c^2+v^2}{(c^2-v^2)^6}\,\{6\,(v\cdot v')^2[v\cdot(g'\times g_0')] + 4\,(v\cdot v')^2[v\cdot(b\times g_0)] \\
&+ 4\,(v\cdot v')\,(v\cdot v'')\,[v\cdot(g'\times g_0)] + \frac{2}{3}\,(v\cdot v'')^2[v\cdot(g\times g_0)] + \frac{9}{7}\,v'^2(v\cdot v')\,[v'\cdot(g\times g_0)] \\
&+ \frac{9}{28}\,v'^4\,[v\cdot(g\times g_0)] + \frac{4}{7}\,(v\cdot v')^2\,[v''\cdot(g\times g_0)] + \frac{8}{7}\,(v\cdot v')\,(v\cdot v'')\,[v'\cdot(g\times g^0)] \\
&+ \frac{8}{7}\,(v\cdot v')\,(v'\cdot v'')\,[v\cdot(g\times g_0)] + \frac{4}{7}\,v'^2\,(v\cdot v'')\,[v\cdot(g\times g_0)] \\
&+ \frac{16}{7}\,\frac{11c^2+v^2}{(c^2-v^2)^7}\,\{\,24\,(v\cdot v')^3\,[v\cdot(g'\times g_0)] + 12\,(v\cdot v')^2\,(v\cdot v'')\,[v\cdot(g\times g_0)] \\
&+ 6\,(v\cdot v')^3\,[v'\cdot(g\times g_0)] + 9\,v'^2\,(v\cdot v')^2\,[v\cdot(g\times g_0)]\,]\,\} \\
&+ \frac{288}{7}\,\frac{13c^2+v^2}{(c^2-v^2)^8}\,(v\cdot v')^4[v\cdot(g\times g_0)].
\end{aligned}
$$

3. In the special case of a magneton of charge $4\pi e$ and moment $4\pi k$ describing a circular orbit at a constant speed and in such a way that the axis of the magneton is always perpendicular to the plane of the orbit, we find that if k is constant the rate of radiation of energy is

$$\frac{8}{3}\,\pi ce^2\,\frac{v'^2}{(c^2-v^2)^2} + \frac{8}{3}\,\pi c^2ek\,\frac{v'^3}{v(c^2-v^2)^3} + \frac{16}{15}\,\pi ck^2\,\frac{v'^4}{v^2}\,\frac{2c^2+3v^2}{2(c^2-v^2)^4}$$

If e, v and v' are given the minimum value of this positive quantity is found to be

$$\frac{2\pi}{3}\,\frac{ce^2v'^2}{(c^2-v^2)^2}\,\frac{c^2+4v^2}{2c^2+3v^2}$$

and is thus about $\frac{1}{8}$ of the rate of radiation from the electric charge alone. Some years ago Dr. W. F. G. Swann expressed to me the desirability of calculating the radiation from an electron which rotates about its axis like a planet while describing a circular orbit. If such an electron can be treated as a magneton to a first approximation the above result is applicable. The fact that the radiation is reduced by rotation may indicate that revolving electrons do rotate.

4. A similar calculation for the case in which the axis of the magneton is tangential to the path indicates that the rate of radiation is greater than that from the electric charge alone.

5. It should be noticed that if $v' = v'' = g'' = 0$ the expression for I vanishes. Hence when an electromagnetic pole and an electromagnetic doublet move together with constant velocity along a rectilinear path, it is possible for the moment of the doublet to change at a constant rate without there being any radiation of energy.

[1] *Electromagnetic Radiation*, Cambr. Univ. Press, 1912: *London, Phil. Mag.* (Ser. 6), 36, 1918, 243.

ON THE DISTRIBUTION OF THE APHELIA OF THE SECONDARY BODIES OF THE SOLAR SYSTEM

By C. D. Perrine

Observatorio Nacional Argentino, Córdoba

Communicated by E. B. Frost, July 12, 1919

The finding of a dependence of orbital eccentricity upon the relative masses of the components led to an examination of the directions of the aphelia of the secondary bodies of the solar system, a resumé of which is the principal object of the present paper.

There is little to guide us as yet in the interpretation of such a dependence. The first explanation which suggests itself is that it is a residual effect of capture. In the comets we have a very exaggerated effect of such a dependence. Their eccentricities are essentially unity and whether they come from interstellar space or from the outer planetary regions they are, for the few journeys which they perform about the sun, captured bodies. This is with reference to the comets with very long period and those with sensibly parabolic orbits. The relations of the orbits of those with comparatively short periods to the outer (and larger) planets establishes the fact that such comets have been captured for the sun by these different planets. We have, therefore, in the solar system at least one class of bodies which has been captured. A study of the orbital characteristics of these in connection with the other secondary bodies (having the sun also for primary) should disclose any similarities which may be significant.

One fact which stands out prominently in connection with this dependence of orbital eccentricity upon relative mass appears to be significant. There are two possibilities aside from that of *no* preference

of eccentricity for relative mass: (a) the preference may be of larger eccentricities for systems with the *smaller* relative masses for the secondaries, or it may (b) be the reverse, a preference of the larger eccentricities for the systems with the *larger* relative masses for the secondaries.

The first of these possibilities is the one of which such widely distributed evidence has been found. Such a condition appears to result naturally from capture.

The second possibility (b), on the other hand, could not result from capture, nor does evidence of such a condition appear to prevail in nature.

It cannot but be regarded as significant, therefore, that of three possibilities the one which is actually found to prevail in nature is the one which would result directly from capture. The evidence is not sufficient to establish from the observed preference the reverse reasoning, viz., that because of this observed preference it must be concluded that binary and planetary systems have resulted from capture. It must be admitted, however, that this observational evidence, which appears to be stronger than any as yet adduced for a process of fission, places a theory of capture in a new light, and requires that it be seriously considered, not only for the solar system but for stellar systems as well.

If the secondary bodies of the solar system have been captured, and providing no force has seriously disturbed the major axes of their orbits since, or disturbed them all equally, we might expect to find a preference of the directions of their aphelia for some particular region of sky. It does not necessarily follow that it would be the region of the apex of solar motion. Differences of direction or velocity in the *relative* motions of the sun and the secondary bodies previous to capture could conceivably cause an apparent preference of their aphelia for almost any part of the sky.

The following four diagrams give in condensed form the results of an examination of the aphelia of the 8 major planets, 821 minor planets, 45 comets with periods of less than 80 years and 364 comets with longer periods and essentially parabolic orbits.

It is impossible to give in a limited space a full summary of the considerations resulting from this examination which has included not only the directions of aphelia but the orbital inclinations, longitudes of the nodes, galactic relations, etc. A full account of the investigation will be published in another place.

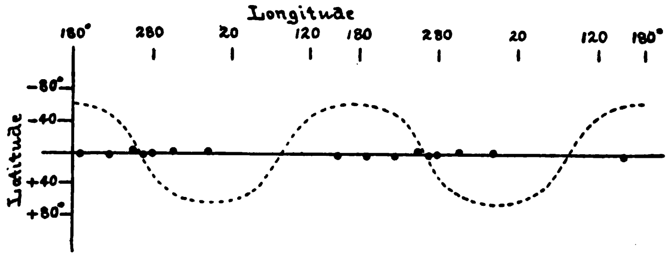


FIG. 1. APHELIA OF MAJOR PLANETS
Dotted line is the Galactic Plane

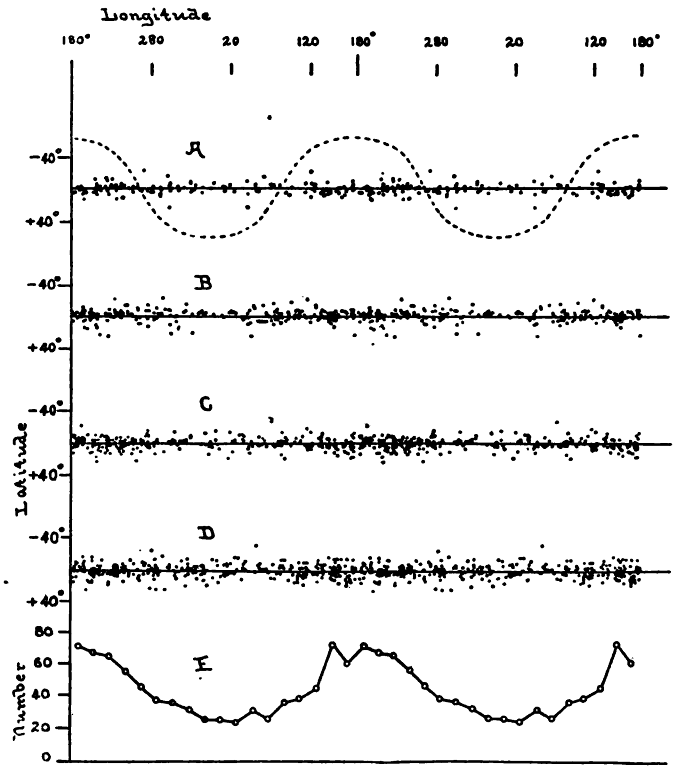


FIG. 2. APHELIA OF 821 MINOR PLANETS

A. Nos. 1 to 120 inclusive
B. Nos. 121 to 320 inclusive
C. Nos. 321 to 560 inclusive
D. Nos. 561 to 821 inclusive
E Frequency curve in longitude (all).

The following is a summary of the conclusions indicated by this investigation:

1. The axes of the orbits of the major planets, minor planets and short-period comets are all closely related to the ecliptic plane. This condition has been known in a general way.

2. The aphelia of the eight major planets are confined to the half of the sky whose center is the longitude of the apex of solar motion. Their longitudes are arranged almost symmetrically with respect to

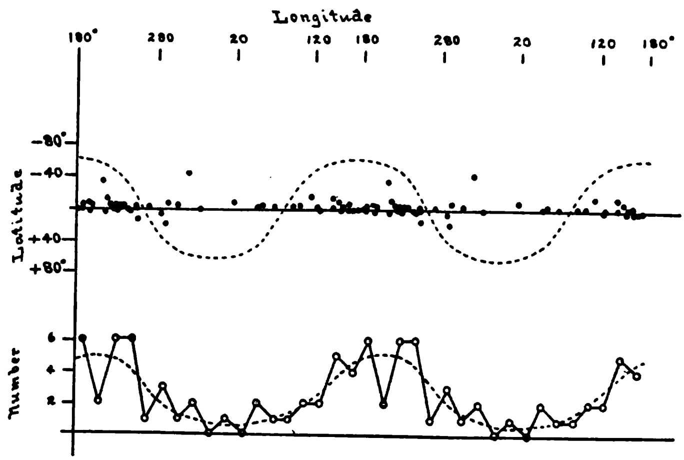


FIG. 3. APHELIA OF SHORT-PERIOD COMETS (UPPER). FREQUENCY CURVE OF SAME IN LONGITUDE (LOWER)

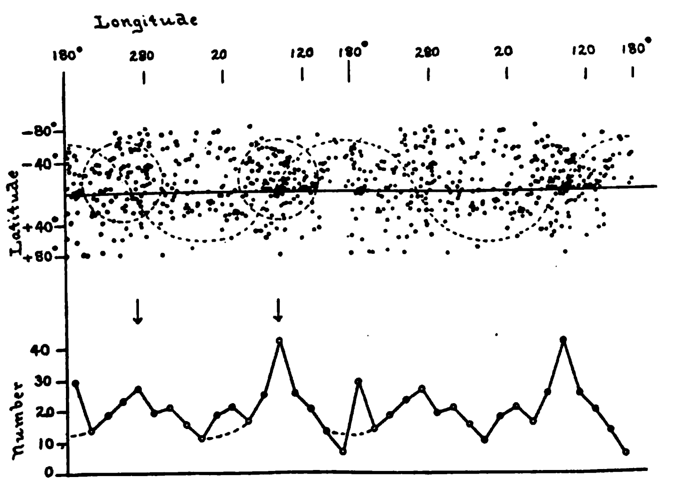


FIG. 4. APHELIA OF 364 PARABOLIC AND LONG-PERIOD COMETS. B.C. 371 TO A.D. 1914. FREQUENCY CURVE IN LONGITUDE (LOWER)

that point. The center of preference in direction of the aphelia of these bodies is, therefore, the intersection of the ecliptic and galactic planes.

3. The aphelia of the minor planets show a consistent and strong preference for longitudes in the region of 195°. The curve of preference

is closely a sine curve with the maximum at 195° and the minimum at 15°. There are indications of variations of a shorter period superposed on this curve which can be explained by more or less harmonic gaps at certain longitudes.

4. The aphelia of the 45 short-period comets show a preference for essentially the same longitude as do the minor planets, viz., 195°, the minimum occurring also at 15°. Considerable irregularities exist in this curve, due largely to the small amount of data. Some of the irregularities, however, appear to exist in the curves of parabolic comets also. The general form of the curve is none the less well marked.

5. The aphelia of the parabolic (and very long-period) comets show a preference for a region near the intersection of the ecliptic with the galaxy which is nearest to the longitude of the antapex of solar motion (90°). A more widely scattered and less pronounced preference is shown for the region of the longitude of the apex of solar motion (270°). There appears to be a tendency to avoid the intermediate galactic regions, between these two regions of preference and also the galactic polar regions.

6. Although the aphelia of the parabolic comets are widely distributed over the sphere, there is a small preference for the ecliptic plane. The deficiency of aphelia is most marked in the region of the northern pole of the ecliptic ($\alpha = 18^h$, $\delta = +67°$).

7. There is a deficiency of small inclinations (to the ecliptic) among the parabolic comets which is nearly normalized if the short-period comets are included. This indicates that these short-period comets have been captured out of the general herd and that their inclinations have not been radically changed since capture.

The distribution of the aphelia of the parabolic comets is what might be expected if they have been captured from bodies which are more plentiful in the galactic regions than elsewhere.

On the theory of probability only a very small proportion out of a mass moving with all directions and speeds would come close enough to be observable from the earth, except such as were moving with nearly the same speed as the Sun when they came within the distance of effective gravitational action. All of such, except those changed into small ellipses by planetary perturbation, will have sensibly parabolic motion. Preponderance of aphelia in the southern ecliptic hemisphere as well as the southerly ecliptic deviation of the maxima near the longitudes of the solar apex and antapex, indicate some sort of preferential motion of these bodies with respect to the Sun before

capture. The general preference of the aphelia of these comets for the longitudes of the solar apex and antapex can scarcely be a relation other than to the sun's motion among the stars. The greater condensation and preference for the longitude of the antapex appears to be real. Such a condition might perhaps follow if the cometary bodies were moving in general more rapidly than the Sun.

To summarize, the short-period comets show a preference of their aphelia for the same ecliptic longitudes as do the asteroids, but radically different from the distribution of the parabolic comets and those of very long period. As the short-period comets are conceded to be captures, has a similar preference of aphelia in the case of the asteroids also resulted from capture or is it due to some other cause? It is conceivable that such a preference might result from gravitational effects of the stellar system. If so why do the major planets show a different preference? The longitude of Jupiter's aphelion is at present approximately 192°, sensibly that of the preferences of the minor planets and short-period comets. Is there any connection? The moderate eccentricity of Jupiter's present orbit does not tend to strengthen a belief in such a relation.

It seems rather more than coincidence that the aphelia of all of the major planets are confined to one half of the sky and that the center of preference coincides so closely with the longitude of the sun's direction of motion in space, notwithstanding the observed revolutions of their lines of apsides, which differ greatly among themselves. Can it be that there are compensations or that our one or two centuries of (accurate) observations are insufficient to disclose gravitational or other effects of the stellar system? or does the mean of the planetary longitudes follow a changing direction of solar motion?

It is perhaps possible to believe that the observed preference of the major planets is coincidence, but not so the preferences of the minor planets and the comets. They are too numerous and their preferences too consistent. These unquestionably indicate a common underlying cause.

The parabolic comets show a very sharp maximum of aphelia at approximately 190° (fig. 4) preceding the one at 270° in some such way as the one at 20° precedes the strongest maximum of all at 90°. A division of the data into two groups shows the same peculiarities, indicating that it is real. This sharp maximum at 190° is perhaps significant, in connection with the similar preference of the minor planets and short-period comets for the same region.

This investigation has suggested a possible physical cause for the position of the ecliptic plane which, so far as I am aware, has not been satisfactorily accounted for. It is purely speculative and I know of no test which would be adequate, particularly as it rests upon the theory of capture which is far from being established. This explanation was suggested by the fact that the ecliptic intersects the galaxy not far from the extremities of the axis of preferential motion. There is some evidence that the two star streams may be in reality parts of a *curved* stream, the so-called axis of preferential motion being in reality a tangent to that curve. In my opinion such a conception best satisfies the observed conditions. Radial velocities indicate maxima to the *south* of both extremities of the so-called ellipsoidal axis. This brings the intersection near 18^h in the general region of what appears to be one star stream.

The centers of preference of the parabolic comets near the longitudes of the apex and antapex of solar motion are also both some 10° or 15° south of the ecliptic. If now the planets were captured, it is not difficult to outline a cause for the position of the ecliptic. Bodies moving with a stellar stream (not to be more definite at present) might be picked up by the Sun and if acted upon by a resisting medium, have their orbits rendered nearly circular. By mutual perturbation their orbits would tend to assume a common or 'invariable plane.' The addition of the masses of the satellites, comets and asteroids would not greatly disturb this plane, once established. This accounts for only one coordinate of the plane. The other we must suppose to be the mean of the group and as far as we know, at present, accidental.

This hypothesis is admittedly speculative and further discussion at this time does not seem warranted.

DISCOIDAL STRUCTURE OF THE LITHOSPHERE[1]

By Bailey Willis

Department of Geology, Leland Stanford University

Communicated by J. M. Clarke, July 15, 1919

Underlying the thesis here presented are the following postulates:

a. The lithosphere is heterogeneous as to density;

b. The lighter and heavier masses are of notable dimensions, varying from a hundred to several thousand kilometers in diameter;

c. Both lighter and heavier masses are distributed throughout continental areas and oceanic basins, but the lighter masses prevail in the continents and the heavier masses beneath the oceans;

d. The prevailing structure of deep-seated rocks is a foliated one and is the result of crystallization in response to appropriate conditions of stress and temperature;

e. Stresses competent to orient foliation in the elastic earth are set up by the effort of lighter and heavier masses to assume positions of isostatic equilibrium. These stresses are general, but they are localized and intensified by unloading and loading of adjacent areas through the process of erosion.

f. Periodic rises of temperature, local recrystallization, and local fusion of moderately deep-seated rocks result from the outward flow of the internal heat of the earth.

g. The strength of rocks increases with increasing pressure, as shown by Adams, but not as rapidly as the load increases from the surface of the earth downward. There is, therefore, a level at which the load equals the strength of any rock. Below that level for the strongest rock the earth is a potentially failing structure, which will fail by recrystallizing or by shearing under a differential stress which equals the critical load.

h. High temperature and fusion are potent factors in producing failure of the rigid lithosphere under its own weight.

It is obvious that these postulates rest upon the work of many investigators, among whom are Hayford and Bowie on gravitation, Dutton, Gilbert, and Barrell on the strength of the lithosphere, Van Hise, Leith, Johnson, and Bowen on schistosity, and Chamberlin on the origin of the earth and its internal heat. The contribution which this paper seeks to make is one toward an explanation of the mechanics of the lithosphere. Its conclusions, if valid, bear upon the growth of mountains, the deformation of superficial rocks, the distribution of volcanoes, the connections between continents, and the permanence of ocean basins.

The argument which leads to the recognition of the discoidal structure of the lithosphere proceeds as follows:

From an early stage of the growth of the earth onward the lighter and heavier masses of the elastic lithosphere have tended toward an equilibrium which may be described as an equilibrium in isostatic adjustment. It is conceivable that the departure from equilibrium may at some time have been such that the stresses set up in the lithosphere were adequate to cause flow and mass movement, until a close approach

to complete isostatic equilibrium was reached. But the present departure from equilibrium is not of such a magnitude, nor is it likely that there has been so great a departure during any stage of geologic time known to us. The stresses due to differences in weight of adjacent masses of lighter and denser material, even when they are intensified by erosion of broad surfaces to a peneplain and by correlative loading of basin margins, are regarded as quite incompetent to overcome the strength of underlying rock masses, whose rigidity increases with load.

Nevertheless, the effort toward isostatic equilibrium produces non-uniform stresses which are long enduring and persistent in direction. In any mass which was lighter than its environment the direction of least stress would be vertically upward and the directions of greater stresses would be horizontal. Under any mass which was heavier than its environment the direction of greatest stress would be vertical and the directions of least stresses would be horizontal. If the lighter and heavier masses be in elastic continuity with one another, the upward stress in the former and the horizontal stress under the latter would combine to form a resultant which would rise in a curve from below upward. Thus the zone embracing the adjacent parts of both columns would be subject to a curved stress, which would proceed outward and upward from under the heavier mass. Stresses of this character appear to be a necessary result of a lack of equilibrium among the masses or elements of the elastic lithosphere.

The law of crystalline orientation in response to stress, as developed by Van Hise, Leith, and Bowen, requires that foliation shall grow in the direction of least stress. If, then, rocks crystallize or recrystallize, through the influence of rising temperature and altering solutions, in adjacent masses which are pervaded by the curved stress due to isostatic inequilibrium, the resulting foliation should be oriented in a curve corresponding to that stress. The foliation should be horizontal under the heavier element, should approach verticality under the lighter element, and should curve upward continuously from under the former toward the surface of the latter.

The quantitative value of the least stress which will orient foliation under the exceedingly complex conditions of metamorphism has not been determined. It must not be confused with the differential stress required to produce mechanical flowage. Adams has demonstrated the latter to be very great. The former is a stress which is within the elastic limit and may safely be regarded as relatively very small.

More important than amount of stress are the characteristics of non-uniformity, as distinguished from hydrostatic pressure, persistence

in direction, and continuity of action during prolonged periods. These conditions are regarded as giving effectiveness to the stresses propagated in the elastic, rigid rocks far beneath the surface by imperfections of isostatic equilibrium.

The postulated directive stresses tending to orient foliation in curved surfaces have coexisted with conditions favorable to recrystallization since an early stage in the growth of the earth, under the planitesimal hypothesis. Under that hypothesis any resulting structure and its effects were characteristic of the growing earth and still pervade its mass from near the surface to the depth to which crystalline textures endure. Also under any other hypothesis of the earth's past history, which recognizes the effects of heterogeneity as to density and the periodicity of dynamo-metamorphism, structures involving curved foliation should result from isostatic stresses.

Upon the preceding reasoning is based the postulate of a discoidal structure of the lithosphere.

The condition for the development of curved surfaces of foliation exists wherever isostatic equilibrium is imperfect. It is obviously most effective where erosion and sedimentation are active. It is not limited to any particular section of the border zone of any lighter or heavier mass which is not in equilibrium, but extends around each such heavier mass and invades the area of each such lighter mass. Since the curves of strain and the resultant foliation dip under the heavier masses from all sides, those masses have a discoidal structure. They may be designated as discs. They are regarded as composed of a series of foliae, which are convex downward and which make up the mass of the disc to a depth of several hundred kilometers. Their outcropping margins will include wider or narrower zones of the lighter structural masses. Between the discs may lie masses which, by reason of the fact that they do not pertain to the heavier disc-shaped masses, belong to the lighter masses. These may be called interdiscs. Their cross section in a vertical plane would present a more or less broadly truncated triangle, whose truncated upper surface would vary from sub-continental to merely isthmian widths.

Thus the lithosphere would superficially exhibit three structural types: (1) the surfaces of the heavier discs, depressed by gravitative adjustment and more or less deeply covered with water or continental sediments. The dominant characteristic of the underlying masses is high density. It is a permanent characteristic and its effects are permanent and continuous. They should not be confused with the temporary effects of orogenic movements, which may also result in super-

ficial depressions; (2) the surfaces of the lighter interdiscs, riding high and covered with the sediments of epicontinental seas, or exhibiting an approximately vertical foliation where denuded to metamorphic rocks; (3) the marginal zones of discs and interdiscs, characterized by extrusion of igneous rocks and orogenic disturbances. Where a broad light mass or structural element lies adjacent to a broad heavy element, as along the Pacific Coast of North America, the zone of marginal deformation may be very wide. Where, on the contrary, two heavy elements are nearly tangent, the zone of disturbance may be exceedingly narrow, as the Isthmus of Panama.

The hypothesis of discoidal structure may best be tested by the facts of the earth's major features and their relations to igneous intrusion and extrusion, to regional, periodic diastrophism, and to orogeny. Certain lines of investigation are suggested in what follows.

Are igneous rocks distributed around deeps, as would be required by the discoidal hypothesis?—Let it be assumed that magmas originate, as postulated by Chamberlin, through local fusion in the outer few hundred kilometers of the lithosphere. Molten material gathering in the foliae beneath a disc would be guided by the foliation toward the surface and would appear in or near the outcrop of the margin of the disc. By postulate any disc is indicated on the surface by an oceanic deep or a continental basin. The relation of volcanoes and also of geologically recent batholiths to oceanic margins has long been recognized. As bathymetric observations accumulate a more detailed relation to individual deeps is noticeable. The most conspicuous examples are the Windward Islands, sometimes called the "Necklace of the Caribbean," and the Alaskan, Aleutian, and Japanese volcanic chain.

Does diastrophism exhibit that periodicity with reference to the larger deeps which we should expect if each deep constitutes a structural unit?—It is well established that the opposite coasts of the North Atlantic have periodically suffered similar and contemporaneous deformation, and the same is true of the encircling lands around the Pacific Ocean.

Have the diastrophic activities of separate great deeps been non-contemporaneous to the extent which would give to each an independent character peculiar to itself?—That the diastrophic activities of the Atlantic and Pacific basins are incommensurate on the geologic time scale is a fact which constitutes the gravest difficulty in the assumption of universal periodicity. If diastrophism be due to an ultimate cause, operating periodically upon the inner mass of the earth, such as the accumulation of internal heat, an explanation of the diverse periodicity of different oceanic basins must be sought through the mechanism

which registers the effects at the surface more quickly in one region or more slowly in another, as the case may be.

Do the mechanics of deformation of the lithosphere, as expressed in the effects of compression, conform to the structures which should follow from the postulated discoidal structure?—Under general tangential stress a spherical shell composed of discs and interdiscs should shorten by riding up of the margins of the discs upon the interdiscs. If the disc were competent to sustain itself by partial bridging, its whole mass would rise; if, as must generally if not always have been the case in the lithosphere, the disc be too wide and too weak to support itself, the margins only would be raised, the mass bending or fracturing as conditions demanded. The marginal zones should exhibit over-thrusts from the basin side and incidental phenomena of folding with minor over- and under thrusting. The facts of mountain structure appear to conform to these requirements. The folded and overthrust structures of the Appalachians and the faulted and upthrust structures of the Pacific ranges of California appear to find an intelligible explanation on the basis of the discoidal theory.

Do the trend lines of mountain chains conform in plan to the postulated discoidal structure?—To answer this question in detail will require an analysis of the earth's surface into discs and interdiscs, but the trend lines of the mountain chains of Europe, Asia, and the two Americas, as developed by Suess, do conform in plan to the obvious depressions within and around those continents. It must be recognized, however, that a long narrow depression filled with sediment may be so compressed and displaced as to become part of the continental interdisc. This is a well known geological occurrence, the transformation of a geosyncline into a mountain range, but it cannot in every such case be assumed that the geosyncline corresponded to a dense underbody. Other mechanical conditions may produce subsidence. With this reservation, however, I believe that the directrices of mountain chains can be shown to wind around discoidal masses.

Does the discoidal theory suggest any solution for the temporary intercontinental land connections which are demonstrated by biologic evidence?—The present isthmian connection between North and South America may be regarded as a typical case of intercontinental bond established by marginal uplift and extrusion of igneous rocks between the Caribbean and eastern Pacific discs. Similar connections may readily be traced between South America and Africa by two different routes. A connection between North America and Europe, via Greenland, Iceland, and England, such as has often been suggested, finds a rational

explanation through its relation to the deeps of the North Atlantic. Oceanica may be intelligently linked up and explained.

If the establishment of isthmian connections be explicable according to the discoidal theory, can their subsequent submergence also be accounted for?—According to the discoidal theory, the source of very large masses of igneous rocks lies in the base of the discs. The extrusion of the melt is accompanied by subsidence of the central portion of the disc, which in turn and in time may be succeeded by slipping down of the elevated margins. The mechanism appears to be workable. It requires, however, fuller elaboration than can here be given to elucidate it. These concepts are obviously consistent with the permanency of oceanic basins and continental masses.

In addition to the tests proposed by these general questions, the theory should be tried by the more exacting requirements of the mechanics of deformation. This has been attempted and not without reasonably satisfactory results. The structure of the Pacific ranges of California has been discussed before the Geological Society of America and the LeConte Club of California, and the paper will shortly be offered for publication.

[1] Abstract of a paper presented to the Geological Society of America at the Pasadena Meeting of the Cordilleran Section.

RECENT SIMULTANEOUS MEASUREMENTS OF THE SOLAR CONSTANT OF RADIATION AT MOUNT WILSON, CALIFORNIA, AND CALAMA, CHILE

By C. G. Abbot

Smithsonian Institution, Washington

Read before the Academy, April 29, 1919

For the past fourteen years, with the exception of the year 1907, the Smithsonian Astrophysical Observatory has observed the solar radiation at its station on Mount Wilson, California, altitude 5700 feet. Measurements are made by means of the pyrheliometer and spectro-bolometer at different hours of the day, so chosen that the intensity of the solar radiation as it would be outside the atmosphere at mean solar distance may be computed therefrom. During the earlier years of the investigation it appeared that the values found vary from day to day, as well as from year to year. In other words, it seemed to be indicated that the sun is a variable star, having a two-fold variation. First, a varia-

tion of long period, having a range of from 3% to 5% during the period of the sunspot cycle. Second, a short interval irregular variation running its course in intervals of a few days, weeks or months, and having a range often as great as 3%, sometimes as great as 7% and even in exceptional cases of 10%.

In the years 1911 and 1912, the reality of this supposed short irregular variability of the sun was tested by observing simultaneously at Mount Wilson and at Bassour, in Algeria, a station about 50 miles south of Algiers, situated at an elevation of about 3600 feet. The results of the two stations on the whole supported one another and indicated that the variations were due to causes outside the earth's atmosphere.

In the year 1918, the Smithsonian Institution sent an expedition, at the cost of the Hodgkins Fund, to Calama, Chile, to obtain during a term of years, daily measurements as far as possible of the intensity of the solar radiation. The station is situated about 150 miles northeast of Antofagasta at an elevation of 7500 feet, in one of the most cloudless regions of the world, where rainfall is almost unknown. The expedition is in charge of Mr. A. F. Moore, Director, assisted by Mr. L. H. Abbot. The observers began actual observing on July 27, and in the six months next following had observed on about 70% of the days.

During a part of this interval measurements were made on Mount Wilson by my colleague, Mr. Aldrich, and the computations have now reached such a stage of advancement that we are beginning to be able to tell that the two stations mutually support one another as to the short irregular periodicity of the sun.

In order to make a proper comparison it is evidently not necessary that the days observed should be consecutive but only that satisfactory observations should be obtained on each day of the comparison at both stations. In order to exhibit the dependence in the clearest possible manner, we may indicate Calama values by abscissae or horizontal distances, and Mount Wilson values by ordinates or vertical distances. If the measurements were without error and there was no variation of the sun, evidently all results should fall upon the same point, whether observed at one station or the other. But if there was a true variation of the sun, the point would be stretched out into a line at 45 degrees to the axes. As all measurements are subject to error it is not to be hoped that the several points corresponding to the several days of observation will all be found strictly upon such a line, but they ought to arrange themselves about it in such a way that the line will give the best mean representation.

This condition is well fulfilled. A change of 1% or less would suffice to bring almost every point on the line. Accordingly we may conclude that the spectro-bolometric determinations of the solar constant of radiation made at Mount Wilson and Calama mutually support one another and indicate a variability of the solar radiation. Since the two stations are situated in different hemispheres of the earth, separated by a distance of upwards of 5000 miles, it must be conceded that the result strongly confirms our view of the short period variability of the sun.

Investigations have already been made by Dr. Clayton and others on the correlation of terrestrial temperatures and pressures with these supposed changes in the sun, and affirmative results have been obtained by them to the question: Do the variations in the sun noticeably affect the terrestrial weather conditions? Dr. Clayton is so strongly convinced of the dependence between the weather and the solar radiation that at his desire the Argentine Government is obtaining daily telegraphic solar radiation reports from the station at Calama on which are based forecasts of the temperature in Argentina. Hitherto the correspondence has been very good, so that Dr. Clayton is very sanguine that a great improvement in forecasting will result from these solar radiation observations. The Brazilian Government has also, at his recommendation, begun comparisons.

If the further studies of the correlations between terrestrial temperatures and solar variations yield similar affirmative results, it will probably be necessary within a year or two to undertake the equipment of several additional solar radiation observing stations in the most cloudless regions of the earth. There should be not less than three additional solar radiation stations, and the cost of conducting them and the station at Calama would absorb an income of from thirty to forty thousand dollars per annum. It is to be hoped that funds for this purpose may become available to the Smithsonian Institution, for with the experience gained by the staff of the Institution it would be able to carry on the research in a way to obtain homogeneous and accurate results.

The choice of sites for additional solar radiation stations involves investigation as to the most cloudless regions of the earth. None of the stations hitherto occupied, not even that at Calama, is as satisfactory as we could wish for. The most serious enemy to the research is the presence of cirrus clouds, or any other form of cloudiness, and it is a grave problem where in the world these are least numerous. The comparison of the observations at the two stations in 1918 shows that no inconsiderable number of the days in which observations were made were unsuitable for the purpose on account of the changes of the atmos-

pheric transparency during the two or three hours necessary to the determination. It is this defect of the uniformity of the atmospheric transparency which requires us to make the measurements at so many stations in order to get a satisfactory result.

Both the Mount Wilson and Calama observations indicate that the solar radiation was above its normal in the year 1918, having an average value of about 1.95 calories per square centimeter per minute. The mean value for many years was 1.93.

ROTATING PROJECTILES FROM SMOOTH-BORE GUNS

By C. G. Abbot

Smithsonian Institution, Washington

Read before the Academy, April 29, 1919

In the late war much use was made of trench mortars. This kind of ordnance consists essentially of a smooth tube with a firing pin at the bottom. The projectile carries a shotgun shell at the rear. It is dropped down the smooth-bore barrel by the soldier and the firing pin explodes the primer on the shotgun shell. This in its turn explodes the charge of propellant which throws the shell into the enemy's lines. Many of the trench mortar shells tumbled end over end as they went over, but some were provided with fins which retained them in approximately steady flight. However the trajectory of these shells was by no means ideal.

It occurred to me to try to secure sufficient rotation to produce steady flight by means of the turbine principle applied to the shells. Unfortunately the research was proposed to me only a few days before the armistice was declared so that by the time preliminary tests were made it was too late for the device to be of war service. However, the results appeared to be so promising that I have made numerous further experiments with a smooth-bore musket of 0.9 inch diameter which had seen service in the Civil War.

The elongated ojival projectile was made in two parts in my experiments, the one a tough steel rear part chambered out to contain the propellant, the other an aluminum nose provided with a steel plug at the rear for screwing into the steel part of the projectile. The steel plug serves the double purpose of closing the chamber and attaching it to the front part. With the aluminium part in front such projectiles are particularly well calculated for tumbling, and invariably did so when fired in the usual manner. At the extreme rear of the steel part of the

projectile a plurality of bell-shaped bores were provided, leading tangentially from the chamber within the projectile to the rear where they emerged into the barrel of the gun. On screwing the aluminum nose with its steel boss into the opening of the chamber the whole presented the appearance of an ordinary elongated projectile except for the bell-mouthed bores visible at the rear. When the gun is fired the propellant within the chamber is ignited by priming contained in the tangential apertures.

The outflow of the gases through the bell-mouthed bores, while they would give some rotation, owing to their friction within the chamber, would fail to give sufficient rotation if it were not that steel pins are inserted across the chamber to form a sort of baffle, so that the outflowing gases may react against this baffle and so tend more strongly to rotate the projectile.

It was feared that the combustion would not be completed, or at least that the gases would still remain in the chamber at high condensation at the time when the projectile left the muzzle of the gun. If this were the case it might easily be that owing to some slightly unsymmetrical construction of the bell-mouthed bores some tendency to deflect the flight of the projectile would be caused thereby by the delayed outrush of the residue of gases. In order to avoid this, the end of the barrel was continued by a tube extending some 10 inches beyond the former muzzle and at the rear of the tube where it joined the barrel there were drilled a number of holes so that the gases might escape therefrom, and relieve the pressure within the chamber of the projectile while still it was subject to guidance from the lengthened barrel of the gun.

Very satisfactory results have been obtained in firing these special projectiles. This method of securing rotation of projectiles appears to be suitable not only for trench warfare but for all varieties of ordnance larger than 1 inch in diameter. Rotations as great as one rotation in 18 calibers have been obtained thereby. It is customary to employ about one rotation in 30 calibers in the ordnance of Europe. The United States employs one rotation in 25 calibers for a large number of its heavy guns. My experiments seem to show that one rotation in 40 or 50 calibers is sufficient to prevent any appreciable deflection of the shells in flight.

The experiments have been conducted within the enclosure of the Astrophysical Observatory, so that it was impracticable to employ heavy charges so as to get very high speeds of flight, but within the range of the experiments I have found by shooting through a succession of paper screens and observing the holes with a theodolite that within the length

of the observatory enclosure there is no measurable lateral deflection of the projectile while in flight. Truly round holes are always left by the rotating projectiles. Very striking results were obtained in an experiment made in Virginia in the presence of a number of Ordnance Officers. Two successive shots were fired at a target in one of which the projectile was made to rotate by means of the included charge and in the other of which the projectile was fired by powder outside of itself. In the one an excellent hit was made, leaving a true round hole, while in the other it happened that in the tumbling of the shell in the air it reached the target exactly broadside. The officers were naturally much impressed by this striking exhibit.

As in most of the investigations we undertake, the same general idea had long before occurred to others. Patents for somewhat similar devices were granted as long ago as the Civil War. So far as I am aware, however, no one hitherto has attained so good a measure of success in applying the turbine principle to projectiles.

MEANS OF MEASURING THE SPEED OF PROJECTILES IN FLIGHT

By C. G. Abbot

Smithsonian Institution, Washington

Read before the Academy, April 29, 1919

In the course of the experiments on rotating projectiles from smooth bore guns I desired to measure the speed of flight in some instances, and as I lacked the usual chronographic apparatus employed by artillerists the following substitute occurred to me and proved very satisfactory after a few trials.

The projectile was fired obliquely across a horizontal beam of light reflected into the observatory from the siderostat. The solar beam was entirely cut off by a diaphragm at the wall of the observatory, except as it passed through two round holes about 6 inches apart. The shot was fired through the left hand one of these holes. About 2 meters inside of the observatory another smaller diaphragm with a small aperture in it was placed opposite to the second or right hand hole. The direction of the gun was so arranged that the shot passed through the hole in this second diaphragm also. About a meter beyond the second diaphragm was placed a double tinfoil screen, the two tinfoils being separated by a sheet of cardboard and connected by wires respectively to a circuit con-

taining an electromagnet which operated a shutter for the purpose which I am about to describe. Opaque cards were placed to cover the left hand hole in the first diaphragm through which the shot was fired and to cover the hole in the second diaphragm through which the shot passed after having accomplished two meters more of its flight. The passage of the shot through the tinfoils beyond closed the electric circuit and operated the shutter which has been mentioned.

The beam from the left hand hole of the first diaphragm passed to a mirror at 45° which reflected it at right angles upon a photographic plate caused to rotate in its own plane by means of an electric motor. The beam from the right hand hole of the first diaphragm, after passing through the second diaphragm was also reflected by the same mirror upon the same photographic plate but diametrically opposite to the first beam with respect to the center about which the plate rotated in its own plane. Between the mirror and the photographic plate was a long slit which restricted the two beams of light to mere streaks extending respectively from the center to the right hand and to the left hand of the photographic plate. Just above the long slit was placed the shutter controlled by the electromagnet.

It will readily be seen that when the gun was fired, first a beam from the left hand hole in the first diaphragm passed to the mirror and was reflected upon the rotating plate. After the projectile had covered the two meters leading to the second diaphragm, the other hole was opened and the second beam began to print upon the other half of the photographic plate. Immediately thereafter the electric circuit was closed, the shutter consequently closed, and so the photographic action stopped before the plate had made a half revolution. The delay in the electric action had no influence on the work provided the shutter closed before the plate made a half rotation. After development the blackened parts of the photographic plate were found to indicate a certain angle through which it had rotated between the time when the projectile opened the first aperture and the time when it opened the second aperture. Having determined the speed of the photographic plate by measuring the speed of the electric motor, the velocity of the projectile became known.

It is easy by this means to determine with some accuracy very great velocities of translation within a very short path. Even one meter path would be quite sufficient.

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 5 OCTOBER 15, 1919 Number 10

SOME PROBLEMS OF SIDEREAL ASTRONOMY*

By Henry Norris Russell

Bulletin of the National Research Council, Number 2

Communicated to the National Academy of Sciences, June 17, 1917

The main object of astronomy, as of all science, is not the collection of facts, but the development, on the basis of collected facts, of satisfactory theories regarding the nature, mutual relations, and probable history and evolution of the objects of study. Before the existing data appear sufficient to justify the attempt to form such a general theory, two policies of investigation may be followed: (1) to collect masses of information, as accurate and extensive as possible, by well tested routine methods, and leave it to the insight of some fortunate and future investigator to derive from the accumulated facts the information which they contain regarding the general problems of the science; (2) to keep these greater problems continually in mind, and to plan the program of observation in such a way as to secure as soon as practicable data which bear directly upon definite phases of these problems.

Much valuable and self-sacrificing work has been done by astronomers who adopted the former policy. In the opinion of many investigators, however, the progress of astronomy would be hastened if fuller consideration were given to the second method of attack, especially with a view to the widest possible coöperation between different observers and institutions.

In the hope that the committees of the National Research Council may be of service in furthering such coöperation, and at the request of the Chairman of the Council, the following survey has been attempted of the general problems of sidereal astronomy, and of investigations which at present promise advances toward their solution.

* This is issued as the first of a series of research surveys prepared under the auspices of the National Research Council.

I. The Individual Stars.—Existing methods of investigation have already put at our disposal a great mass of information regarding the physical characteristics of the stars—mass, density, luminosity, color, spectrum, temperature, and so on. The central problem of stellar astronomy may be formulated as follows: From the existing data, and from all further data which may be secured by methods new or old, to deduce a theory of stellar evolution, that is, of the changes in the temperature, density, brightness, spectrum, and other observable characteristics of a star with the progress of time, and of the dependence of these changes upon those factors which are invariant for a given system, such as mass, angular momentum and chemical composition. Such a theory must satisfactorily represent the observed properties of the general run of the stars, and the relative abundance of the different types, and must be capable of extension so as to account for the exceptions to the usual rules.

Among the subsidiary problems whose solution is bound up with that of the main problem are (*a*) that of the evolution of binary systems, whether by fission, tidal action, or otherwise; (*b*) that of the causes and mechanism of variable brightness among the stars; (*c*) that of the source of the energy which the stars radiate into space in such enormous amounts.

These problems of stellar astronomy are mainly physical in character, though some phases, such as (*a*) are mainly dynamical.

II. The Galactic System.—The great majority, if not all, of the visible stars appear to belong to an assemblage limited in space, either by regions nearly void of stars, or by absorbing material which conceals whatever may be immersed in it. Within this galactic system we may investigate the distribution of the stars in space, and its variation for stars of different spectral type, absolute magnitude, etc., the motions of the stars (including the Sun), and the phenomena of preferential motion ('star-streaming') in certain directions, and the dependence of these motions upon spectral type, absolute magnitude, etc.; and the association of the stars into sub-groups, or clusters, and the motions of these clusters.

All these studies lead up to a single ultimate problem, which may be defined as the representation of the present positions and motions of the stars as a stage in the history of a dynamical system (whether in a steady state or not) and the deduction of the presumable history of the system in the past and the future. Among the subsidiary problems connected with this are (*a*) the existence, character, distribution and gravitational influence of possible dark or absorbing matter in space; (*b*) the

relation between the age or evolutionary stage of a star and its position and motion within the galactic systems. The latter connects the problems of stellar and galactic evolution in such a way that any notable advance in the solution of one is likely to be of aid in that of the other, while an unfounded assumption regarding either will probably confuse the discussion of both.

III. Clusters and Nebulae.—So little has been known of these objects until very recently that the problems which they present can hardly yet be coördinated into a single statement. Among the most obvious are:

(1) The relations of clusters and nebulae to the galactic system. It now appears probable that the galactic system is very much larger than was supposed a few years ago, and that not only the irregular clusters, and the gaseous nebulae, both planetary and extended, but also the globular clusters, and probably the Magellanic Clouds, lie within its confines. But the relations of the spiral nebulae are still uncertain.

(2) Motions and dynamical relations within clusters, especially globular clusters, and explanation of the law of distribution of stars in such clusters.

(3) Nature of the gaseous nebulae, especially of 'nebulium,' and cause of their luminosity. Internal motions in gaseous nebulae.

(4) Nature of spiral nebulae, and explanation of the rapid motions of their parts.

In all these cases a persistent attempt should be made to account for the observed phenomena by means of the known properties of matter and forces of nature, and the existence of unknown forces should be postulated only if there is apparently no escape from the necessity of doing so.

It may now be profitable to survey rapidly the different fields of astronomical investigation, and consider the bearing of various researches —some now under way, some practicable at present, and others desirable if means for effecting them can be devised—upon these general problems.

1. Spectra.—It seems to be increasingly clear that the master-key to these problems, so far as they have yet been formulated, lies in the investigation of the spectra of the stars and other bodies, and the correlation of their other characteristics with the spectra. Fortunately, the spectra are among the few characteristics which can be investigated independently of any knowledge of the distances of the various bodies,— and, indeed, of the distances themselves, except for the limitation arising from the faintness of most of the remoter objects.

(*a*) Two fundamental facts appear upon the study of the lines of stellar spectra. The first is that almost all of the thousands of lines which have been observed are identifiable as those of known elements, and can be reproduced under conditions which can be realized in terrestrial laboratories. The few outstanding exceptions are yielding year by year. The recent identification of the G band in the solar spectrum as due to hydrocarbons,[1] and of the bands of ammonia[2] and water-vapor[3] in the ultra-violet, leaves very few 'unknown' solar lines of any importance. Nor are there any of great account in stellar spectra, except in stars of the fourth type (Class N) and in the Wolf-Rayet and 'early' helium stars.

So many of the lines in the latter have recently been found to be identical with those given in the laboratory by familiar elements (such as hydrogen, oxygen, carbon, and helium), under unusually intense electrical excitation[4] that there is good reason to hope that further researches in this direction may account for those which still remain, and even solve the long-standing riddle of the origin of the characteristic nebular lines (which are associated with the Wolf-Rayet lines in the nuclei of planetary nebulae and in new stars at certain stages). The spectrum of the solar corona, however, still remains an isolated problem.

(*b*) The second great fact is that the vast majority of stellar spectra fall into a single, continuous, linear sequence, which forms the basis of the Harvard system of classification, now generally adopted. Almost all the spectra which did not obviously belong to this sequence have been brought into connection with it by the recent work of Wright,[5] connecting the gaseous nebulae with the Wolf-Rayet stars at the head of the series, and that of Curtiss and Rufus,[6] which shows that the small but definite classes R and N form a sort of side-chain, branching from the main sequence near the other end, at class G (or perhaps K). Miss Cannon's experience[7] in classifying over 200,000 spectra shows that the objects that do not fall into the sequence, thus extended, are almost vanishingly rare.

The general characteristics of this sequence are now well established, and the types which were selected, by a sort of survival of the fittest, in the evolution of the Harvard classification prove to have been surprisingly well distributed along the series. With the aid of the quantitative methods of classification developed by Adams and Kohlschütter,[8] the precise classification of any spectrum of which a good photograph with suitable dispersion is available should be an easy

matter, even in the interval between G and K5, where the differences between consecutive types are least prominent. The publication of a detailed descriptive 'key' with good reproductions of photographs of spectra of each successive class would however be a great boon to isolated workers.

Of much greater importance is the devising of some method for photographing the spectra of stars fainter than the tenth magnitude—which are now about at the limit of accessibility. Long exposures with the objective prism are greatly embarrassed by difficulties in guiding, but the problem is doubtless soluble in some way, and ought to be solved.

(*c*) There is now good reason to believe that the differences between the main classes of spectra arise from differences in the effective surface temperatures of the stars, and that differences in their other physical characteristics play only a minor rôle in the spectra, but reveal themselves in differences in detail, formerly described as 'peculiarities' when they were noticed at all. The investigation of these finer differences is to-day the most promising field in stellar spectroscopy.

What valuable results may be obtained was shown by Hertzsprung's[9] work on Miss Maury's 'c-stars' (with unusually sharp spectral lines) which prove to be of greater real brightness than any other class of stars so far known; and later, and still more remarkably, by Adams' and Kohlschütter's[10] discovery that the absolute magnitudes of stars (of the 'later' spectral classes, at least) can be predicted with surprising accuracy from the relative intensity of a few pairs of lines in their spectra. The data for stars of great luminosity are still scanty, but should be easily obtainable, using the hundreds of spectrograms now available at the great observatories, and determining the mean absolute magnitude of groups of stars, which the spectroscopic method indicates as being of similar brightness, by means of their parallactic motions. When this has been done, our knowledge of the distribution of the naked-eye stars in space will be very greatly advanced.

The careful comparison of the spectra of pairs of stars otherwise similar, but known to differ in other characteristics than absolute magnitude, may yield results of importance. Many recognizable spectral 'peculiarities' too, such as the diffuseness or sharpness of the lines, the presence of bright lines, the abnormal strength or weakness of certain lines, etc., have as yet been very incompletely studied, especially as regards their relation to other characteristics of the stars. For example, it should be possible to distinguish between widening of spectral lines due to a star's rotation, (which would affect all lines alike), and

widening due to physical conditions in its atmosphere (which are likely to affect some lines more than others).

(*d*) Another promising field is found among the reddest stars. Curtiss makes the very interesting suggestion that the division of the spectral series into the branches G–K–M and G–R–N (or perhaps K–R–N) may be due to differences of chemical composition[11]—since it is known that the surface temperatures of these stars are low enough to permit the formation of chemical compounds. If this is true, the strength of the characteristic bands of titanium oxide or of carbon should depend upon the relative proportions of these elements, and show little correlation with the color index, or the extension of the spectrum in the violet, which depend primarily on the temperatures. There is already considerable evidence that this is actually the case, and it may be remarked that the star Epsilon Geminorum, which is of spectral class G5 has a color index (+1.52) almost equal to that of Classes M or R.[12] This star may be in the situation anticipated by Curtiss, in which an exact chemical equilibrium between carbon and titanium oxide suppresses the bands of both.

Photography of the spectra of bright stars in the red, and even the near infra-red, is now practicable, and Merrill[13] has already obtained results of great interest and promise. Investigation of the spectra of the brightest stars with high dispersion is also profitable, as is shown by the work of Adams[14] upon the pressures which probably prevail in the atmospheres of Sirius, Procyon, and Arcturus. Fortunately, the stars brighter than the second magnitude afford examples both of giant and dwarf stars of almost every spectral class.

2. (*a*) Almost equal in importance to the line absorption in stellar spectra is *the distribution of intensity in the continuous background*. The most complete and satisfactory method of studying this would be the direct measurement of the energy carried by different wave-lengths, but this has not yet been proved practicable. A first step has however been taken by Coblentz,[15] who has not only measured the total energy radiation of more than a hundred stars, but in some cases the percentage transmitted by a water cell, thus providing our first knowledge of stellar radiation in the infra-red. With the great reflectors just completed, the determination of spectral energy curves for the brightest stars may be possible.

The distribution of energy in the luminous region of the spectrum is however readily determinable. For the brighter stars, spectro-photometric methods can be employed, as in the visual work of Wilsing

and Scheiner,[16] and the photographic investigations of Rosenberg.[17] Fainter stars, down to the sixteenth magnitude, at least, can be reached by the determination of *color indices*.

(*b*) In order that these color indices may be capable of full utilization, it is necessary, first, that trustworthy and homogeneous scales of visual, photographic and photovisual magnitudes be established over the whole range of about 47 magnitudes from the Sun to the faintest observable stars. This problem, which is fundamental in all stellar photometry, is already well advanced toward solution. But in the second place, it is necessary that the physical meaning of the units of magnitude should be precisely known; that is, that the 'luminosity curve' which expresses the relative sensitiveness of the photometric receiver for equal energy of different wave-lengths should be exactly determined. And, above all, it is essential that this luminosity curve should be independent of the brightness of the stars under observation. These last two conditions are at present very imperfectly satisfied, if at all. Very little is known about the luminosity curves of the standard plates and apparatus used in the determination of photographic and photovisual magnitudes, and nothing at all about the luminosity curves of the eyes of the 'standard observers' at different observatories,—except that they must be very different under the conditions prevailing at Harvard and at Potsdam.[18] It is certain that the Purkinje effect alters the form of the visual luminosity curve as the brightness of the illumination varies, probable that this affects the visual comparison of the brightness of stars of widely different magnitudes, and uncertain whether, and to how great an extent, similar photographic influences exist.[19]

The direct determination of the luminosity curves for the principal instruments and methods employed in the determination of photographic and photovisual magnitudes would be neither difficult nor laborious. For visual observations it can be derived indirectly, if direct measures prove difficult. To make these investigations at once is urgently desirable, for the present bases of the scales of stellar magnitude are not permanent. The photographic and photovisual scales depend on the properties of present commercial types of rapid plates, which may not be manufactured a few years hence if improvements are devised; and the visual scales are based on the characteristics of the eyes of observers some of whom have already retired from active work.

Such an investigation would also establish a connection between the scales of stellar magnitude and the physical units of measurement of light in the laboratory (which are now defined in terms of a definite

luminosity curve), and would enable us to express our stellar photometric data in absolute units.

It is also desirable that methods for measuring the brightness of the stars with red and ultra-violet light should be developed, with careful determination of the luminosity curve in each case, and of the color equation which (for normal stars) makes it possible to reduce color-indices obtained on any of these systems to a standard scale.

The determination of the colors of faint stars by other methods affords a promising field, as is shown by the success of the method of effective wave-lengths,[20] and of that of exposure ratios,[21] recently developed at Mount Wilson.

Such a determination of exact scales of magnitude and color index is evidently a necessary condition for the full utilization of the great mass of material which is in process of collection concerning the numbers of stars of different magnitudes, their concentration towards the Galaxy, etc.

(*c*) The statistical investigation of the relations between color index and spectral type, and between both and absolute magnitude, have already opened up possibilities of estimating the distances of stars far too remote to be reached in any other way. Such investigations should be extended, with special reference to stars of great and small absolute brightness, and to those having peculiar spectra.

Closely connected with this is the question of possible selective absorption of light in space. Shapley's results,[22] and the theoretical work of L. V. King,[23] appear to negative the existence of any general absorption of this sort. But local selective absorption may occur, and it would be well worth while to study intensively the color indices and spectra of stars in regions where the existence of absorbing matter is suspected, such as Barnard's dark lanes in Scorpius. It is interesting in this connection to note that the three most abnormally yellow stars of Class B (ζ, o and ξ Persei)[24] lie within 5° of one another, in a region full of diffuse nebulosity.[25] A survey of the stars in this region for color-index and spectral type would be well worth while.

(*d*) Another interesting problem is presented by the extreme infrequency of very red stars. Color-indices up to about +1.8 on the Harvard scale are fairly common; but greater values are very unusual, and are practically confined to the 'side chain' which includes Class N. In this subsidiary sequence the color-indices increase to about +4, as might be expected as a result of decreasing temperature; but in the main series, ending in Class M, this does not happen. Are all the stars of Class M of about the same temperature, or is an increase of redness in Classes Mb and Mc masked by increasing absorption in the

red end of the spectrum? There are certainly very heavy absorption bands in the red in these spectra; and further evidence in favor of this hypothesis is found in Coblentz's measures of Alpha Herculis,[26] which show this star, of Class Mc, radiates far more heat in proportion to its light than do stars of Class Ma, and also in Hertzsprung's[27] observation that the very faint dwarf stars of Class Mb are not nearly as red as their small luminosity, and probable low surface brightness, would lead one to suppose. A careful study of the color indices, and, if possible, of the spectral energy curves, of the stars of Classes Ma, Mb, and Mc is much to be desired. The extraordinarily red stars S Cephei[28] and +43°53,[29] which have color indices exceeding five magnitudes, should be included in such a study.

3. One other stellar characteristic which may be investigated without knowledge of distance is *variability of brightness*. If we really understood the causes of stellar variability, we should probably have advanced a long way towards the solution of the whole problem of stellar evolution, if not have solved it completely. But, in spite of the great number of variable stars, the variety of the phenomena which they represent, and the accuracy with which they can now be observed, the humiliating admission must be made that no even tolerably satisfactory theory of the causes of the variation exists, except for the eclipsing variables, and in this case it is based on the proposition that, except for the accident of eclipse, the components are not variable at all!

Successful attack upon the problem of intrinsic stellar variation will probably demand the correlation of all the data that can be brought together from every accessible source. In the case of regular variables, precise light curves are of importance, and many stars still await investigation,—some of them visible to the naked eye, and long known to be variable. The new photometric methods of precision—especially the photoelectric cell—have opened a wide field in the study of bright stars with small variation, in which important results have already been obtained,—notably by Stebbins[30] and Guthnick,[31]—and more may be anticipated.

(*a*) Former suspicions of changes in form of the light curves appear to have been unfounded in the case of eclipsing variables; but similar changes are believed with good reason to exist among Cepheid variables.[32] To prove their reality—still more to discover their laws—demands very precise observations, preferably by two or more observers at different places and the same time.

(*b*) Changes in color, as well as in brightness, appear to be the general —perhaps the invariable, rule among eclipsing variables, and especially

among Cepheids—the star being always redder at minimum than at maximum. More recent observations show that changes in the spectrum go hand in hand with the others.

In the case of eclipsing variables, these changes arise from a difference in spectral type between the components, and it is found that stars separated by an interval less than their own diameters, and therefore very probably of the same origin and age, may have spectra differing as widely as those of Sirius and Arcturus.[33] Observations of such systems, when the eclipse is total, provide the only direct method at present existing for studying the relations between spectral type, color index, surface brightness, and density, which are of fundamental importance. The determination of the spectral type of the fainter components of such systems, though often very difficult, on account of their extreme faintness, deserves special effort.

(*c*) The concomitant variations in brightness, color, and spectrum, which Shapley[34] has shown to occur in every Cepheid variable that has been properly investigated, indicate very strongly that the proximate cause of the changes in all three is a periodic variation in the surface temperature of the stars. Shapley's suggestion[35] that these differences in temperature arise from some sort of internal changes, perhaps of the nature of periodic oscillations in the radius, density, temperature, etc., appears to be the best which has been yet made; but there are still grave difficulties in explaining how such pulsations should in all cases produce the very distinctive form of the light curve, with its rapid rise and slow fall, and still greater trouble in accounting for the variations in radial velocity, which show so remarkable a relation, both in amplitude and phase, to those in light. It is in fact still doubtful whether these stars are really binary systems or not. Intensive studies of a number of these variables, including the greatest practicable variety of representative cases, would be well worth while.

(*d*) Still less is known concerning the very numerous variables of long period, and the roughly periodic and irregular variables. In the observation of their changes in brightness, amateur observers may obtain results of much value, and, under the admirable coöperative schemes organized by the American and British Associations of Variable Star Observers, they are at present furnishing a great mass of valuable information. Very little is known regarding changes in the spectra of long-period variables, except that they often exist,[36] especially as regards the bright hydrogen lines which are usually present at maximum. Observations of the color indices of these variables are also much to be desired. Certain peculiar variables, such as R Coronae and SS Cygni,

are typical of small but definite groups, whose variation, though quite distinctive, is entirely unpredictable. The spectra of the stars of the first of these groups are similar to one another, and unlike anything else.[36] Those of the second group are also peculiar, and appear to be variable.[36] Both present problems as alluring as they are difficult. The spectra of other peculiar variables also deserve investigation.

(*e*) New stars are usually pretty fully observed while they remain bright, but work remains to be done in following at shorter intervals the changes during their later stages. The recent work of Adams and Pease[37] indicates that they settle down into Wolf-Rayet stars; but, according to Miss Cannon,[38] the spectrum of the Nova in Corona, fifty years after its outburst, is now of class K. No one seems yet to have followed up Hertzsprung's interesting suggestion[39] that stars of very small absolute luminosity should be investigated for variability. Abundant material for a photographic study must exist in the Harvard collection.

4. Knowledge of the *distances* of the stars is indispensable in the solution of many problems. The nearer ones, to a distance of thirty parsecs or so, are now accessible to direct measures of *parallax*, and great activity prevails in photographic observation for this purpose, in accordance with a wide and well-considered plan of coöperation.

In my opinion, however, the greatest need in parallax work at present is the investigation and elimination of the systematic errors which are still present in the best work, as is shown by the too frequent appearance of large discordances—sometimes amounting to more than 0".05—between the results of different observers, although the probable errors derived from the internal agreement of each observer's plates are of the order of ±0".01. The intercomparison of the results of various observers for the same stars is hardly a sufficient test for the absence of systematic error, especially as all are using nearly the same method of observation. The only secure control is afforded by observing stars whose parallaxes can be predicted, from other considerations, with greater accuracy than they can be observed. This demands prediction with a probable error not exceeding ±0".005. Fortunately, several groups of stars exist for which such prediction is possible. The most prominent of these consists of those stars of spectrum B which are between 60° and 120° from the solar apex. If the parallaxes of these stars are computed on the assumption that their individual proper motions are entirely due to the solar motion, the resulting errors will correspond to a probable error of less than one-third of the parallaxes themselves—that is, to about ±0".002. The

stars of Kapteyn's Scorpius-Centaurus group[40] would be ideal objects, if they were not too far south.

For fainter stars, eclipsing and Cepheid variables are available. Of the 90 eclipsing variables whose parallaxes were estimated by Russell and Shapley,[41] 69 are fainter than the eighth magnitude, and the mean parallax of these is 0".002, while only ten per cent exceed 0".004. For the Cepheids of similar brightness, the parallaxes estimated by Hertzsprung and Shapley[42] are even smaller.

When once the systematic errors have been tracked to their source and eliminated, an extensive program of observation can be undertaken with security. Much duplication of observations is desirable, for it is obviously better that the parallax of a star should be determined from the mean of three or four short series of as many different observatories than by a series with a single instrument, however long and elaborate. Certain objects for which especially accurate parallaxes are desirable should be observed at as many places as possible. Examples are binary stars, stars differing in absolute magnitude from the bulk of those of the same spectral class, or from the values predicted by the spectroscopic method, stars with exceptionally rapid motions in space, planetary nebulae, etc. Attempts to determine by direct observation the mean difference in parallax between classes of stars with small parallaxes (for example, those of the third and fourth magnitudes, taken as a whole) should, in my judgment, be deferred until the systematic errors have been thoroughly cleaned out.

5. Knowledge of parallax leads at once to that of *absolute magnitude*, which, in the interest and importance of its systematic relations to other characteristics of the stars, stands second only to spectral type.

(*a*) The relations between the two afford a very interesting study, which has led Hertzsprung[43] and Russell[44] to the recognition of the two series of 'giant' and 'dwarf' stars, coincident in class B, but gradually drawing apart among the redder stars until, as Adams' spectroscopic results have recently confirmed,[45] they are completely and widely separated in class M. If Russell's views are correct, the existence of these two series is the key to the problem of stellar evolution. In any case, their existence must be accounted for, and will be of importance in testing any theory. The securing of additional data, especially regarding the absolute magnitudes of individual giant stars, is much to be desired. It is of importance to determine not only the mean absolute magnitude of the giant and dwarf stars of each spectral class (whenever the two are separated) but the dispersion of the individual values about the mean. Only when the latter is known can the results

of statistical investigations be cleared from the effects of the egregious observational preference for the brighter and remoter stars.

(*b*) Kapteyn[46] has obtained fairly good values of the dispersion among the various divisions of Class B, and provisional values for Class A; and Russell[47] has given rough estimates for the dwarf stars, and a still rougher one for the giants of Class M: but further work is greatly needed. Adams' spectroscopic method offers an easy solution of the problem, as soon as his present provisional scale of absolute magnitudes for the giant stars has been revised with the aid of studies of the parallactic and peculiar motions of groups of stars whose spectra indicate that they are similar in real brightness. Strömberg[48] has already shown in this way that Adams' mean absolute magnitude for all the giant stars, taken together, is substantially correct; but there is evidence that the provisional estimates for the very brightest stars (such as the Cepheid variables) make them considerably too faint.[49]

(*c*) The existing evidence indicates that the majority of the stars of any given spectral class are confined within surprisingly narrow limits of absolute magnitude (provided that the giants and dwarfs can be treated separately). But there are exceptions of great interest. For example, Kapteyn[50] has shown that β Orionis is some eight magnitudes brighter than the average for its class (B8); and the faint companions of Sirius[51] and o^2 Eridani[52] have spectra of class A, although they are at least eight magnitudes fainter than normal stars of this class. Exceptional brightness is probably explicable by unusual size or mass; but the two exceptionally faint stars (which are known to be of normal mass for stars of their brightness) present a real puzzle. Something about the physical conditions in these stars must be very unusual, and they should be studied with the greatest attainable detail. Other such objects may be found among the faint stars of large proper motion.[95]

6. Beyond the limit of direct measures of parallax, our main reliance must be placed on *proper motions*, which are of fundamental importance in the study of the galactic system.

The brighter stars have already been cared for by Boss, and those down to magnitude 7.5 are under discussion. The fainter stars can best be investigated by photography, carrying the work to objects as faint as can be reached with large instruments, in accordance with Kapteyn's 'Plan of Selected Areas' or some equivalent. For this purpose, it is essential to have a set of reference stars, distributed uniformly over the sky, and of suitable brightness to serve as photographic standards, and to make the observations strictly differential with respect to these, using them not merely as reference points for position when reducing a

single plate, but as reference points for proper motion when comparing two plates of different epochs. The observations of these reference stars must at present be made with meridian circles; but the proposed methods for determination of absolute positions of the stars by photography deserve careful study and trial.

Pending the completion of such a program, the investigation of the proper motions of faint 'optical' companions of bright stars, such as has been made by Comstock,[53] furnishes our best source of information concerning the proper motions of faint stars, but is complicated by systematic errors in the early measures. A survey of the whole heavens for stars of large proper motion is very desirable. In this case it is legitimate to treat the general 'background' of stars as at rest, and the observations can be very rapidly made, with the blink microscope or similar appliances. Early plates are probably already available for almost, if not quite, the whole of the heavens. Such an investigation is likely to yield important information concerning the stars of very small absolute luminosity—as is shown by Barnard's[54] and Innes's[55] recent remarkable discoveries—and should be extended to the faintest accessible stars.

Comparison of measures of plates taken at different epochs (still treating the bulk of the stars as fixed) will yield much information about proper motions of moderate size. This has already been done on an extensive scale with plates of the Astrographic Catalogue.

Special investigations should be made to determine at an early date the proper motions of all stars belonging to certain interesting classes for which early determinations of position are available—for example, binaries, variables, and stars having peculiar spectra.

7. The study of the *radial velocities* of the stars is intimately associated with that of the proper motions. The determination of radial velocities with the slit spectroscope has been brought to a high degree of perfection, but the separate investigation of each one of the many thousands of stars which are now accessible would involve an enormous amount of labor. The development of some method by which radial velocities could be determined *en masse* with the objective prism would be a great boon. If some absorbing medium giving sharp and well distributed lines in the blue and violet could be found, the problem would become simple; and other solutions are doubtless possible.

It is also desirable that some method be devised for obtaining, at least approximately, the radial velocities of stars possessing spectra with very diffuse lines. At the present time, no radial velocities have been published for some of the very brightest stars, on this account.

In extending the list of observed radial velocities, much advantage has been gained by a policy of selective observation of classes of stars of special interest—such as stars of unusually large and small proper motion, absolute magnitude, and the like, variable stars, and stars of the rarer spectral types. A similar investigation of double stars showing evidence of physical connection would be worth while.

8. Statistical discussions of the motions of the stars and of the Sun, and their relation to spectral type, etc., offer an extensive and very intricate field. Among the matters demanding further investigation may be mentioned the reason for the differences in the direction and velocity of the solar motion derived from stars of different spectral types, and from proper motions and radial velocities separately; the origin of the constant term in radial velocities (Campbell's K term); the existence of tendencies toward common motion among the stars in particular regions of the sky; the dependence of the mean peculiar velocities of the stars upon spectral type and absolute magnitude, and the real cause of this dependence (possibly a correlation between large velocity and small mass); the true nature of preferential motion, and whether it really gives evidence of the existence of two physically different 'streams;' the dependence of preferential motion upon spectral type, absolute magnitude (the latter an unworked field) and perhaps upon the region of the sky considered; the devising of a rapid method for the detection of moving clusters, and the identification of their members; and so on. The discussion of most of these problems should be based simultaneously on proper motions and radial velocities. Results derived from either one alone may fall into errors which the combination of both would detect.

One practical matter deserves specific mention. When it appears desirable to exclude certain stars from a statistical discussion (for example, those of very large proper motion), the limits of exclusion should be clearly and precisely stated. Neglect to do so may cause great trouble to other workers who wish to make a comparison with their own results, and has sometimes led to very serious errors of interpretation.

9. Another set of data of fundamental importance depend upon relations involving the *masses* of the stars. Here there appears the grave difficulty that nothing at all can at present be found out concerning the mass of a star unless it is double. There are plenty of double stars, to be sure; but what certainty have we that they are similar in mass to stars which are not double? Only an indirect answer is possible, by means of the statistical comparison of single and double stars with respect to as many characteristics as may be—absolute magnitude,

spectrum, color, radial velocity, proper motion, distribution in space, etc. (bearing in mind that the limits of telescopic resolution restrict our knowledge of the remoter pairs). But Eddington's recent theoretical researches[82] lead to the hope that it may some day be possible to estimate the mass of any star when its absolute magnitude and spectral type are accurately known (using the data for double stars as a guide).

(*a*) As regards the determination of the masses of individual stars, it should be borne in mind that, for statistical purposes, a pair in which the relative motion of the components is known, though the motion in angle may be only a few degrees, is very nearly as valuable as one which has completed a revolution—while a pair for which the relative motion is unknown is of no use at all. The slowly moving pairs which are often, but inaccurately, described as 'fixed,' possess an importance exactly analogous to the stars of small proper motion, and give us invaluable information about those stars which are bright in proportion to their mass—the giant stars, in fact. Now that the discovery of double stars is apparently in sight of completion, it is to be hoped that more attention may be given to the problem of determining the relative motion in as many systems as possible.

(*b*) The existing data suffice to show that the masses of the stars differ from one another less than any other of their characteristics—the whole range among well determined masses being from 20 times the Sun's mass to one-sixth of the Sun's, which may be compared with a range in luminosity of at least ten million fold. For this very reason, very careful observations are required to enable us to say with certainty that one star is more or less massive than another. It appears certain that the stars of spectrum B are unusually massive,[56] and there is sufficient evidence to show that, in general, stars of great luminosity are more massive than those of small absolute brightness, and that, among the dwarf stars, those of 'later' spectral type are of smaller average mass.[57] But there are very few cases in which we can be sure that a given star is more or less massive than the average for its type.

It is very desirable to determine how great is the range of difference among the masses of stars of similar spectral class or absolute magnitude. Extremely precise determinations of parallax will be needed if this problem is to be solved, but the effort will be well worth while. Sufficiently reliable values of the mean masses of stars of different groups have already been determined, to make it possible to estimate the parallaxes of all but the nearer binaries and 'physical pairs' more accurately than they can at present be observed.[57] This should be of aid in the interpretation of other statistical studies of double stars,

such as the proportion of double stars among all the stars of a given magnitude, the relative numbers of close and wide pairs, etc.

The determination of the relative masses of the components of binary systems will soon also be possible in many cases which have previously been somewhat neglected.

When a sufficient number of accurate determinations of mass have been made, a detailed study of the spectra of stars differing in mass should be made, in the hope of finding peculiarities depending directly on the mass, which might make it possible to estimate the masses of isolated stars.

(*c*) A great number of *spectroscopic binaries* await investigation, and more are continually being discovered. In the determination of orbits, preference should be given to those which show the spectra of both components, as it is only in this case that definite information can be obtained about the masses. Eclipsing and Cepheid variables are also worthy of special attention, and also stars of large proper motion, or others which appear to be dwarf stars.

It is very desirable that some method should be found for observing the spectrum of the secondary component when it is too faint to be directly seen. Perhaps Koch's spectromicrometer might furnish a solution. Favorable cases for trial, in which the brightness of the invisible secondary spectrum is known, may be found among eclipsing variables.

10. The *densities* of stars can so far be determined only when they are eclipsing variables. In this case, when both spectra can be photographed, the *diameters* of the components can also be found. Several systems of this sort, which have not yet been investigated spectrographically, are within the reach of existing instruments.

If, however, the relations between spectral type, color index, and surface brightness can be so well determined that it is possible to estimate the last of the three when the other two are known, it will then be possible to determine the densities of all visual binary stars, the linear diameters of all stars of known parallax, and the angular diameters of all the stars in the sky. The known eclipsing variables should afford sufficient material for a first investigation of the problem, if only sufficiently accurate information can be obtained regarding the color-equation of the visual and photographic methods of observation which have been employed at various observatories.

11. All that can be said at present regarding the *internal constitution* of the stars depends on Eddington's theoretical work,[32] which indicates that, in the stars of low density, the mass should be greatly condensed toward the center—the central density being 54 times the mean den-

sity. But the problem is capable of investigation by observation. There are many close eclipsing pairs in which the components are ellipsoidal in form, as is proved by variability of the Beta Lyrae type. In such systems the lines of apsides of the orbits should advance, at a rate depending on the masses, dimensions, and internal constitution of the components. If the last is like that of Jupiter or Saturn, the advance of periastron should be rapid. What little evidence there is indicates a slower motion, and hence a very strong central condensation; but more intensive studies are necessary before definite conclusions can be drawn. There are several systems for which the necessary data concerning the dimensions and forms of the orbits and the stars are accessible to suitably planned observations,—notably α Virginis and *U* Herculis. A careful study of such stars, by means of simultaneous photometric and spectroscopic observations, would be remunerative.

The singular and so far inexplicable changes which occur in the periods of most eclipsing variables, and so far have defied prediction, also deserve extended study; and Eddington has recently called attention to the fact that secular changes in the periods of Cepheid variables are likely to give a clue to the rate of stellar evolution.[58] The first scanty evidence points to a very extended time scale.

12. In the investigation of *star-clusters*, measures of position, for the purpose of detecting future proper motions, are obviously a duty to posterity. There is little chance that anything more than the motion of the clusters as a whole will be perceptible in our generation, and only measures of the utmost attainable accuracy and freedom from systematic error are likely to be of use to the astronomers of the future. Of far more promise are studies of the distribution of the stars within the clusters, their magnitudes, and, above all, their color indices. Such investigations, in Shapley's hands,[59] have given us for the first time a true conception of the distances and magnitudes of the globular clusters. Students of the subject are eagerly awaiting the detailed publication of the evidence on which he bases his conclusion that the apparent avoidance by these clusters of the region within 1500 parsecs of the galactic plane is due to a real absence of clusters from this region, and not to obscuration by absorbing matter.

The variable stars in clusters also deserve further attention. Those so far discovered appear to belong to the Cepheid type, which is natural, as these seem to be actually the brightest of all variables. Long period and eclipsing variables may yet be discovered among the fainter stars.

Good work can still be done also upon the irregular clusters,—as is shown by Trümpler's[60] study of the outlying members of the Pleiades.

One of the most attractive of unexplored fields is the investigation of the *Magellanic Clouds*. The small amount of work which has been done, mainly on the Smaller Cloud, has led to the discovery of a remarkable relation between the periods and absolute magnitudes of the variables in the Cloud,[61] to the estimate that its distance is 20,000 parsecs,[62] and to the discovery that the nebulae within it, and probably the Cloud as a whole, have a very high radial velocity.[63] The great instruments which are now being erected in the southern hemisphere may well be actively directed toward this region.

13. (*a*) Foremost among the many problems presented by the *gaseous nebulae* is the cause of their luminosity. In spite of our ignorance of the origin of the characteristic nebular lines, the appearance of such lines as λ 4686 in the spectra of nebulae, and of the Wolf-Rayet spectrum in their nuclei, suggests that in them "we are presented" (in Fowler's words)[64] "with phenomena which result either from the effects of powerful electrical actions or of very elevated temperatures." Though such conditions may easily enough exist in the nuclei, it is very hard to see how high temperatures can prevail throughout the whole volume of a nebula.* There are several possible ways out, however.

The electrical action may be a bombardment of the outer region by corpuscles emitted from the nucleus. Or perhaps the luminosity of the gases is fluorescent, like that of the sodium or bromine vapors studied by Wood.[66] Or, as Fabry has recently suggested,[67] we may have to do with a body which absorbs and emits radiation only in narrow regions of short wave-length, and may therefore attain a very high temperature in thermal equilibrium with the radiations from a distant, but still hotter, source. To determine the true explanation among these and many other possibilities may tax the resources of both experimental and theoretical spectroscopy.

The association of gaseous nebulae with stars of 'early' spectral type might be anticipated on any of these theories. For such stars are very hot bodies, and would be the most powerful sources both of corpuscular and ultra-violet radiation. Hence the association of these stars with nebulae does not prove that the stars originate from the nebulae. It is entirely conceivable that, on the contrary, the nebulae, as visible objects, owe their existence to the radiation of the stars, and are their offspring, and not their parents. Some gaseous nebulae, however, are not near bright stars, and the nuclei of planetary nebulae appear to be

* Fabry's calculated temperature of 15,000° for the Orion Nebula,[65] as he points out, is liable to be diminished by an unknown amount on account of the widening of the lines of the spectrum by turbulent motion of the nebular matter in the line of sight.

comparable with some of the faintest stars in luminosity. Clearly, nothing final can be said on this subject until we know what it is that shines in the gaseous nebulae, and why. It may be remarked, however, that the wide-spread assumption that the origin of the stars is to be sought in the visible nebulae appears to have had very little solid basis. All classes of nebulae except the extended gaseous nebulae have already been excluded from consideration as observational knowledge increased.

(*b*) A few nebulae, like those in the Pleiades,[68] appear to shine by light reflected from neighboring stars, and Slipher's spectroscopic work is steadily adding to the list of his discoveries in this field. Hertzsprung[69] has shown photometrically that the brightness of the nebulosity in the Pleiades is entirely consistent with the reflection hypothesis. Similar studies of other nebulae, and especially of the remarkable variable nebulae recently observed by Slipher,[70] would be of value.

Barnard's long continued researches[71] have made it highly probable that there exist many ***dark nebulae***, revealed only by the effects of their opacity in concealing whatever lies beyond them. It is highly significant that the most remarkable of these dark regions is obviously directly connected with one of the nebulae which shines by reflected light,—that surrounding Rho Ophiuchi[72]—and that the whole mass is comparatively near us in space, at a distance of 100 to 150 parsecs. If such masses of practically opaque material are scattered through the galactic and extra-galactic regions at distances comparable with this, the resulting absorption of light must play a very important rôle in limiting the apparent extent of the universe. If this absorption is of the type which is produced by dust, or even by particles of the size of the drops of water in ordinary clouds, it will affect all wave-lengths to substantially the same extent, and be much more difficult to detect than the gaseous scattering, increasing for the shorter wave-lengths, which several investigators have sought for, but whose existence Shapley has apparently disproved.[22] It seems appropriate to remark in this connection that absorption independent of the wave length seems ***a priori*** much more likely to occur than the other, since the same quantity of matter in the form of a fog is incomparably more effective than in gaseous form, (compare the opacity of a few meters of cloud with that of all the rest of the atmosphere) and also since most forms of matter are likely to be in the solid or liquid state at the temperatures prevailing in interstellar space.

(*c*) The forms of nebulae—especially of planetary and ring nebulae—deserve careful study. As Campbell suggests,[73] it is difficult to account for them without assuming the existence of some repulsive force which

counteracts the attraction of the nucleus. He suggests light-pressure—which would fit in well with views of the origin of the luminosity such as are suggested above. In such a case we should anticipate that most of the light of the nebula would come from the nucleus, and this appears to be usually, though not always, the case.

(*d*) Measures of the radial velocities of nebulae have already shown that the planetary nebulae, as a class, are moving in space much more rapidly than the stars;[74] that there exist internal motions within them, usually of a rotational character, but sometimes more complicated;[73] and that, in order to keep the moving material from flying away into space, the total masses of the nebulae must be very considerable, and probably a good deal larger than those of the stars.[73] Much remains to be done in the investigation of these motions, and in their interpretation. The proper motions of planetary nebulae, and perhaps in some cases the internal motions of the nebular material, can be determined by comparison of suitable photographs, and it is probable that in a decade or two we shall obtain in this way a fair idea of the distances and real dimensions of these bodies. Observations for parallax on some of the larger and presumably nearer planetary nebulae are also desirable.

The extended gaseous nebulae should be examined spectrographically to see whether turbulent motions exist in others, as they do in the great nebulae of Orion;[75] and it would be worth while to compare photographs of some of those which show sharp details, in the hope of detecting proper motion, either of the whole or of parts.

Investigations of the distribution within the gaseous nebulae of the substances which give the different spectral lines may be made by photography either with absorbing screens or with slitless spectroscopes, and promise information regarding the conditions prevailing in the nebulae, and the mutual relations of the lines of unknown origin.

14. (*a*) The *spiral nebulae* have been shown by recent investigations to be the most extraordinary objects in the heavens. Their enormous radial velocities—first detected by Slipher[76]—and the almost equally rapid internal motions within them,[77] put them in a class by themselves. Further measures of these motions are needed; and, when the radial velocities of a sufficient number of spirals, well distributed over the heavens, are known, it may be possible to determine definitely the direction and rate of the motion of the Sun (and presumably of the whole galactic system) with respect to the system of nebulae. The provisional determination by Young and Harper,[78] from very scanty data, indicates for the motion of our system the enormous velocity of 600 kilometers per second.

(*b*) As van Maanen[79] and others[80] have shown, the proper motions of some spiral nebulae—both of the mass as a whole and of the condensations in the arms relatively to the centre—are apparently large enough to be determined by the careful comparison of plates taken only a few years apart. This opens up another wide field of study, and will make it possible before long to determine the mean parallax of many such nebulae by comparison of the proper motions and radial velocities of their nuclei. There is also reason to hope that the distances of some individual nebulae, which are seen at a suitable angle, can be determined by comparing the radial and transverse components of motion along the arms. Enough is already known to convince us that the distances of these nebulae must be measured in thousands of parsecs, and their diameters in parsecs, and that direct measures for parallax are utterly hopeless.

(*c*) Photometric measures, both of the total light of the spirals and the relative brightness of their parts, would be of value, especially if accompanied by determinations of color. Seares[81] has recently shown that the outer convolutions are far bluer than the centre—which is the part that shows the spectrum of solar type. Spectroscopic observations of these outer regions, if possible, would be of great interest. Another matter calling for further study is the nature of the dark bands which cross many nebulae which appear to be spirals seen edgewise, and look as if they were due to the interposition of opaque material in the outer regions of the nebula.

(*d*) The distribution of spiral nebulae in the heavens—so utterly different from that of any other objects—may be explainable when their real distribution in space is even partially known. It is hardly time as yet to consider the greater question of their real nature, except to note, with van Maanen,[79] that, unless they are in process of very rapid dissipation into space, their masses must be exceedingly great.

15. Finally, it must not be forgotten how important a place *theoretical investigations* will occupy in the solution of the larger problems of sidereal astronomy. The increasing observational data are already furnishing just those guides which point the skilled mathematician in the right direction, and these indications have been very successfully followed, especially by certain members of that 'Cambridge school' which combines keen mathematical analysis with a thorough knowledge of modern physics. Results of remarkable generality have already been obtained.

In the field of stellar evolution, Eddington[82] has worked out in detail the importance of radiation pressure in determining the conditions of internal equilibrium of the stars, and the approximate equality in

brightness of the giant stars of all spectral types has found a simple explanation.

If the conclusion that the luminosity of a giant star is a function of its mass, but not of its temperature or age, is confirmed, and the nature of the function fixed by observation, the problem of determining the masses of stars which are not double will in many cases be solved.

Jeans,[83] discussing the problem of the figures of equilibrium of a rotating mass of compressible fluid, has already reached conclusions which not only bear upon the origin of double stars, but have suggested an entirely new and very stimulating conception of the nature of spiral nebulae, as huge rotating masses of gas, which, becoming unstable at the edge under the influence of their own rotation and the attraction of the neighboring stars, throw off matter from their periphery in streams of such enormous size that they may divide into 'nuclei' large enough to form ordinary stars upon condensation.

In the field of galactic astronomy, Schwarzschild[84] has developed powerful methods for handling the statistical material which must be our main guide, and Jeans[85] and Eddington[86] have shown that 'star streaming' demands no unknown forces for its explanation, but is probably interpretable dynamically, as a property of a system of stars in motion under their own gravitation—although the existence of 'streaming' appears to indicate that the galactic system is not in a 'steady state.' Eddington[87] has shown that the similarity of distribution of the stars in different globular clusters presents a problem by no means simple, though of much interest.

Almost the whole of this work has appeared within the last three years, and further notable advances may be anticipated. Indeed, almost as these words are written, comes the first installment of an important paper by Eddington[88] on the oscillations of a gaseous star, which may afford the long-sought solution of the problem of Cepheid variation.

Among other specific problems awaiting discussion may be mentioned the question whether the tidal interaction of two compressible and slowly condensing bodies can cause an originally small eccentricity to increase to the very large values which are found in many visual binaries, and some spectroscopic binaries as well; and, if this proves to be impossible, how the systems in question can have originated;[89] the origin and laws of the complicated changes which occur in the periods of many eclipsing binaries; and the equilibrium and motions of the constituent parts of planetary and spiral nebulae.

Mention should also be made of the work of Nicholson[90] on the interpretation of unknown lines in the spectra of nebulae and of the

solar corona as arising from hypothetical atoms of very simple structure—which has successfully met the test of prediction—and of the development of the theory of general relativity, which has already been used by deSitter[94] to set a superior limit to the whole quantity of matter in the universe, and may have important applications in future.

16. Of more fundamental nature, and obvious importance, is the unsolved problem of the source of the energy which the stars are continually radiating at so rapid a rate. It is becoming increasingly plain that the gravitational energy liberated by contraction from infinity would not nearly suffice to maintain the Sun's radiation during geological time[90] (according to even the more conservative estimates of the latter); yet the mere continuous existence of life on the Earth is evidence that the Sun has not merely kept on shining throughout this interval, but has not changed in brightness by more than one magnitude, at the outside. In the case of some giant stars, contraction from infinity would hardly suffice to furnish the energy which they have radiated during historic time.[91] There appear to be two ways out of the difficulty; either the stars do not radiate heat in all directions to space at the same rate as they do towards the Earth, or else they have some unknown and exceedingly great supplies of internal energy. The first alternative, however, seems to be excluded by the fact that the amount of heat which the Earth receives from the Sun, and loses again by radiation into space, is not greatly, and probably not at all, inferior to that which a black body of the same size and temperature as the Earth's effective radiating surface would radiate to an enclosure at the absolute zero.[92] There seems therefore no escape from the conclusion that the heat radiated by a star can not be provided by contraction. What the source of the energy may be, how it is converted into heat in the body of the star, and where it goes after passing from the star's surface into the ether, are at present the greatest of all the unsolved problems of astronomy.

[1] Newall, Baxandall and Butler, *Monthly Not. Roy. Astron. Soc., London* 76, 1916, (640).

[2] Fowler, A., *Proc. Royal Soc., London*, A, 94, 1918, (470).

[3] Fowler, A., *Ibid.*, A, 94, 1918, (472).

[4] Fowler and Brooksbank, *Monthly Not. R. A. S., London*, 77, 1917, (511–517).

[5] *Astrophy. J., Chicago*, 40, 1914, (466–472).

[6] Curtiss, R. H., *Popular Astronomy, Northfield, Minn.*, 25, 1917, (279–285).

[7] Cannon, Miss A. J., *Ibid.*, 24, 1916, (656).

[8] Adams, W. S., these PROCEEDINGS, 2, 1916, (143).

[9] Hertzsprung, E., *Z. Wiss. Photog., Leipzig*, 3, 1905, (429–435).

[10] Adams, W. S., these PROCEEDINGS, 2, 1916, (147–156).

[11] See Ref. 6; also Rufus, W. C., *Pub. Astron. Obs. Univ. Michigan, Ann Arbor*, 2, 1916, (143).

[12] King, E. S., *Ann. Harvard Coll. Obs., Cambridge*, **76**, 1915, (119).
[13] Merrill, P. W., *Popular Astron., Northfield*, **25**, 1917, (661).
[14] Adams, W. S., *Astroph. J., Chicago*, **33**, 1911, (64–71).
[15] Coblentz, W. W., *Lick. Obs. Bulletin, Berkeley, Cal.*, **8**, 1915, (104–120).
[16] Wilsing and Scheiner, *Publ. Astrophys. Obs., Potsdam*, **19**, 1909, (5–221).
[17] Rosenberg, H., *Astron. Nach. Kiel*, **193**, 1912, (357–370).
[18] See *Harvard Annals*, **64**, 126–127.
[19] King, E. S., *Ibid.*, **76**, 1915, (189).
[20] Hertzsprung, E., *Astrophys. Jour.*, **42**, 1915, (92–110).
[21] Seares, F. H., these PROCEEDINGS, **2**, 1916, (521).
[22] Shapley, H., these PROCEEDINGS, **2**, 1916–17, (12–15) and **3**, (267–270).
[23] King, L. V., *Trans. Roy. Soc. Canada, Ottawa*, **9**, 1915, (99–103).
[24] Color index of ζ Persei + 0.16; color index of ξ Persei + 0.03; King, E. S., *Harvard Annals*, **76**, 1915, (118). Color index of *o* Persei; similar to that of a star of spectrum F., Wilsing and Scheiner, Potsdam Publ. **19**, 1909, (64).
[25] Barnard, E. E., *Astrophys. Jour.*, **41**, 1915, (253–258).
[26] Coblentz, W. W., *Lick Obs. Bull.*, **8**, 1915, (121).
[27] Hertzsprung, E., *Astrophys. Jour.*, **42**, 1915, (352–356).
[28] *Harvard Circular, No. 188*, 1915.
[29] Cannon, Miss A. J., *Harvard Annals*, **91**, 1918, (274), (star No. 1546).
[30] Stebbins, J., *Pop. Astron.*, **25**, 1917, (657), and earlier papers.
[31] Guthnick, P., *Vierteljahrsschrift d. Astron. Gesellsch.*, **53**, 1918, (169).
[32] Shapley, H., *Astrophys. Jour.*, **43**, 1916, (180).
[33] Cannon, Miss A. J., *Pop. Astron.*, **25**, 1917, (314).
[34] Shapley, H., *Astrophys. Jour.*, **44**, 1916, (273–291).
[35] Shapley, H., *Ibid.*, **40**, 1914, (448–465).
[36] See numerous notes by Mrs. W. P. Fleming, *Harvard Annals*. **56**, (209–212).
[37] Adams, W. S. and Pease, F. G., these PROCEEDINGS, **1**, 1915, (391).
[38] Cannon, Miss A. J., *Harvard Annals*, **76**, 57.
[39] Hertzsprung, E., *Astrophys. Jour.*, **42**, 1915, (118).
[40] Kapteyn, J. C., *Ibid.*, **40**, 1914, (118–126).
[41] Russell, H. N., and Shapley, H., *Ibid.*, **40**, 1914, (422–423).
[42] Shapley, H., *Ibid.*, **48**, 1918, (282–287).
[43] Hertzsprung, E., *Zeitschr. f. Wiss. Phot.*, **3**, 1905, (442).
[44] Russell, H. N., *Pop. Astron.*, **22**, 1914, (275–294 and 331–357).
[45] Adams, W. S., these PROCEEDINGS, **2**, 1916, (157–163).
[46] Kapteyn, J. C., *Astrophys. Jour.*, **47**, 1918, (255–275).
[47] Russell, H. N., *Observatory, London*, **37**, 1914, (167–169).
[48] Strömberg, G., *Astrophys. Jour.*, **47**, 1918, (14).
[49] For example, the mean absolute magnitude given by Adams, (*Mt. Wilson Contribution*, No. 142) for the four Cepheid variables and eight other stars of Miss Maury's classes *c* and *ac* which, appear in his list, is + 0.3; and there is abundant evidence from the parallactic motions that the mean absolute magnitude of the stars of this peculiar class is at least as bright as − 2, and probably brighter.
[50] Kapteyn, J. C., *Astrophys. Jour.*, **47**, 1918, (263).
[51] Adams, W. S., *Publ. Astron. Soc. of the Pacific*, **27**, 1915, (236).
[52] Adams, W. S., *Ibid.*, **26**, 1914, (198).
[53] Comstock, G. C., *Astron. Journal, Albany*, **25**, 1907, (119).
[54] Barnard, E. E., *Ibid.*, **29**, 1916, (181).
[55] See summary in *Monthly Not. Roy. Ast. Soc.*, **78**, 1918, (304).
[56] Ludendorff, H., *Astron. Nach.*, **189**, 1911, (145–155).
[57] Russell, H. N., *Pop. Astron.*, **25**, 1917, (666).
[58] Eddington, A. S., *Monthly Not. Roy. Ast. Soc.*, **79**, 1918, (19).

[59] Shapley, H., *Astrophys. Jour.*, **48**, 1917–18, (89–124, 154–181).
[60] Trümpler, R., *Pop. Astron.*, **26**, 1918, (9).
[61] Leavitt, Miss H., *Harvard Circular*, **173**, 1912.
[62] Shapley, H., *Astrophys. Jour.*, **48**, 1918, (155).
[63] Wilson, R. E., *Pub. Ast. Soc. Pacific*, **27**, 1915, (86).
[64] Fowler, A., *Monthly Not. Roy. Ast. Soc.*, **77**, 1917, (516).
[65] Fabry, *Astrophys. Jour.*, **40**, 1914, (256).
[66] Wood, R. W., *Physical Optics*, 2d ed. (New York, 1911), p. 583.
[67] Fabry, *Astrophys. Jour.*, **45**, 1917, (269–277).
[68] Slipher, V. M., *Lowell Observatory Bulletin*, **2**, 1912, (24).
[69] Hertzsprung, E., *Ast. Nach.*, **195**, 1913, (449).
[70] Slipher, V. M., *Lowell Obs. Bull.*, **3**, 1918, (63).
[71] Bannard, E. E., *Astrophys. Jour.*, **49**, 1919, (1).
[72] Slipher, V. M., *Lowell Obs. Bull.*, **2**, (155).
[73] Campbell, W. W., *Pub. Ast. Soc. Pacific.*, **29**, 1916, (284).
[74] Campbell, W. W. and Moore, J. H., *Ibid.*, **27**, 1915, (240).
[75] Fabry, *Astrophys. Jour.*, **40**, 1914, (241).
[76] Slipher, V. M., *Pop. Astron.*, **23**, 1915, (23).
[77] Slipher, V. M., *Lowell Obs. Bull.*, **2**, 1914, (66).
[78] Young and Harper, *Jour. Roy. Ast. Soc. Canada*, **10**, (134.)
[79] van Maanen, A., *Astrophys. Jour.*, **44**, 1916, (210–228).
[80] Kostinsky, S., *Monthly Not. Roy. Ast. Soc.*, **77**, 1917, (233).
[81] Seares, F. H., these PROCEEDINGS, **2**, 1916, (553).
[82] Eddington, A. S., *Monthly Not. Roy. Ast. Soc.*, **77**, 1916–17, (16 and 596); also *Astrophys. Jour.*, **48**, 1918, (205).
[83] Jeans, J. H., *Observatory*, **40**, 1917, (60).
[84] Schwarzschild, K., *Astron. Nach.*, **190**, 1912, (361), and **198**, 1914, (217).
[85] Jeans, J. H., *Monthly Not. Roy. Ast. Soc.*, **76**, 1915–16, (70 and 552).
[86] Eddington, A. S., *Ibid.*, **74**, 1913, (5), and **75**, 1916, (366), and **76**, 1916, (37).
[87] Eddington, A. S., *Ibid.*, **76**, 1916, (572–585).
[88] Eddington, A. S., *Ibid.*, **79**, 1918, (2).
[89] Nolke, F., Abh. Nat. Ver. Bremen, **20**, Teil 2, 1911, (193–220). Jeans, J. H., *Observatory*, **42**, 1919, (40).
[90] Shapley, H., *Pub. Ast. Soc. Pacific.*, October, 1918, (283–298).
[91] Eddington, A. S., *Monthly Not. Roy. Ast. Soc.*, **79**, 1918, (19).
[92] Abbot, G. G., these PROCEEDINGS, **4**, 1918, (104–106).
[93] Nicholson, J. W., *Monthly Not. Roy. Ast. Soc.*, **76**, 1916, (415); **78**, 1918, (349), and earlier papers.
[94] deSitter, W., *Ibid.*, **78**, 1917, (25).
[95] As this paper goes to press, van Maanen, (*Pub. Ast. Soc. Pacific*, **31**, 1919, (42)) announces the discovery of a star of type F, and absolute magnitude 14.3 on Kapteyn's scale. If the surface brightness of this object has the value which is ordinarily associated with its color and spectrum, its diameter must be comparable with that of the Earth. These faint stars are evidently going to present a problem of great interest and difficulty.

ON THE SPECTRAL LINES OF A PULSATING STAR

By Harlow Shapley and Seth B. Nicholson

Mount Wilson Observatory, Carnegie Institution of Washington

Communicated by G. E. Hale, July 21, 1919

The hypothesis that the periodic variations in light and spectrum of a Cepheid variable are due to radial expansion and contraction of a single stellar body demands a behavior in the periodic oscillation of the spectral lines which is decidedly different from the behavior of the absorption lines of an ordinary spectroscopic binary star. In the latter case the line-shifts show, in accordance with the Doppler effect, the orbital motion of the star as a whole; in the former case the displacements of the spectral lines depend on the radial velocity of the part of the stellar disk from which the light emanates. The radial velocity due to a spherical pulsation diminishes from a maximum value at the center of the stellar disk to zero at the limb; the shifts, therefore, are not true bodily displacements of the normal spectral line, as in the case of orbital motion, but represent a broadening toward the violet while the star expands and toward the red while it contracts. Obviously a lack of symmetry as well as a widening is produced; but if the total displacement, due to radial movement of the absorbing layers, is of the same magnitude as the inherent width of the lines, it is clear that the asymmetrical broadening may not be readily noticed or measured. The object of this note is to examine in some detail the character of the spectral lines of a pulsating star.

For a star of unit radius undergoing alternate expansion and contraction symmetrically about the center, the radial velocity, in terms of its maximum value, for any point on the visible hemisphere is $v = \sqrt{1 - r^2}$, where r is the distance from the center of the apparent disk. Hence

$$r = \sqrt{1 - v^2} \tag{1}$$

The usual law of darkening, which has been found satisfactory in the case of the sun and eclipsing stars[1] and has subsequently been derived by Jeans[2] in his theory of stellar photospheres, may be written

$$\frac{J}{J_0} = 1 - x + x\sqrt{1 - r^2}$$

where J/J_0 is the surface intensity at the distance r from the center in terms of the central intensity, and x is the so-called darkening

coefficient. Then the total light of a star is given by

$$L = J_0 \int_0^{2\pi} \int_0^1 (1 - x + x\sqrt{1 - r^2})\, r dr d\theta$$

Introducing (1) and integrating once

$$L = 2\pi J_0 \left[\int_0^1 v(1 - x)\, dv + \int_0^1 v^2 x dv \right] \tag{2}$$

Suppose that from any given element of surface the width of the absorption lines is very small compared with the displacement due to radial motion of the element; and let the amount by which the continuous spectrum is decreased by a given absorption line be $L' = L$ *times* a constant. The equation of the intensity curve of a line in the spectrum of the whole surface would then be $\frac{I}{I_0} = \frac{dL'}{d\lambda}$ which may be written $\frac{dL'}{dv}$ because of the essentially linear relation (throughout very short intervals of the spectrum) between wave-length, λ, and velocity: and hence, from (2),

$$\frac{I}{I_0} = 2\pi \left[v(1 - x) + v^2 x \right] \tag{3}$$

where I_0 is the intensity for maximum absorption ($v = 1$) divided by 2π and is independent of the degree of darkening.

For a uniform disk $x = 0$ and

$$\frac{I}{I_0} = 2\pi v \tag{4}$$

while for one darkened to zero at the limb, $x = 1$, and

$$\frac{I}{I_0} = 2\pi v^2 \tag{5}$$

These intensity curves are plotted in figure 1. For a star of the spectral character of the sun, x is $\frac{3}{4}$ in the neighborhood of $\lambda = 4400$ A. Therefore

$$\frac{I}{I_0} = \frac{\pi}{2}(v + 3v^2)$$

and the intensity curve lies between those for uniform and completely darkened disks, but much nearer the latter, as shown by the broken curve in figure 1.

It is obvious from the above discussion that darkening at the limb will aid in concealing whatever asymmetry there may be in the spectral

lines of a pulsating star. From the eclipsing stars that most resemble the typical Cepheid variable stars in density, spectral type, and absolute luminosity, there is evidence in certain cases that the light diminishes more rapidly toward the limb than required by the above law. On the other hand, however, the relative intensity in the violet region of the continuous spectrum of Cepheids and other giant F-type stars

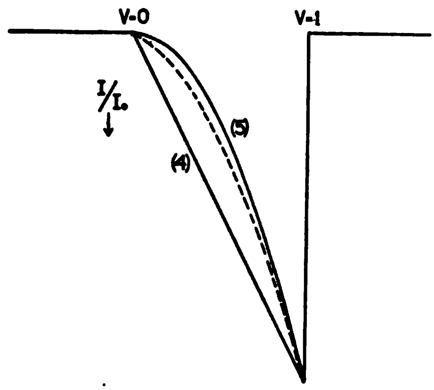


FIG. 1

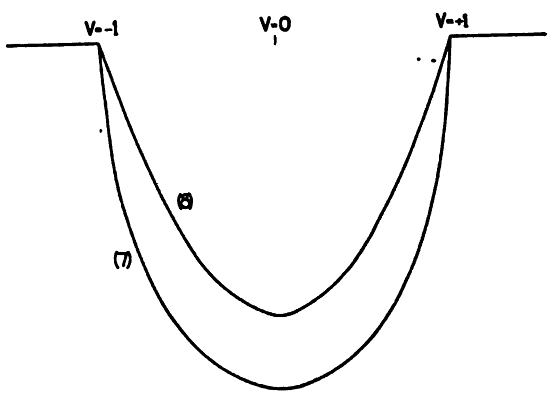


FIG. 2

may be urged against the assumption of excessive darkening. In any case, in view of the existence of large numbers of strong absorption lines throughout the spectra, it may be accepted that Cepheid variables are considerably darkened at the limb. For them the coefficient x may be reasonably assumed to lie between one-half and unity.

The preliminary assumption that spectral lines are inherently infinitesimal in width is of course untrue. Even if the radiation concerned

were strictly monochromatic the spectral lines would have a finite width since the resolving power of a spectrograph is not infinite. As is well known, however, the light giving rise to an absorption line is by no means monochromatic. Its distribution may be represented satisfactorily by

$$\frac{I'}{I'_0} = e^{-k^2\lambda^2} = e^{-k^2v^2/c^2} = e^{-k^2(v-v_n)^2} \qquad (6)$$

where v_n is the value of v for intensity I', corresponding to maximum absorption. The form of the resulting line, as reproduced by the spectrograph, has been discussed in detail by Wadsworth.[8]

Rotation of the star will also contribute to the widening of spectral lines. Since v, the projection of the rotational velocity in the line of sight, is numerically equivalent to the distance on the stellar disk from the axis of rotation, we may write $r^2 = v^2 + y^2$. Therefore the total light of the star is

$$L = \int\int J\, dv\, dy = J_0 \int_{-\sqrt{1-v^2}}^{+\sqrt{1-v^2}} \int_{-1}^{+1} [\,(1 - x) + x\sqrt{1 - v^2 - y^2}]\, dv\, dy$$

and

$$\frac{I}{I_R} = 2\,(1 - x)\sqrt{1 - v^2} + \frac{\pi x}{2}\,(1 - v^2)$$

In this case I_R is a function of the degree of darkening, being the intensity for maximum absorption (when $v = 0$) divided by $2\,(1 - x) + \frac{\pi x}{2}$.

Then for $x = 0$

$$\frac{I}{I_R} = 2\sqrt{1 - v^2} \qquad (7)$$

and for $x = 1$

$$\frac{I}{I_R} = \frac{\pi}{2}\,(1 - v^2) \qquad (8)$$

For comparison with figure 1, the intensity curves of the absorption lines for a rotating star (assuming monochromatic radiation and infinite resolving power) are shown in figure 2.

It can be shown without difficulty that if for an average Cepheid the period of rotation is less than two months the widening of the lines due to rotation will be decidedly greater than the widening due to pulsation.

Obviously, then, rotation of the star, resolving power, and the inherently finite width will all affect the appearance of the absorption lines, independently of changes due to pulsation. If the resolution is

not too small, however, and the widening due to rotation is not too large compared with the normal width of the line, these three factors will not appreciably affect the *form* of the line given by (6). The narrowest lines observed in stellar spectra are in width rarely less than 0.2A, which is of the same order as the semi-amplitude of the shift for a typical Cepheid variable.

The true form of a line in the spectrum of a pulsating star is therefore not as given in figure 1, but may be approximated by integrating (6) over the whole surface of the star. Any given incremental annulus on the stellar disk has a velocity v_n and contributes an incremental spectral line whose intensity curve is given by (6):

$$I' = I'_0\, e^{-K^2(v-v_n)^2}$$

The intensity of this line for maximum absorption, I'_0, occurs for $v = v_n$; and hence from (3),

$$I'_0 = I_n = 2\pi\, I_0 \left[v_n\,(1-x) + v_n^2\, x\right] \qquad (9)$$

Substituting (9) in (6)

$$I' = 2\pi\, I_0 \left[v_n\,(1-x) + v_n^2\, x\right] e^{-K^2(v-v_n)^2}$$

and summing up these incremental lines for all chosen values of v_n from 0 to 1, we obtain, as closely as we care to compute, the intensity curve from the whole surface:

$$I'' = F\,(v, x) = \sum_{v_n=0}^{1} I'$$

Writing $c\lambda = v$ from (6), and $I''_0 = 2\pi I_0$, we have, in the limit, for the intensity of the built-up absorption line at any point λ,

$$\frac{I''}{I''_0} = f\,(\lambda, x) = \int_0^1 \left[\,v_n\,(1-x) + v_n^2\, x\right] e^{-K^2(c\lambda - v_n)^2}\, dv_n \qquad (10)$$

Equation (10) has been integrated mechanically for different values of x and different ratios of line width to displacements; figures 3a and 3b show the resulting intensity curves for line widths of about 0.4A and 0.2A, respectively, adopting in both cases a radial velocity that would yield a measured displacement of about 0.3A. From these curves we obtain the results of the following table:

	WIDTH OF UNDISPLACED LINE	WIDTH OF DISPLACED LINE		MEASURED DISPLACEMENT		ACTUAL DISPLACEMENT AT CENTER OF DISK
		$x=\frac{1}{2}$	$x=1$	$x=\frac{1}{2}$	$x=1$	
From figure 3a..........	0.34A	0.40A	0.38A	0.30A	0.32A	0.40A
From figure 3b..........	0.17A	0.23A	0.22A	0.32A	0.33A	0.40A

The last three columns of the table show that, if the pulsation hypothesis is correct, the measured range in velocity is about 75% of the actual variation at the center of the disk.

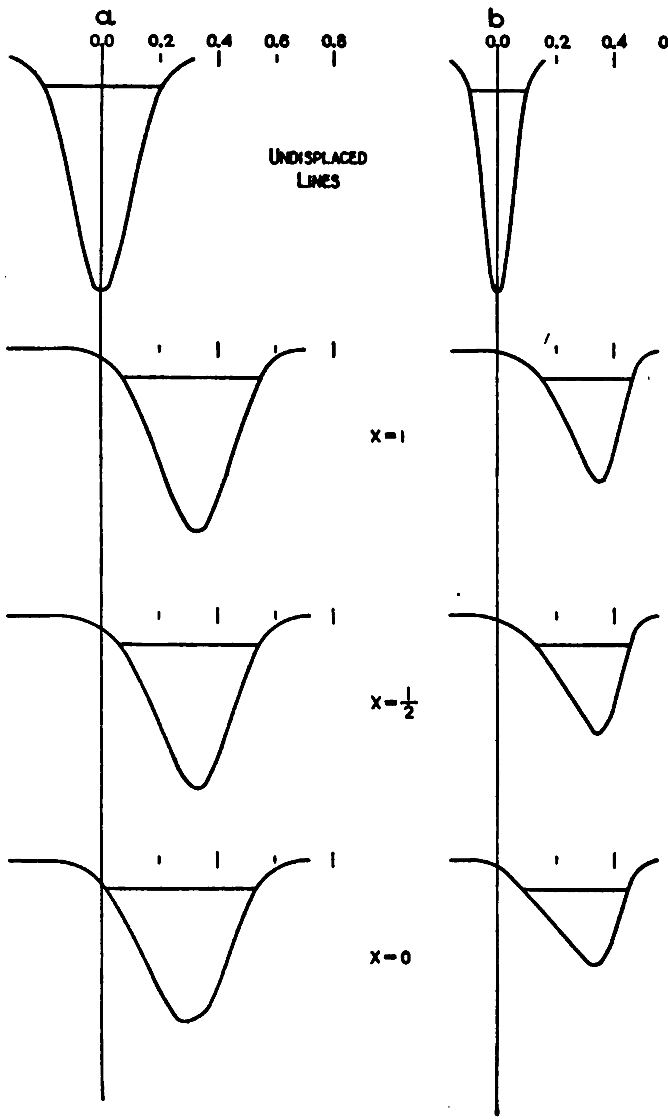


FIG. 3

There is considerable uncertainty in measures of the widths of absorption lines by ordinary methods, but it has been assumed that the measured widths lie between intensities of ½ I'_0, where I'_0 is the in-

tensity at maximum absorption of the undisplaced spectral line. It has also been assumed that the setting for the position of a line is such that the area under the intensity curve is divided equally and that the effect of absorption which is less than $\frac{1}{8} I'_0$ will not be appreciated; the results, however, are but slightly affected by the value assumed, provided it is within reasonable limits.

The lines of the spectrum of δ Cephei are much wider at minimum than at maximum light (and velocity).[4] Those shown in figures 3a and 3b are of the same width as observed in the case of δ Cephei at minimum and maximum, respectively, when a stellar spectograph of high dispersion and resolving power is used. The change observed in line width, according to results by Adams and by Young,[5] appears to have twice the period and about three times the amplitude of the variation which would be produced in a pulsating star by the distribution of velocity over the stellar disk; this latter variation, therefore, although it might be measurable if isolated, would probably be completely masked by the larger variation due to other causes.

Since practically all of the asymmetry is above the limit $\frac{1}{8} I'_0$ any deviation from a symmetrical form of the absorption lines would at all phases of the pulsation be inconspicuous on a spectrogram, and the lines would appear to be shifted bodily. The asymmetry would be most pronounced at maximum when the lines are comparatively narrow; but even then it seems doubtful if it could be observed if we are right in assuming that the lines for a Cepheid at maximum are best represented by curves such as those shown in figure 3, with values of x between $\frac{1}{2}$ and 1.

We conclude from these results that it is doubtful if either the periodic broadening or the slight asymmetry of the lines, due to a distribution of velocity over the stellar disk in accordance with the pulsation hypothesis, could be observed by any method thus far used.

[1] Shapley, Harlow, *Mt. Wilson Contr.*, No. 99, *Astrophys. J., Chicago, Ill.*, 41, 1915 (291–306), pp. 292–295.

[2] Jeans, J. H., *Mon. Not. R. Astr. Soc., London*, 78, 1917 (28–36), p. 35.

[3] Wadsworth, F. L. O., *Phil. Mag., London*, Series V, 43, 1897 (317–343).

[4] Adams, W. S., *Observatory, London*, 42, 1919 (167–168).

[5] Young, R. K., *J. R. Astr. Soc. Canada, Toronto, Can.*, 13, 1919 (45–54).

ETHYLENE CHLORHYDRIN

By M. Gomberg[1]

Chemical Laboratory, University of Michigan

Communicated, July 30, 1919

Within a few days after the so-called "Mustard Gas" was introduced (July 12–13, 1917) as a means of offence, it was definitely identified as β-β-dichlorethylsulphide. There was good reason to believe that it had been manufactured from ethylene chlorhydrin, according to the method described some thirty years previously by V. Meyer.[2] The reactions involved in that method are these: (1) Ethylene chlorhydrin in solution reacts with sodium sulphide and gives, in good yield, dihydroxyethylsulphide, which is non-poisonous; (2) this product gives on treatment with concentrated hydrochloric acid the highly toxic β-β-dichlorethylsulphide:

$$2\ HOCH_2CH_2Cl + Na_2S = (HOCH_2CH_2)_2S + 2\ NaCl$$
$$(HOCH_2CH_2)_2S + 2\ HCl = (ClCH_2CH_2)_2S + 2\ H_2O$$

The problem however was,—how to get the chlorhydrin itself? From the practical standpoint, a process based upon the additive reaction between ethylene and hypochlorous acid seemed most promising, notwithstanding the facts that this acid could only resist in concentrations of 1 to 3%, and that the best yield of chlorhydrin by this method was known to be not more than 30% of the theory.

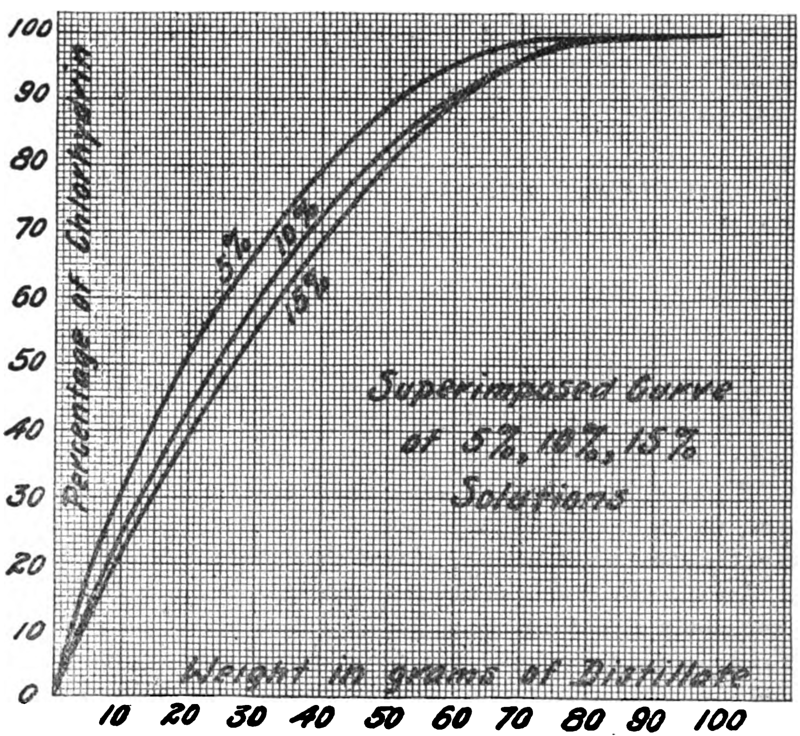
The reaction between chlorine and water was employed as the source of hypochlorous acid. As is well known, this reaction yields extremely little hypochlorous acid, the equilibrium favoring largely the left-hand side of the equation. Nonetheless, it was found that if ethylene and chlorine in equimolecular amounts are passed into cold water and the mixture is well stirred, chlorhydrin, and not ethylene chloride, is the principal product. This is explained on the assumption that reaction II proceeds with considerably greater velocity than reaction I.

$$\begin{array}{cccccc} \mathrm{I} & HOH + Cl & \rightleftharpoons & HOCl + HCl & \mathrm{II} \\ & C_2H_4 \downarrow & & \downarrow C_2H_4 & \\ & C_2H_4Cl_2 & & ClC_2H_4OH & \end{array}$$

While a concentration up to 15% of chlorhydrin can be attained in this way, in practice it has been found advisable to stop with a concentration of 7 to 8%. The progress of the reaction is ascertained by distilling a small sample of the product and determining its refractive index, that of water being 1.333 while that of chlorhydrin was found to be 1.442.

It was established that not only is it unnecessary to keep on neutralizing the hydrochloric acid produced with the progress of the reaction, but that in fact it is inadvisable to do so.

Separation of chlorhydrin.—Pure chlorhydrin boils at 128°C. It was found, however, that when mixed with water in proportion of 42.5 parts of the former and 57.5 parts of the latter, the two form a constant boiling mixture which distills at 95.8°. Consequently, aqueous solutions of chlorhydrin poorer than 42.5% tend to give on distillation initial fractions approaching in composition the constant boiling mixture; solutions richer than 42.5% give as the final fractions pure chlorhydrin. With the addition of salt or calcium chloride to the liquid to be distilled, concentrations of chlorhydrin up to 80% can be readily obtained. The diagram shows what happens when 5, 10 and 15% solutions of chlorhydrin are subjected to distillation.



Chlorhydrin is miscible with water in all proportions. It has, however, been found that it can be salted out from its aqueous solutions, provided that the proper conditions as to initial concentration, etc., are

observed. Solutions containing 70% of chlorhydrin can readily be obtained in this manner.

It was also found possible to extract the chlorhydrin from its aqueous solutions by some immiscible solvent, benzene suggesting itself as the best from the practical standpoint. Thus, by a judicious combination of the three methods,—distillation, salting out and extraction,—chlorhydrin could readily be obtained in any desired concentration and purity.

[1] The investigation was done under the auspices of the Bureau of Mines, War Gas Investigations Department, December 1917–July, 1918. The paper in detail, approved for publication by Major-General William L. Sibert, Director of the Chemical Warfare Service, U. S. A., will appear in the *Journal of the American Chemical Society.*

[2] Meyer, V., *Berlin, Ber. D. chem. Ges.*, **19**, 1886, (3260). Clarke, H. T., *London, J. Chem. Soc.*, **101**, 1912, (1583).

STUDIES OF THE CONSTITUTION OF STEEL

By Edward D. Campbell

Chemical Laboratory, University of Michigan

Communicated by M. Gomberg, July 30, 1919

Researches on steel, considered as a solid solution, were carried on in this laboratory by the late J. W. Longley prior to 1876.[1] Other investigations of iron and steel were carried on in the interval between 1876 and 1891, at which latter time the present writer began a series of researches on the chemical constitution of the carbides of iron. The results of these investigations have gone to show that the carbides found in steel have a much more complex structure than would be indicated by the commonly accepted formula Fe_3C. The idea that the carbides of iron and many other metals can best be studied if the carbides are regarded as metallic substitution products of hydrocarbons was advanced[2] in 1896. In addition to recognizing the complex molecular constitution of the carbides, the assumption has been made that the atomic relations existing between the carbides or other solutes dissolved in iron are essentially the same as those which exist between the molecules of substances in aqueous solution and the water in which they are dissolved. Some further evidence in support of the hypothesis of the unity of mechanism of all solutions without regard as to whether the solvent is solid or liquid, metallic or a non-conductor of electricity, is given in two papers, one of which will be read at the Autumn meeting of the Iron and Steel Institute and the other at a forthcoming meeting of the Faraday Society.

In the first of these it is shown that from twelve samples of steels, including straight carbon steel, high silicon, high phosphorus, manga-

nese, five nickle steels, chrom-steel, chrom-tungsten and chrom-molybdenum steel, most of the carbon and a considerable proportion of the sulphur may be removed by heating bars of the metal in a slow stream of moist hydrogen, the temperature being maintained for from four to twelve days between 950°C. and 1000°C. Since the elements other than carbon and sulphur are not affected by hydrogen, this method of treatment affords a very satisfactory means of changing the carbide concentration without material change in composition, since the percent of sulphur is usually quite low in commercial steels.

The experimental data given in the second paper, "The Solution Theory of Steel and the Influence of Changes in Carbide Concentration on the Electrical Resistivity" demonstrate clearly that Benidicks' Law,[3] "Equiatomic concentrations in iron possess equal resistivities," is not tenable. It is the molecular, not the atomic concentration in metallic solutions, which determines the electrical resistivity, just as it is the molecular and not the atomic concentration which determines conductivity in aqueous solutions.

The fallacy contained in the conclusion drawn by Le Chatelier[4] that chromium, tungsten and molybdenum have but slight influence on the electrical resistance of steel, may be explained by the fact that when these elements are present in steel, they form with carbon complex carbides, so that the molecular concentration of the carbides is little if any greater than if chromium, tungsten and molybdenum were absent. If, however, the carbon is removed, the chromium, tungsten or molybdenum will itself combine with or dissolve in the iron, each thus producing an increase in electrical resistivity nearly equal to that which would be produced by an equal atomic concentration of carbon alone.

The force-field theory of solution developed by E. C. C. Baly[5] by a series of investigations of the action of light on aqueous solutions, may be applied to solid solutions in metals and, assuming the unity of mechl anism of these and of aqueous solutions, can be made to give a rationa-explanation for thermal and electrical resistivity, as well as for the thermo-electromotive properties of solutes in solid solutions.

Many other properties of solid solutions will also probably be found capable of explanation by the force-field theory.

[1] On the Relationship of Structure, Density and Chemical Composition of Steel, *Amer. Chemist*, **7**, Nov. 1876, (175–178).

[2] Campbell, E. D., *Amer. Chem. J.*, **18**, 1896, (719–723).

[3] *Zs. physik. Chem.*, **40**, 1902, (545).

[4] Le Chatelier, *Paris, C. R. Acad. Sci.*, **126**, 1898, (1709, 1782).

[5] Baly, E. C. C., *J. Amer. Chem. Soc., Easton, Pa.*, **37**, 1915, (979).

INHERITANCE OF QUANTITY AND QUALITY OF MILK PRODUCTION IN DAIRY CATTLE

By W. E. Castle

Bussey Institution for Research in Applied Biology, Forest Hills, Mass.

Communicated, August 1, 1919

In 1911 the late T. J. Bowlker undertook at his farm in Framingham, Mass., an experimental study of inheritance in dairy cattle by the modern method of crossing pure breeds and looking for a recombination in the second crossbred generation of the characters differentiating the breeds crossed. The breeds which he selected for study were the Holstein-Friesian and the Guernsey, one supreme among dairy breeds in quantity production, the other of very high rank as regards quality of milk produced. It was his belief that if quantity and quality of milk production were independently inherited characters, it should be possible to combine them in a single race by the method of crossbreeding, in accordance with Mendel's law. The desired recombination of qualities, if attainable, would be of much importance to the dairy industry, and at any rate the knowledge whether such recombination is attainable would be a valuable contribution to science. With rare insight into the difficulties surrounding the problem and the proper method of attacking it, Mr. Bowlker planned his experiment on a considerable scale. He had a herd of some 40 pure-bred registered cows, about two-thirds of them being Holsteins, the rest being Guernseys. He also had a pure-bred registered bull of each breed, and in the pedigrees of these bulls excellent blood-lines were represented. He decided to cross-breed the entire herd, mating the Holstein cows to the Guernsey bull and the Guernsey cows to the Holstein bull. In this way what are known as reciprocal crosses were made between the two breeds. In all about 140 F_1 calves were produced between the years 1912 and 1919. As ıast as the F_1 heifers attained suitable age they were bred to F_1 bulls in order to secure the desired F_2 generation, in which combination of characters might be expected. About 35 living F_2 calves have been produced in this way of which the heifers have been saved for milking tests, for which, however, they are still too young.

Mr. Bowlker did not live to see the completion of his experiment but died in February, 1917. His family undertook to complete the unfinished work, with such scientific advice as I could give them, but practical difficulties having arisen which make it impossible to carry the work

further on the Bowlker farm, arrangements have been made for turning the entire project over to the University of Illinois, where experts in dairying and genetics will study the results in hand and such as may be forthcoming. At the generous suggestion of the new owners of the herd I am making this preliminary statement of results obtained at the Bowlker farm.

Throughout the experiment the herd has been under the same system of management and in charge of the same superintendent, Mr. Leon Haley. The care and feeding have been such as are given to ordinary farm herds and the cows have been milked twice a day. Pure breds and crossbreds have been kept in the same barn and treated alike in every respect. The milk from each cow has been weighed at each milking and these records are now available for study. Occasional butter-fat tests have been made for each cow but these are fewer than could be wished and make it possible only approximately to estimate the total butter fat production of each cow.

Both the F_1 and the F_2 cross-bred calves have proved vigorous and have grown well. Some data have been accumulated on the relative growth-rates of the pure-bred and the cross-bred calves but these are insufficient as yet to lead to any definite conclusions. It should be observed, however, that the F_1 cows have calved for the first time at a slightly earlier age than the cows of either pure breed kept on the farm, a bit of indirect evidence that in vigor and early maturity they conform to the usually high standard of cross-breds. The recorded observations on weight of milk given by pure bred and by F_1 cows and the estimated amounts of butter-fat contained in this milk are summarized in tables 1, 2 and 3. Only data for comparable ages and lactation periods (first and second) are included in the tables. The column 'age' shows the approximate age of the cow at the beginning of the lactation period. The column 'time' shows the duration in months of the lactation period. If no entry is made in this column it will be understood that the lactation period covered a full year. The column 'total pounds milk' shows the amount of milk produced during the twelve months or less of the lactation period. No account is made in this summary of milk produced after the cow had been in lactation for more than twelve months.

Table 2 shows that 25 pure-bred registered Holsteins calving at an average age of 2.8 years gave in their first lactation period an average of 7673 pounds of milk. The amount of butter-fat in the milk can be stated for only 8 of the 25 cows those which were still in the herd when the systematic taking of butter-fat tests for individual cows was undertaken. For these 8 cows the estimated butter-fat percentage is 3.4.

Since this agrees closely with Roberts'[1] finding for the breed in general, it will be assumed to hold for the entire group of 25 cows. On this basis the average butter-fat production for the Holstein cows in their first lactation period would be 261 pounds.

TABLE 1

MILK AND BUTTER-FAT PRODUCTION OF F_1 COWS

COW	FIRST LACTATION PERIOD					SECOND LACTATION PERIOD			
	Age	Time	Total pounds of milk	Fat	Total fat	Age	Time	Total pounds of milk	Total fat
		months		*per cent*			*months*		
*Alice	2⅓		8,070	4.0	323				
Annie	2¾	9	(5,460)	4.3	235	3½		7,536	324
Bessie	3⅓	10	(6,297)	4.0	252				
Blackfoot	2¼		7,755	4.1	318	4⅙		10,125	415
*Daisy	3⅓	11	(7,661)	4.3	329	4⅙		9,375	403
Dora	2¾		7,908	4.2	332	4		11,538	484
Ellen	3⅙	9	(6,096)	4.2	256	4⅙	11½	(8,949)	376
Emerald	2½		8,517	4.1	349				
Emma	2¾	7	(3,768)	4.0	151	3⅔	9½	(7,080)	283
Fairiel	2¼		4,740	4.6	218				
Fanny	2⅝	9	(5,325)	4.0	213	3¾		9,135	365
*Freckles	2⅙		7,773	4.0	311	3⅓		8,340	333
*Grace	3	11	(5,796)	4.3	249				
Inka	2⅓		7,881	4.3	338				
Jane	2¼		7,809	4.5	351	4⅓	11	(8,646)	389
Julia	2⅝	10½	(6,498)	4.6	279	4		8,835	406
*Kate	2¾		7,689	4.5	346				
Lavaine	2½		5,550	3.7	205				
*Lilvene	3⅙	11	(6,000)	4.1	246				
Lilac	2⅙	11	(7,542)	3.8	286	3⅙		6,489	246
*Mary	2¾		(8,199)	4.0	327	4½	11	(8,871)	355
Nancy	2½		4,899	4.3	210				
Peach	2⅓		8,508	3.8	323				
Pearl	2½		7,446	3.7	275				
Peggy	2½		6,507	3.8	247				
*Ruby	2½	9	(4,440)	3.7	164				
Sapphire	3⅙		6,618	4.0	264				
Susan	2⅙	10½	(6,168)	4.1	252				
Sylvene	2⅙		6,075	4.0	243				
Tessa	2½	11½	(5,700)	4.4	250	3½		7,710	339
Twisk	2½		6,252	3.8	237				
Average	2.61	11.14	6,612	4.08	270	3.88		8,663	363

In table 3 are summarized the milk production records of 8 pure-bred registered Guernsey cows which also formed a part of the Bowlker herd. They calved for the first time at an average age of 2.7 years and gave in a first lactation period of from seven and one-half to twelve months

an average of 4617 pounds of milk. The recorded butter-fat tests of these cows are too few to form an adequate basis for estimates of their individual butter-fat production. But such tests as were made agree sufficiently well with Roberts' estimate of 5.0% for the breed. On this

TABLE 2

MILK AND BUTTER-FAT PRODUCTION OF PURE-BRED REGISTERED HOLSTEIN COWS

COW	FIRST LACTATION PERIOD					SECOND LACTATION PERIOD			
	Age	Time	Total pounds of milk	Fat	Total fat	Age	Time	Total pounds of milk	Total fat
		months		*per cent*			*months*		
H. S. Clotilde	$2\frac{1}{2}$		7,765	3.5	271	$3\frac{5}{8}$	$10\frac{1}{2}$	(9,803)	343
H. S. Inka	3		9,089	3.4	309	$4\frac{1}{4}$	8	(8,866)	301
H. Perfecta Clotilde	$3\frac{1}{4}$	$9\frac{1}{2}$	(8,030)	3.2	257	$4\frac{1}{4}$	11	(10,025)	320
H. S. Judy Colantha	$3\frac{1}{4}$		8,875	3.6	319	$4\frac{1}{2}$	11	(9,455)	340
H. S. Katharine	$2\frac{3}{4}$		8,247	3.2	264	$4\frac{1}{2}$	10	(9,263)	296
H. S. Pietertje	$2\frac{1}{2}$	$9\frac{1}{2}$	(6,887)	3.2	220	$3\frac{1}{2}$	9	(8,631)	276
H. S. Twisk	$2\frac{1}{2}$	9	(7,701)	3.4	261	$3\frac{1}{2}$		10,312	350
H. S. Wachusett	$2\frac{1}{2}$	10	(6,481)	3.6	233	$3\frac{1}{2}$		10,813	392
H. M. Cre. DeKol	$2\frac{3}{4}$	$8\frac{1}{2}$	(5,432)		Av. 267	$3\frac{2}{3}$	11	(7,619)	Av. 327
H. S. Erie Colantha	$2\frac{1}{2}$		6,417			$3\frac{1}{2}$		10,470	
H. S. Erie Vernon						$3\frac{2}{3}$		10,248	
H. S. V. Julipana	$3\frac{1}{3}$		8,582						
H. S. Landor	$2\frac{11}{12}$		7,725						
H. M. Segis	$2\frac{5}{6}$	$9\frac{1}{2}$	(8,900)			$3\frac{3}{4}$		10,327	
H. S. Vale Judy	3		10,101						
H. S. Veldoor	$3\frac{1}{4}$	$10\frac{1}{2}$	(7,445)			$4\frac{1}{6}$	10	(9,555)	
H. Cre. Mercedes DeKol	$3\frac{1}{6}$	$8\frac{1}{3}$	(8,928)			$4\frac{1}{3}$	9	(8,186)	
H. M. Judy Colantha	$2\frac{2}{3}$		5,493			$4\frac{1}{6}$	$7\frac{1}{2}$	(6,170)	
H. L. Whetstone DeKol	$2\frac{11}{12}$	$9\frac{1}{3}$	(5,230)						
H. A. Cre. DeKol	3		7,479			$4\frac{1}{3}$	$11\frac{1}{2}$	(8,961)	
H. I. Colantha DeKol	$2\frac{1}{3}$		7,692			$3\frac{3}{4}$	$9\frac{1}{2}$	(7,490)	
H. Tessa Netherland	$2\frac{5}{6}$		9,699			$4\frac{1}{2}$	$10\frac{1}{2}$	(9,190)	
H. C. Plum	$2\frac{5}{6}$		8,628			$4\frac{1}{4}$		11,989	
H. S. Dexter Erie	$2\frac{1}{2}$		7,945						
H. Judy Cre. DeKol	3	$9\frac{1}{2}$	(7,399)			4	$11\frac{1}{2}$	(12,041)	
Average	2.825	11.0	7,673	3.4	261	4.0		9,475	322

basis the average butter-fat production of the 8 Guernseys in their first lactation period would be 231 pounds.

Table 1 shows the observed milk and estimated butter-fat production of 31 F_1 cows which calved at the average age of 2.6 years, one-tenth of a year younger than the Guernseys, two-tenths of a year younger than the Holsteins. They produced an average of 6612 pounds of milk.

This is 1061 pounds less than the average yield of the pure Holsteins, but is 1995 pounds more than the average yield of the pure Guernseys. In other words, the average yield of an F_1 cow exceeds the half-way point between Holsteins and Guernseys by 467 pounds.

As regards butter-fat content of the milk, several different tests have been made of the milk of each F_1 cow and on the basis of these tests rest the estimated percentages given in the table. They indicate an average butter-fat content of 4.08%, which is slightly less than the intermediate between 3.4, the butter-fat percentage of pure Holsteins, and 5.0, the butter-fat percentage of pure Guernseys, for the exact intermediate is 4.2%. But the high quantity production of the F_1 cows more than makes up for the slight deficiency in the quality of their milk, so that

TABLE 3

MILK AND BUTTER-FAT PRODUCTION OF PURE-BRED REGISTERED GUERNSEY COWS

COW	FIRST LACTATION PERIOD			SECOND LACTATION PERIOD		
	Age	Time	Total pounds of milk	Age	Time	Total pounds of milk
		months			*months*	
Hailey All Fawn			5,014	$4\frac{1}{3}$	11	(5,558)
H. D. Keepsake	$2\frac{2}{3}$	9	(3,825)	$3\frac{2}{3}$		7,664
H. D. Lady	3	8	(2,897)	$3\frac{5}{6}$		4,852
H. H. Dairymaid	$2\frac{5}{6}$		6,942	$4\frac{1}{12}$		7,093
Hailey Gloria	$2\frac{1}{3}$	$11\frac{1}{2}$	(5,548)	$3\frac{7}{12}$	$11\frac{1}{2}$	(6,084)
H. Modena Maid	3	$11\frac{1}{2}$	(6,060)	4	$8\frac{1}{2}$	(3,983)
Rockbottom Keepsake	$2\frac{5}{12}$	9	(4,529)	$3\frac{1}{2}$	$10\frac{1}{2}$	(4,209)
Rockbottom Leading Lady	$2\frac{5}{12}$	$7\frac{1}{2}$	(2,121)	$3\frac{1}{3}$	$10\frac{1}{3}$	(5,307)
Average	2.72	10.0	4,617	3.79		5,593

the average total butter-fat production of each cow is 270.2 pounds, which is 9 pounds more than the average butter-fat production of the pure Holsteins, and 40 pounds more than the estimated butter-fat production of the pure Guernseys. Accordingly on the basis of the records for their first lactation period the F_1 cows seem to surpass either pure breed in butter-fat production and to be better than an intermediate between the pure breeds as regards quantity of milk produced.

A second lactation period has been completed by 13 of the 31 F_1 cows listed in table 1. They of course produce more milk in the second lactation period than in the first, and so comparison should be made with the second lactation period of pure-bred Holsteins and Guernseys, data for which are given in tables 2 and 3. Roberts[1] has shown that the butter-fat percentage of a cow's milk does not change materially with age, so

that we assume it to be for each cow the same in the second lactation period as in the first.

Table 2 shows that 20 pure-bred Holsteins of average age 4.0 years, when the second calf was born, produced an average of 9475 pounds of milk in the second lactation period. Table 3 shows that 8 pure-bred Guernseys of average age 3.8 years when the second calf was born, produced an average of 5593 pounds of milk in the second lactation period. Table 1 shows that the 13 F_1 cows being of average age 3.9 years when the second calf was born, produced an average of 8663 pounds of milk in their second lactation period. This is 812 pounds less than the average production of the pure Holsteins, but is 3070 pounds more than the average production of the pure Guernseys and *exceeds the intermediate between the pure breeds* by 1129 pounds. The deviation is of the same sort as in the first lactation period but is even more striking. As regards quantity production it is clear that the F_1 cows are better than an intermediate between the breeds crossed.

In butter-fat production, using the same percentage estimates as were used for the first lactation period, the Holsteins average 322 pounds, the Guernseys 280 pounds, and the F_1 cows 363 pounds, an excess for the F_1 cows of 41 pounds over the pure Holsteins and of 83 pounds over the pure Guernseys. Again the F_1 cows show superiority over either pure breed in butter-fat production, while in quantity production of milk they are nearer to pure Holsteins than to an intermediate between the pure breeds.

This result is comparable with that observed in the inheritance of size and other quantitative characters both in animals and in plants. In such cases F_1 usually surpasses more or less the intermediate between the races crossed owing to the superior vigor commonly possessed by cross-bred organisms. But F_2 usually varies about a strict intermediate between the races crossed and such we may anticipate will probably be the outcome in this case unless quantity production and quality production to some extent depend on different genetic factors, the point which Mr. Bowlker set out to investigate. If independent genetic factors are really concerned in quantity production and in quality production respectively, their recombination in F_2 may be expected in animals which are both high producers and which give rich milk and such animals should be capable of transmitting this desirable combination of characters to their descendants. Such a result would realize Mr. Bowlker's fondest hopes, and would open great possibilities for the systematic improvement of dairy cattle.

A word should be said as to the theoretically important question whether the reciprocal crosses differ in their results. No consistent difference is observable. Eight of the 31 F_1 cows (indicated by an asterisk in table 1) had Guernsey dams while 23 had Holstein dams. In their first lactation period the cows with Guernsey dams gave less milk but more butter-fat, average 6953 pounds of milk, 287 pounds of butter-fat. But in the second lactation period, three cows with Guernsey dams give a little more than the average amount of milk with exactly the average amount of butter-fat. It seems unlikely, therefore, that any sex-linked factors are concerned in the case.

[1] Roberts, Elmer. Correlation between the percentage of fat in cow's milk and the yield. *J. Agric. Res.*, **14**, No. 2, July 8, 1918.

STUDIES OF MAGNITUDES IN STAR CLUSTERS, X. SPECTRAL TYPE B AND THE LOCAL STELLAR SYSTEM

By Harlow Shapley

Mount Wilson Observatory, Carnegie Institution of Washington

Communicated by G. E. Hale, August 26, 1919

In an earlier communication[1] it has been suggested that the properties of star-streaming and the observed decrease of stellar density with increasing distance from the sun, as well as other observational phenomena, can be explained if the sun is shown to be not far from the center of a large physically-organized star cluster or cloud, which is imbedded in a typical stellar region of the galactic system. Evidence of the existence of such a local group, intermingled with stars of the galactic field, was found from various sources; in particular, the distribution of the brighter stars of spectral type B directly supported the hypothesis that a much flattened local system, of limited extent, surrounds the sun asymetrically, with its equatorial plane inclined some 12 degrees to the plane of the Milky Way.[2] Until recently we have commonly supposed that this stellar assemblage, which is now called the local cluster, constitutes the major part of the known sidereal system; at present we hold it to be a very minor part of the galactic organization.

The completion at the Harvard College Observatory of the first three volumes of the *Henry Draper Catalogue* of stellar spectra permits examination of the distribution of the fainter B-type stars as a graphical test of the existence, dimensions, and inclination of the local system. Miss Cannon has kindly supplied the proof sheets of the third volume of the catalogue in advance of publication. The region covered by

these three volumes is that most important for the present problem, since the spectra of all B stars (down to magnitude 8 or 8.5 and many fainter ones) from zero to nine hours in right ascension are included, and in the later hours of right ascension the stars of the Milky Way and those of the local cluster are much less clearly separated on the face of the sky. This difference in separation in opposite hemispheres results from the fact that the sun is some distance to one side of the central plane of the local cluster, thus giving a conspicuous 'dip' to the encircling inclined belt of cluster stars, which minimizes the separation from the galactic equator in Scorpio and Ophiuchus and exaggerates it the most in Taurus and Orion.

The amount of the apparent separation from the Milky Way for any given galactic longitude naturally depends on the average distance (brightness) of the cluster stars concerned. For those not more than 200 parsecs distant from the sun, the maximum separation from the galactic equator is about 17° in Orion and Taurus, as indicated by the broken curve in all the accompanying figures. For stars at the distance of 400 parsecs, which appears to approximate the equatorial radius of the cluster, the maximum separation is a little less than 14 degrees, as shown by the lightly dotted curve in figure 2.

Right ascension and declination are rectangular coordinates in figure 1, which shows the course of the Milky Way throughout the first thirteen hours of right ascension. The data are taken from *Harvard Annals*, **50,** which was the chief source of information regarding stellar spectra before the appearance of the *Henry Draper Catalogue*. Each plotted point in figure 1 shows the position of a star of spectral type B (the subdivisions B8 and B9 are not classed with the other B's—a customary procedure in statistical discussions of spectra). The full curve represents the galactic equator, that is, the central line of the Milky Way after correction for the small dip due to the sun's position slightly to the north of the galactic plane. The broken curve shows the projection of the plane of the local cluster. The determination of this curve was based upon stars brighter than magnitude 5.5 (*Mt. Wilson Contr.*, No. 157, p. 21); if the stars to the sixth magnitude, as plotted in figure 1, had been used in the computation, it is very probable that the curve would be slightly nearer to the galactic equator.

It is clear at once from figure 1, that these B-type stars are affiliated with the local cluster. We may reasonably assume that about twenty of them, along the galactic plane, may be actual members of the galactic field, and that the other three hundred are members of the subordinate local system, quite unrelated to the plane of the Milky Way. On the

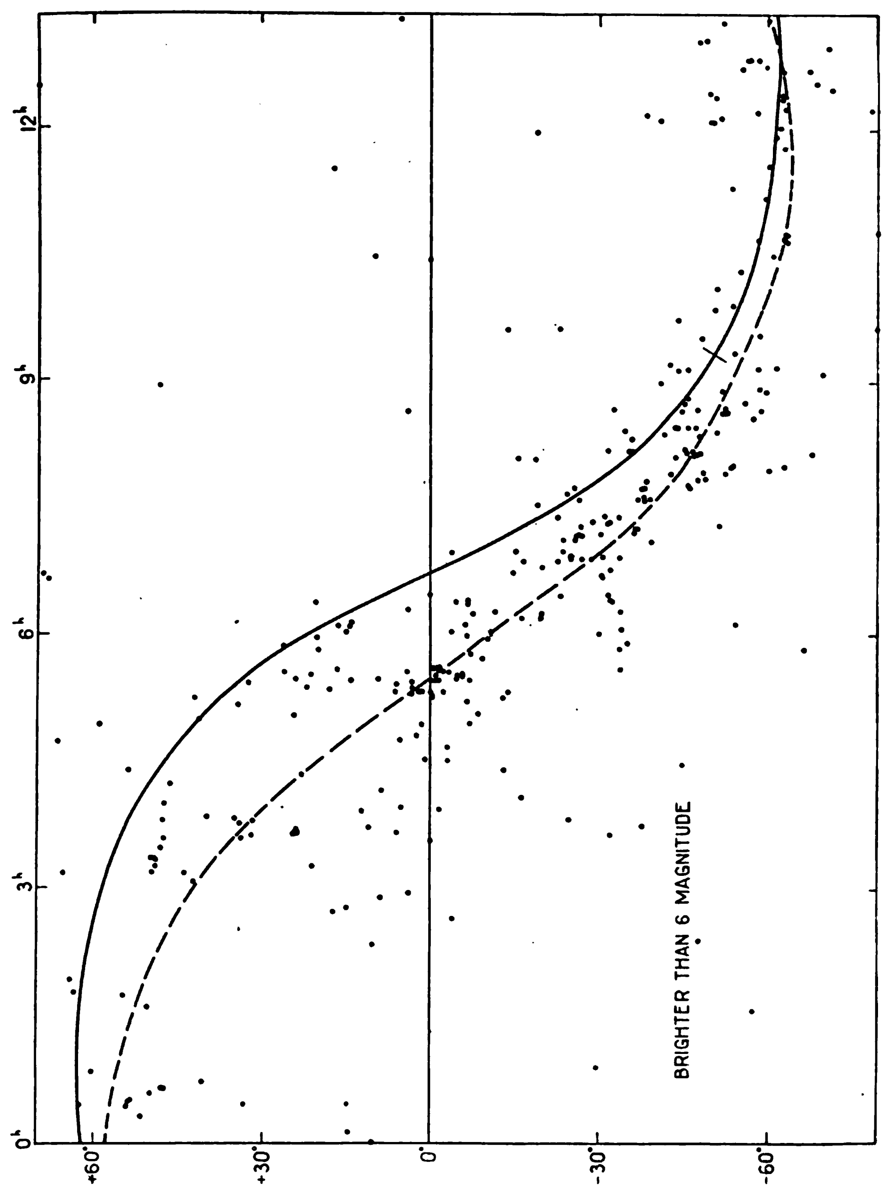


Fig. 1. Distribution of B0–B5 stars, from *Harvard Annals, 50*. In all three figures ordinates are declination, abscissæ are right-ascension. The curves are described in the text.

east side of the Milky Way (in the part of the sky under consideration) there are scarcely any bright B-type stars; on the west there are hundreds of them. The remarkable inequality gives rise to what was called, in an earlier paper, the 'Secondary Galaxy.' This sub-ordinate Milky

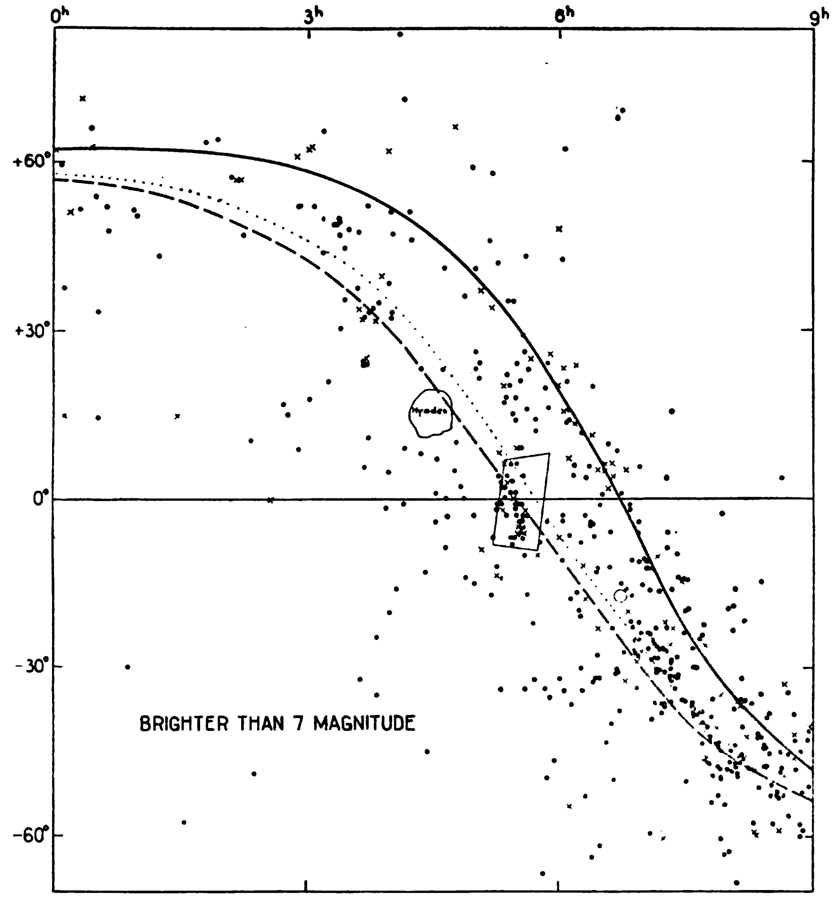


Fig. 2. Distribution of bright B stars, from the *Henry Draper Catalogue*. Crosses refer to sub-types B0, B1, and B2 (and undefined B's); the dots refer to sub-types B3 and B5. The curves are described in the text. The large circle shows the position of Sirius; the trapezoid indicates the constellation Orion. The direction of the center of the local cluster falls near the lower right-hand corner of the diagram.

Way, though well defined by star analyses throughout its complete circuit of the sky, to the naked eye is most clearly seen in the region mapped in figure 1, where it stands out some 10 to 20 degrees along the west side of the Milky Way from Perseus southward through Taurus, Orion, and Canis Major to Puppis and Carina. Not only the very

bright stars of types B and A throughout this region may be assigned to the cluster, but also the rich belt of stars of all types with magnitudes from three to seven. Across from Orion and Canis Major, on the other side of the Milky Way and at the same distance from the galactic equator, we find in Cancer and Hydra no counterpart of this richness in

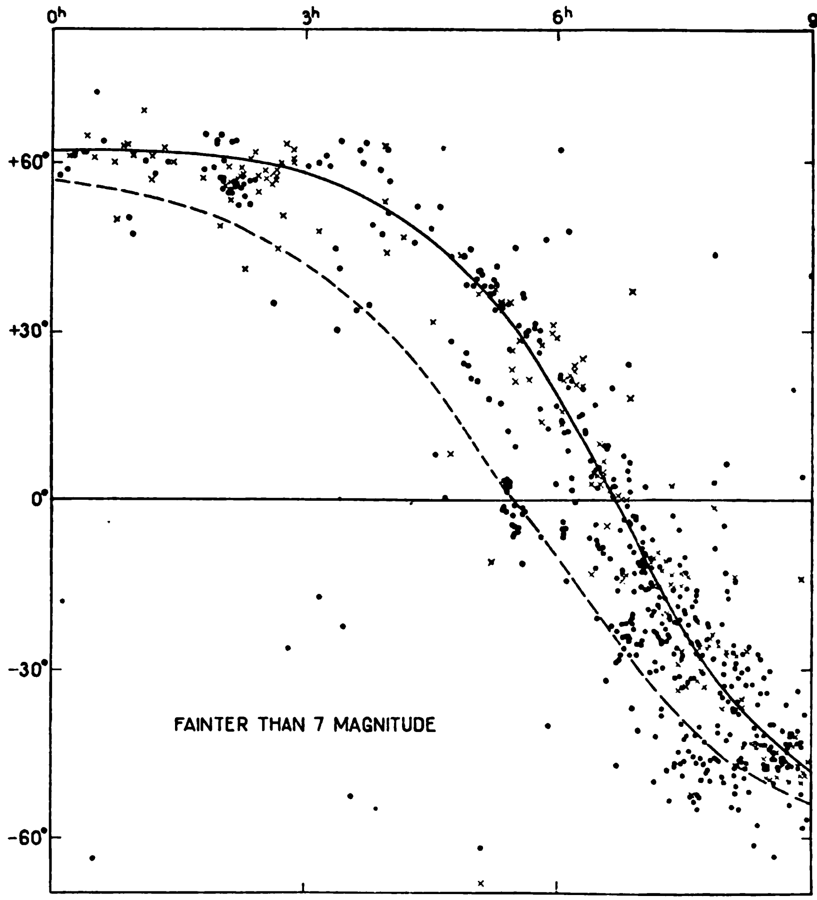


Fig. 3. Distribution of faint B stars, from the *Henry Draper Catalogue*. The wide extent of the bright double cluster in Perseus is shown by the points near declination + 60° and right ascension 2^h.

naked-eye stars; and the intervening constellation Monoceros is almost destitute of stars brighter than the sixth magnitude, although it lies along the central line of the Milky Way and is surpassingly bright in telescopic stars of the far-extending galactic segment.

From the recent studies by Walkey, Charlier, and Strömberg it is known that the center of the stellar system that immediately surrounds

the sun is in the general direction of the constellation Carina. The approximate galactic longitude of this center (which is to be taken as the center of the local cluster rather than that of the galactic system) is indicated in figure 1 by the short line across the galactic equator. A greater frequency of stars is to be noted in this direction, especially of the fainter objects plotted in figures 2 and 3.

We recall that Strömberg's researches,[8] through showing that stars of types F, G, and K appear to have motions related to the same center as that determined for the B-type stars, may be taken as strongly supporting the hypothesis of a local cluster which involves all types of stars. The observed decrease of stellar density with distance from the sun, for all the principal spectral classes, is also almost conclusive evidence that the phenomena of the local cluster are not confined to the B stars alone. It appears highly probable, therefore, that the inclination and extent now under investigation refer to the cluster as a whole, and not only to these blue giant stars which, because of their uniformity in absolute brightness and their apparently high concentration to dynamically central planes, are of such high value in outlining a stellar system.

The available volumes of the new catalogue of spectra cover less area than shown in figure 1, but the extension to the seventh magnitude (fig. 2) adds definitely to our knowledge of the cluster. Stars of the seventh magnitude are 1.6 times as far away as those of the sixth when the intrinsic brightness is the same. Increasing the depth of our survey by that amount has therefore brought in many B-type stars of the galactic field, objects concentrated to the galactic equator, and it has not introduced a great many stars of the local system. Hence, there is a suggestion in figure 2 that we are attaining the edges of the local cluster when we go out to the B-type stars of the seventh magnitude.

Extending the survey much deeper into space by including the stars fainter than magnitude 7 (fig. 3), we find that outside the relatively small domain of the local cluster the galactic equator has definitely established itself as the central circle for B-type stars.

The very high concentration of these distant stars to the Milky Way is a striking illustration of the great depth of the galactic system when compared with its extent perpendicular to the galactic plane. Probably the most important inference from figures 2 and 3 is, however, that the galactic field is continuous in the immediate environs of the local cluster—that the latter is not a large group or cloud distinctly and distantly isolated in space from other stellar regions. Similar evidence of a surrounding and intermingling galactic field may be deduced from the distribution of Cepheid variables, N-type stars, planetary nebulae, and similar special classes of sidereal objects.

In figures 2 and 3 the crosses indicate the position of stars of the sub-types B0, B1, and B2, and the dots refer to the sub-types B3 and B5. Stars of the first group of sub-types are believed to be intrinsically brighter than those of the second. For a given apparent magnitude, therefore, their distances are greater—possibly even twice as great. It is to be noted that the crosses have completely left the secondary circle in figure 3, indicating that the local cluster is not large enough to include objects as distant as B1 stars must be when they appear fainter than magnitude 7.

[1] Shapley, Harlow, these PROCEEDINGS, 4, 1918, (224–229); *Mt. Wilson Communications*, No. 54. See also *Mt. Wilson Contr.*, No. 157, and a small modification of the original statement of a star-streaming hypothesis, *Mt. Wilson Contr.*, No. 161, section IX.

[2] For other properties of the local cluster and for a discussion of the peculiar value of B-type stars in describing its extent, form, and orientation, reference may be made to *Mt. Wilson Contr.*, No. 157, part II. and No. 161, section IX.

[3] Strömberg, Gustaf, *Astrophys. J., Chicago, Ill.*, 47, 1918, (7–37); *Mt. Wilson Contr.*, No. 144.

INFLUENCE OF IONS ON THE ELECTRIFICATION AND RATE OF DIFFUSION OF WATER THROUGH MEMBRANES

By Jacques Loeb

The Rockefeller Institute for Medical Research, New York

Communicated, August 27, 1919

1. When pure water is separated from a watery solution by a strictly semipermeable membrane more molecules of water will diffuse through the membrane from the pure solvent to the solution than will diffuse simultaneously in the opposite direction; and this difference in the rate of diffusion of water in the two opposite directions will be the greater the higher the concentration of the solution. When the solution is put under pressure, the number of molecules of water diffusing in the unit of time from the solution into the pure solvent will be increased and if this pressure reaches a certain value the number of molecules of water diffusing simultaneously in opposite directions through the membrane will become equal. We may therefore define the osmotic pressure of the solution as the additional pressure which has to be applied to the solution in order to cause as many molecules of water to diffuse from the solution to the pure solvent as will diffuse simultaneously in the opposite direction.

van't Hoff's theory of osmotic pressure assumes that this quantity depends exclusively upon the concentration of the solution. For some

time it had been known that in certain cases water is able to diffuse from solutions of higher to solutions of lower concentrations, and it has been suggested by a number of authors, e.g., Girard,[1] Bartell,[2] Bernstein,[3] and others that these phenomena are due to electrical forces caused by the presence of electrolytes in solution. They assume on the basis of the experimental and theoretical work done by Quincke, Helmholtz, Perrin,[4] and others on electrical endosmose that differences of electrical potential on both sides of the membrane influence the rate of diffusion of water through the membrane.

The writer has recently investigated the influence of electrolytes on the rate of diffusion of water through collodion bags prepared in a definite and uniform way and bathed over night in a 1 per cent gelatin solution. The bags had the shape of Erlenmeyer flasks with 50 cc. contents and were closed by rubber stoppers which were perforated by a glass tube with a bore of 2 mm., the tube serving as a manometer.

When such a bag was filled with watery solution and was dipped into distilled water the level of the water in the manometer rose owing to the fact that more water diffused from the pure water into the solution than diffused simultaneously in the opposite direction, as was to be expected. It was found that the initial rate of diffusion of water was influenced in an entirely different way by electrolytes and non-electrolytes. The solutions of non-electrolytes, e.g., sugars, influenced the initial rate of diffusion of water through the membrane in proportion to their concentration and this influence began to show itself when the concentration of the sugar was above M/64 or M/32. Sugar solutions of lower concentrations than M/64 caused no rise in the manometer. We will call this effect of the concentration of the solute on the initial rate of diffusion the gas pressure effect. Solutions of electrolytes show this gas pressure effect also, but it commences at somewhat higher concentrations than M/64, namely at M/16 or even M/8. Solutions of electrolytes of a lower concentration than M/16 or M/8 have a specific influence on the rate of diffusion of water through the membrane which is not found in the case of non-electrolytes.

When we separate a watery solution of an electrolyte of a concentration below M/16 from pure water by a collodion membrane, the water molecules diffuse through the membrane as if they were electrically charged—positively or negatively according to the nature of the ions present—and as if they were attracted electrostatically by ions of one sign and repelled by ions of the opposite sign. When we used solutions of electrolytes theoretically isosmotic with a M/64 cane sugar solution

it was found that the influence of the nature of the electrolytes on the rate of diffusion of water through a collodion membrane could be expressed in the following two rules:

(1) Neutral solutions of salts possessing a univalent or a bivalent cation influence the rate of diffusion of water through a collodion membrane as if the water particles were charged positively and were attracted by the anion and repelled by the cation of the electrolytes; the attractive and repulsive action increasing with the number of charges of the ion and diminishing inversely with a quantity which we will designate arbitrarily as the 'radius' of the ion. The same rule applies to solutions of alkalies.

(2) Solutions of neutral or acid salts possessing a trivalent or tetravalent cation influence the rate of diffusion of water through a collodion membrane as if the particles of water were charged negatively and were attracted by the cation and repelled by the anion of the electrolyte. Solutions of acids and of neutral salts with monovalent or bivalent cation when rendered sufficiently acid obey the same rule.[5]

Thus the rate of diffusion of water into a neutral solution was considerably greater when the solution was M/128 NaCl than when it was M/64 cane sugar; and when different sodium or potassium salts were compared it was found that the rate increased with the valency of the anion of the salt in solution, sulfates and oxalates acting more powerfully than chlorides and nitrates, and citrates and ferrosulfocyanides more powerfully than sulfates and oxalates. The rate of diffusion of water was less when the solution was M/192 $CaCl_2$ than when it was M/64 cane sugar, and the same was true for all solutions of neutral salts with bivalent cation and monovalent anion. The attraction of M/128 solutions for water increased in the order $Li < Na < K$, showing the influence of the 'radius of the ion.' Solutions of alkalies like NaOH or KOH acted similarly to solutions of NaCl or KCl. All this was to be expected if water particles behaved as if they were positively charged, being attracted by the anion and repelled by the cation of the electrolyte.

In the case of electrolytes falling under rule 2, water particles behave as if they were negatively charged and attracted by the cation and repelled by the anion of the electrolyte with a force increasing with the number of charges of the ions. Thus solutions of Al_2Cl_6 attract water very powerfully, solutions of $Al_2(SO_4)_3$ of the same theoretical concentration act much more feebly and solutions of aluminium citrate have practically no more influence on the rate of diffusion of water than cane sugar solutions of the same concentration. When we render M/128 so-

lutions of NaCl acid (by dissolving the salt in M/1024 HCl) the water particles diffusing through the membrane are negatively charged and are attracted by the Na ion of the solution. This is supported by the fact that M/192 $CaCl_2$ dissolved in M/1024 HCl (or HNO_3) attracts water much more powerfully than does M/128 NaCl of the same hydrogen ion concentration; and the attraction of M/512 Al_2Cl_6 of the same hydrogen ion concentration for water is still more powerful than that of $CaCl_2$. All this is intelligible on the assumption of an electrostatic influence of the ions upon negatively electrified particles of water—no matter what the nature or source of electrification of water may be. That water is indeed electrified in the sense expressed in the two rules was proved directly by experiments on electrical endosmose.

The collodion membranes are not only permeable to water but also to crystalloids in solutions. It could be shown by analytical experiments that the phenomena expressed in the two rules were not due to differences in the rate of diffusion of the solute. The reader will find a full description of these experiments in a recently published paper.[6]

3. The two rules mentioned before were based on experiments with solutions of about the same gas pressure, namely that of a M/64 sugar solution. When we compare the osmotic effect of different concentrations of sodium (or Li, K, NH_4) salts within the limit of M/8192 to about M/16 we find a curious phenomenon. In these experiments the solution of the electrolyte was put inside the collodion bag and the latter was dipped into a beaker with pure water. It was found that under these conditions the initial rise (i.e., the rise in the first ten or twenty minutes) of water in the collodion bag increased rapidly with the increase of the concentration of the solution, this initial rise reaching a maximum when the concentration of the electrolyte was about M/256. With a further increase of the concentration of the electrolyte the initial rate of diffusion of water from pure solvent into the solution dropped rapidly, reaching a minimum at about a concentration of M/16. We therefore notice the paradoxical fact that M/256 solutions of all these electrolytes attract water more powerfully than M/16 solutions of the same electrolytes. Thus in the case of a neutral solution of sodium oxalate the level of water rose in the manometer of the flask in twenty minutes to about 100 mm. when the concentration of the solution was M/1024, to 220 mm. when the concentration was M/256, but only to 100 mm. again when the concentration was M/16. While I do not wish to make any assumption concerning the source of the electrification of water and the mechanism by which

the electrolytes influence the rate of diffusion of water through a membrane it will simplify the presentation of my results if it is permitted to ascribe them to the attraction and repulsion of the charged particles of water by the ions. With this reservation we may say that the rise in the first part of the curve, in concentrations from 0 to about M/256, being caused by the prevalence of the attractive action of the anion upon the negatively charged particles of water, while the drop in the curve, when the concentration increases beyond M/256, is due to the fact that with a further increase in the concentration the repelling effect of the Na ion upon the water particles increases more rapidly than the attractive effect of the oxalate anion. With concentrations above M/16 or M/8 the gas pressure effect of the solution commences to prevail over the electrostatic effects of the ions and the rate of diffusion of water rises again with increasing concentration.

We may state incidentally that all these phenomena can be observed just as well in collodion membranes which have not been treated with gelatin, so that the gelatin plays no rôle in the action of the membrane on solutions of neutral or alkaline salts.

4. This influence of the concentration of electrolytes upon the rate of diffusion of water explains the phenomenon of negative osmosis. It had been known for more than fifty years through the experiments of Dutrochet and Graham which were recently confirmed by Flusin,[6] that solutions of certain acids, like tartaric and oxalic, when separated from pure water by a membrane of pig's bladder produce a negative osmosis, i.e., water diffuses from the solution to the pure solvent. This is, of course, exactly the reverse of what one should expect on the basis of van't Hoff's theory. The writer has investigated this phenomenon and he found that it holds with certain exceptions for all the acids and all the alkalies, *and that this expulsion of water from the solution (the so-called negative osmosis) occurs in exactly the same range of concentrations where the drop in the electrostatic attraction of sodium salts for water occurs, namely in concentrations from about M/256 to about M/8.* Collodion flasks were filled with distilled water and submersed in beakers filled with solutions of acids or alkalies of different concentrations. The volume of water in the flask instead of diminishing increased when the concentration of the acid was between M/256 and M/16 or M/8 and the increase was the more considerable the higher the concentration of the acid or alkali within the limits mentioned.

Our first rule states that in the presence of salts and alkalies with univalent and bivalent cation water diffuses through the collodion mem-

brane as if its particles were positively charged. On the basis of this statement the initial expulsion of water by solutions of alkalies in concentrations above M/256 should be due to the fact that above a concentration of M/256 the repelling action of the cation upon the positively charged water particle increases more raipdly with increasing concentration of the elctrolyte than the attractive action of OH ions. When the cation is bivalent, e.g., when we use $Ca(OH)_2$ or $Ba(OH)_2$ as alkalies, the repelling action of the Ca and Ba ion with their two charges is so much stronger than the weak attractive action of the OH ions that water diffuses more rapidly from solution into pure water than pure water can diffuse into the solution, and the level of the water rises (during the first twenty minutes or more) in the pure solvent and falls in the solution. It harmonizes with our suggestion that the negative osmosis does not exist in the case of alkalies with monovalent cation like NaOH or KOH.

If the solution is an acid, water diffuses through the membrane as if its particles were negatively charged, being attracted by the H ion and repelled by the anion of the acid. In concentrations of acid of M/256 to about M/8 the solution expels during the first twenty minutes water into the pure solvent and the more the higher the concentration. This negative osmosis should be due to the fact that within this range the repelling action of the anion of the acid upon negatively charged water particles increases more rapidly with the concentration than the attractive action of hydrogen ion.

This suggestion is supported by the fact that the rate in which the acid solution expels water increases for the strong acids with the valency of the anion. Thus the amount of negative osmosis is small or negligible in the case of HCl or HNO_3, is considerable in the case of H_2SO_4 and oxalic acid, and still greater in the case of H_3PO_4. The phenomenon is not noticeable in the slightly dissociated acetic acid.

5. The definition of osmotic pressure as given in the introduction of this paper suggests that in the case of strictly semipermeable membranes the permanent osmotic pressure of the solution should show a simple relation to the difference in the initial rate of the diffusion of water in the two opposite directions through a membrane. Our experiments have shown that the initial rate of diffusion of water through a collodion membrane is determined for lower concentrations of electrolytes to a large extent by the electrostatic effects of the ions present and we should expect that these electrostatic effects would also influence the permanent osmotic pressure of solutions of electrolytes. Collodion membranes which are

permeable for solutions of crystalloids are impermeable for solutions of colloidal salts, e.g., the salts of gelatin. It can be shown that the permanent osmotic pressure of solutions of gelatin salts is influenced by the electrostatic action of ions in a similar way as is the rate of diffusion of water. I have found that water diffuses into neutral solutions of Na, K, Ca or Ba gelatinate as if the particles of water were positively charged. Since Ca and Ba (and all the other bivalent cations) have a greater repelling effect on positively charged particles of water than Li, Na, K or NH_4, the initial rate of diffusion of water should be greater when gelatin (in a 1% solution) is in combination with a monovalent cation than when it is in combination with a bivalent cation. I have tested this idea and found it confirmed. When we separate 1% gelatin solutions from distilled water by a collodion membrane the initial rate of diffusion of water into the solution is a little over twice as great when the gelatin exists in the form of sodium gelatinate than when it exists as calcium gelatinate, both having the same hydrogen ion concentration. The permanent osmotic pressures of the two types of solutions show also approximately the same ratio, being about or a little over twice as great in the case of sodium gelatinate as in the case of calcium gelatinate of the same concentration of gelatin as well as of hydrogen ions. Li, K, and NH_4 gelatinate behave like sodium gelatinate while Mg, Sr, and Ba gelatinate behave like calcium gelatinate.

Water particles behave like negatively charged particles in the presence of gelatin salts in which the gelatin is a cation, e.g., gelatin chloride or gelatin sulfate. If 1% solutions of these two gelatin salts of the same hydrogen ion concentration are prepared and put into collodion bags, water diffuses twice as rapidly into the gelatin chloride solution as into the gelatin sulfate solution. The osmotic pressures of the two solutions vary also as about 1:2 or a little less.[7]

The full report of the experiments on the influence of the concentration of electrolytes on the electrification and rate of diffusion of water through membranes will appear in the *Journal of General Physiology*.

[1] Girard, P., *Paris, C. R. Acad. Sci.*, **146**, 1908, (927); **148**, 1909, (1047–1186); **150**, 1910, (1446); **153**, 1911, (401); La pression osmotique et le mechanisme de l'osmose, *Publications de la Société de Chimie-physique*, Paris, 1912.

[2] Bartell, F. E., *J. Amer. Chem. Soc.*, **36**, 1914, (646). Bartell, F. E., and Hocker, C. D., *Ibid.*, **38**, 1916, (1029–1036).

[3] Bernstein, J., *Elektrobiologie*, 1912.

[4] Perrin, J., *J. Chem. Physique*, **2**, 1904, (601); **3**, 1905, (50).

[5] Loeb, J., *J. Gen. Physiol.*, **1**, 1918–19, (717).

[6] Flusin, G., *Ann. chim. phys.*, **13**, 1908, (480).

[7] Loeb, J., *J. Gen. Physiol.*, **2**, 1919, No. 1.

NATIONAL RESEARCH COUNCIL

MEETING OF THE INTERIM COMMITTEE

At the National Research Council Building, May 20, 1919, at 9.30 a.m.

Present: Messrs. Bancroft, Hussey, Leuschner, Mathews, Merriam, Washburn, and Woods.

Mr. Merriam, Chairman of the Council, in the Chair.

The minutes of the meetings of the Interim Committee of April 22 and April 28 were approved as circulated.*

In behalf of the committee consisting of Messrs. Walcott (Chairman), Cross, Dunn, and Merriam, appointed May 8 to carry out plans and to make arrangements to move the offices of the National Research Council to more suitable quarters, the Chairman reported favorably on the availability of the Navy League Building.

Moved: That if the Navy League Building be leased for the use of the National Research Council a committee of three be appointed to consider apportionment of space for the departments of the Council. (*Adopted.*)

Appointed: Messrs. Yerkes (Chairman), Leuschner, and Mathews. At the request of Mr. Yerkes, Mr. Mathews was subsequently designated as chairman of the committee.

Moved: That Mr. H. O. Wood be appointed Assistant to the Secretary of the National Research Council from May 16, 1919, to June 30, 1919. (*Adopted.*)

Moved: That Mr. Albert L. Barrows the recently appointed Secretary of the Divisions of States Relations and Educational Relations of the National Research Council, be authorized to stop on his way to Washington from California to consult with a number of state scientific committees and with research committees in educational institutions. (*Adopted.*)

Mr. Yerkes, as Chairman of the Committee on Organization of the Government Division, consisting of Messrs. Bancroft, Nichols, and Yerkes presented a report which was provisionally approved and later, May 27 and June 17, adopted by the Executive Board and the Council of the National Academy of Sciences in the following form:

Organization of the Government Division

1. The Division shall consist initially of

(a) a representative of each of the following bureaus with the provision that other bureaus may be added:

* The Interim Committee has power to act between meetings of the Executive Board. Heretofore its actions have been reported as part of the next following meeting of the Executive Board.

Department of State
- Foreign Intelligence Division

Department of the Treasury
- Public Health Service

Department of War
- General Staff Corps
  - Military Intelligence Division
- Surgeon General's Office
- Corps of Engineers
- Ordnance Department
- Signal Corps
- Coast Artillery Corps
- Air Service
- Chemical Warfare Service

Department of Justice
- Division of Investigations

Post Office Department

Department of the Navy
- (Office of Naval Operations)
  - Office of Naval Intelligence
- Bureau of Navigation
  - Naval Observatory
- Bureau of Ordnance
- Bureau of Construction and Repair
- Bureau of Steam Engineering
- Bureau of Medicine and Surgery

Department of the Interior
- Patent Office
- Bureau of Education
- Geological Survey
- Reclamation Service
- Bureau of Mines

Department of Agriculture
- Weather Bureau
- Bureau of Animal Industry
- Bureau of Plant Industry
- Forest Service
- Bureau of Chemistry
- Bureau of Soils
- Bureau of Entomology
- Bureau of Biological Survey
- Office of Public Roads and Rural Engineering

Department of Commerce
- Bureau of the Census
- Bureau of Standards
- Bureau of Fisheries
- Coast and Geodetic Survey

Department of Labor
- Information and Education Service

and the

Smithsonian Institution

(b) the Chairman of the Research Information Service who shall serve as Secretary of the Division.

2. That the name of a suitable representative of each of the foregoing bureaus be presented to the President of the United States in accordance with Article V, Section 3, of the Organization of the National Research Council, for designation by him for service with the National Research Council on the Government Division.

3. That the President of the United States be asked to designate Heads of Government Divisions, or suitable representatives thereof, to be appointed as members of the National Research Council and assigned to the Government Divisions.

4. That the Government Division be represented on each Division of Science and Technology by one member.

5. That this official representative of the Government to serve on a Division of Science and Technology be nominated by the Executive Committee of the Government Division in conference with the Executive Committee of the Division of Science and Technology on which he is to serve.

6. That further desirable cooperative relations between the Division of Government Relations and the several Divisions of Science and Technology be established through Article III, Section 6, of the new organization.

Mr. Yerkes presented the following nominations of representatives of Government Departments for membership on the Research Information Service:

Labor Department, Royal Meeker
Agriculture Department, Carl L. Alsberg
Commerce Department, S. W. Stratton
Interior Department, Van H. Manning
War Department, Major General Marlborough Churchill
Navy Department, Rear Admiral Albert P. Niblack
State Department, Wesley C. Frost
Treasury Department, J. W. Schereschewsky

Moved: That the names of the foregoing representatives of government departments be presented to the President of the United States in accordance with Article V, Section 3, of the Organization of the National Research Council, for designation by him for service with the National Research Council on the Research Information Service. (*Adopted.*)

On further recommendation of Mr. Yerkes, it was

Moved: That Dr. Milton J. Greenman, of the Wistar Institute of Anatomy and Biology of Philadelphia, be elected a member at large of the Research Information Service with the recommendation to the President of the National Academy of Sciences that he be appointed a member of the National Research Council and assigned to the Research Information Service. (*Adopted.*)

Mr. Leuschner, the Acting Chairman of the Division of Physical Sciences presented the following committees for approval:

Committee of five consisting of Messrs Millikan, Moore, Moulton, Slaught, and Dickson (Chairman) to consider the proposal of a Mathematical Dictionary and report to the Executive Committee of the Division.

Committee of three consisting of Messrs. Leuschner (Chairman), Bumstead, and Fulcher to make recommendations regarding abstracts, bibliographies, and monographs.

Mr. Leuschner, the Acting Chairman of the Division of Physical Sciences reported that in accordance with action taken by the Executive Board of the Council on April 15, 1919, which provides for a joint committee of the Divisions of Chemistry and Chemical Technology and of Physical Sciences on a plan for the publication of critical tables of physical and chemical constants, Messrs. Bumstead and Millikan had been nominated by the Division to serve on the joint committee. (*Approved*)

On recommendation of the Acting Chairman of the Division of Physical Sciences, it was further

Moved: That the Division of Educational Relations be requested to coöperate with the Division of Physical Sciences and that of Chemistry and Chemical Technology, in formulating proper plans whereby a research survey in Physics and Chemistry might be made in educational institutions and report back to the Division. (*Adopted.*)

Moved: That the principle of appointing committees for the organization and stimulation of research in various subjects of physics be approved, with the provision that each member of the division become responsible for one or more subjects, which he shall choose in consultation with the Acting Chairman of the Division. (*Adopted.*)

Moved: That Mr. H. O. Wood be appointed Acting Secretary of the American Section of the proposed International Geophysical Union. (*Adopted.*)

On recommendation of the Chairman of the Council it was

Moved: That the Chairman of the Research Fellowship Board be a member of the Division of Educational Relations. (*Adopted.*)

Mr. Washburn, the Acting Chairman of the Division of Chemistry and Chemical Technology presented the following recommendations:

That a Committee on Sewage Disposal be formed to undertake the following projects:

1. To prepare a research survey of the field with special reference to possible methods of sewage disposal which will recover the valuable oils, fats, and fertilizer constituents of the sewage; this survey to include statistical data and a discussion of the economic aspects of the subject as well as the scientific ones.

2. To outline a series of basic research problems necessary to the further extension of our knowledge of the possibilities of recovering and utilizing the valuable constituents of sewage.

3. To ascertain what researches are already in progress in the country in connection with this problem, and if it seems desirable, to prepare a list of sewage experiment stations or similar organizations whose equipment and staffs may possibly be utilized in connection with any project of coöperative research which it seems wise to undertake.

4. To prepare general plans and estimates of cost of establishing a sewage experiment station to study new methods of treating sewage for the recovery of its valuable constituents and to work in close coöperation with investigators now engaged or who in the future may be interested in undertaking physical, chemical, or bio-chemical investigations on various aspects of the problems which present themselves.

5. To investigate particularly the Rice Process and the Miles Process of sewage treatment and to make recommendations to the Division as to what action, if any, the Council might take with reference to research or development in connection with one or both of these processes. (*Adopted.*)

That the Committee on Sewage Disposal be constituted with the following personnel:

Professor G. C. Whipple, Professor of Sanitary Engineering, Harvard University, Chairman.

Major C. G. Hyde, (Professor of Sanitary Engineering, University of California) Surgeon General's Office, War Department, Washington, D. C.

Lt.-Col. Edward Bartow, Sanitary Corps, U. S. A., A. E. F. Water Analysis Laboratory, U. S. A. P. O. No. 702 (?) (Chief of the Illinois State Water Survey and Professor of Sanitary Chemistry, University of Illinois.)

Lt.-Col. W. D. Bancroft, (Chairman Division of Chemistry and Chemical Technology, National Research Council), American University Experiment Station Washington, D. C.

Professor E. B. Phelps, care of Dr. Charles North, 30 Church Street, New York.

Mr Martin H. Ottner, 105 Judson St., Jersey City, N. J.

Dr. W. W. Garner, Physiologist, Bureau of Plant Industry, U. S. Department of Agriculture, Washington, D. C. (*Adopted.*)

That the title of the Committee on Specific Heats of Explosive Materials, authorized at the meeting of April 21, be changed to the Committee on the Thermal Properties of Explosive Materials. (*Adopted.*)

That the Chairman of the Division of Chemistry and Chemical Technology be authorized to expend $150.00 for the purchase of reference books. (*Adopted.*)

Moved: That the question of preparing a list of the reference books belonging to the National Research Council be referred to the Research Information Service. (*Adopted.*)

Moved: That the hotel bills connected with attendance on the meetings of April 14–15, 1919, and of April 26–27, 1919, of the members of the Division of Biology and Agriculture be approved, but that this action on the part of the Interim Committee shall not be considered as a precedent. (*Adopted.*)

Moved: That the Interim Committee approve the bills for railroad fare and sleeper for the members of the Executive Committees of the Divisions of Physical Sciences and of Biology and Agriculture for meeting to be held the latter part of May, 1919. (*Adopted.*)

The following recommendations were submitted by Mr. Woods in behalf of the Division of Biology and Agriculture:

That the present Committee on Bibliography of the Division of Biology and Agriculture be continued with the addition of two members to be appointed by the Chair, and that this committee be instructed to coöperate with Mr. Yerkes of the Research Information Service. (*Adopted.*)

That the Fertilizer Committee of the Division of Biology and Agriculture be temporarily continued with Mr. J. B. Lipman as Chairman. (*Adopted.*)

That the Committee on Protein Metabolism in Animal Feeding of the Division of Biology and Agriculture be continued until a new general committee is established. (*Adopted.*)

That the present Committee on Salt Requirements of the Division of Biology and Agriculture be continued, and that Dr. Crocker be substituted for Dr. Woods on this Committee. (*Adopted.*)

The Interim Committee adjourned at 12.15 p.m.

PAUL BROCKETT, *Assistant Secretary.*

MINUTES OF JOINT MEETING OF THE EXECUTIVE BOARD OF THE NATIONAL RESEARCH COUNCIL WITH THE COUNCIL OF THE NATIONAL ACADEMY OF SCIENCES

At the National Research Council Building, May 27, 1919, at 9.30 A.M.

Present: Messrs. Bancroft, Carty,* Dunn, Flinn, Howell,* Hussey, Leuschner, Manning, Mathews, Merriam, Millikan, Pearl,* Ransome,* Walcott,* Washburn, Woods, Yerkes.

Mr. Walcott in the Chair during the transaction of the business of the Council of the Academy.

Mr. Merriam in the chair during the transaction of the business of the National Research Council.

The minutes of the regular meetings of the Executive Board of the National Research Council of April 15 and May 13, of the special meeting of the Executive Board of May 8, of the joint meeting of the Executive Board of the National Research Council and the Council of the National Academy of Sciences of April 30, and of the Interim Committee of the National Research Council of May 20, were approved as circulated, with certain amendments which have been included in the minutes of these meetings as previously reported in the Proceedings. Matters concerning solely the National Academy of Sciences are omitted from these minutes. Report on certain other matters which have not, as yet, been brought to a definite conclusion will be included in the minutes of future meetings.

A report from the Editorial Board of the Proceedings of the National Academy of Sciences was presented by its chairman, Mr. Raymond Pearl. The following extract concerns the National Research Council:

A special joint committee of the Editorial Board of the Proceedings on the one hand, and the National Research Council on the other hand, met and considered the general question of the future relations of the National Research Council and the Proceedings. The special committee was constituted as follows: For the Editorial Board. the Chairman, the Managing Editor, and Dr. Day; for the National Research Council, Doctors Hale, Merriam, and Yerkes. The general policy worked out by this joint committee was agreed to by the Editorial Board. It was in effect this, that the Proceedings of the National Academy of Sciences should become in a broader sense than had been true in the past, representative of the activities of the National Research Council, as well as of the National Academy It was understood that the National Research Council should offer to the Proceedings as official communications of the Research Council of the National Academy of Sciences such material, (intended for publication as bulletins of the Council) as the Council considered to be of permanent scientific value in the Proceedings. It was felt that in view of the proposed enlargement of the scope of the Proceedings to represent the activities of the National Research Council that some changes in the typography of the Proceedings might be desirable and it was agreed that changes looking to the improvement of the Proceedings in this regard would be inaugurated as soon as agreement on the matter had been reached between the conferees from the two interested bodies.

Moved: That the report be referred to the Editorial Board with power. (*Adopted.*)

* Members of the Council of the National Academy of Sciences.

The Council of the Academy adjourned at 10.30 a.m.

The Council of the National Academy of Sciences and the Executive Board of the National Research Council met in joint session at 10.35 a.m.

Mr. Dunn, as Chairman of the Nominating Committee for members at large of the Executive Board, presented a report of progress, stating that four members-at-large of the Executive Board had already been nominated on April 30 and had been appointed by the President of the Academy and that the Committee desired to recommend the appointment of the following additional members-at-large: Messrs. Van H. Manning, Director Bureau of Mines, S. W. Stratton, Director Bureau of Standards, and John J. Carty, Chief Engineer American Telephone and Telegraph Company, 195 Broadway. The Secretary was requested to cast the ballot for the names presented. The Secretary cast the ballot for the nominees and they were declared elected for recommendation for appointment by the President of the National Academy of Sciences to membership in the Council. Mr. Dunn stated that five additional members at large remained to be nominated and that the Committee had in mind to nominate only three of the remaining five in the immediate future.

Moved: That the expenses of Mr. Albert Barrows incident to visiting various state scientific committees and research committees in educational institutions on his way to Washington, be paid from the funds of the Division of Educational Relations, and that Mr. Barrows' salary begin with his services to the Council in this connection. (*Adopted.*)

Moved: That one thousand (1000) copies of the War Organization of the National Research Council pamphlets be printed. (*Adopted.*)

In behalf of the Research Fellowship Board Mr. Leuschner, Acting Secretary of the Board, reported the following appointments to National Research Fellowships for the ensuing year:

E. F. Barker (Physics)
A. E. Caswell (Physics)
F. E. E. Germann (Physics)
Leonard B. Loeb (Physics)
George P. Paine (Physics)
Robert A. Patterson (Physics)
E. Russell Bichowsky (Chemistry)
Emmet K. Carver (Chemistry)
W. D. Rodebush (Chemistry)
Geo. Scatchard (Chemistry)
Warren C. Vosburgh (Chemistry)
Lansing S. Wells (Chemistry)

Mr. J. C. Merriam, Chairman, made the following announcement:

That he had continued the original Committee on Nominations of officers and members of the Executive Board, Messrs. Hale, Millikan, and Noyes, for the nomination of three vice chairmen and that this committee had made the following nominations:

First Vice Chairman—Mr. C. D. Walcott
Second Vice Chairman—Mr. Gano Dunn
Third Vice Chairman—Mr. R. A. Millikan

Moved: That the Secretary cast the ballot for Messrs, C. D. Walcott, Gano Dunn, and R. A. Millikan for First, Second and Third Vice Chairmen respectively, of the National Research Council. (*Adopted.*)

The Secretary reported that he had cast the ballot in accordance with the foregoing resolution and Messrs. Walcott, Dunn, and Millikan were declared First, Second, and Third Vice Chairmen respectively.

The Chairman reported the receipt of a communication from Secretary Redfield stating that he would be pleased to have the National Research Council represented at the World Cotton Conference to be held at Atlantic City, May 28.

Moved: That Mr. Johnston and Mr. H. E. Howe be appointed delegates to represent the National Research Council at the World Cotton Conference to be held at Atlantic City, May 28, and that this appointment carry with it reimbursement from the funds for the general maintenance of the Executive Board for expenses of travel and subsistence. (*Adopted.*)

With reference to the proposed establishment of a popular journal of science it was

Moved: To pass the following resolution, only members of the Council of National Academy of Sciences voting:

WHEREAS, at the meeting of the National Academy of Sciences on April 30 the following motion was adopted:

"That the Council of the National Academy of Sciences be authorized to proceed with arrangements for a popular journal of science, provided arrangements be made for properly financing the journal, and for safeguarding the scientific quality of its contents,"

Resolved by the Council of the National Academy of Sciences that the Executive Board of the National Research Council be requested to assist the Council of the National Academy of Sciences by investigating and reporting back on the feasibility of a popular journal of science, and by securing pledges of funds. (*Adopted.*)

Moved: That Mr. Vernon Kellogg be invited to prepare as promptly as possible for the National Research Council a report on the feasibility of establishing a popular journal of science, on a plan for the organization and direction of such a journal, and on estimates of necessary expenditures in connection therewith. (*Adopted.*)

Mr. Washburn reported that in accordance with the resolution passed by the Executive Board on May 8, 1919, an informal conference concerning the continuation of the Committee on Explosives Investigations, had been held with representatives of the Army, the Navy, and the Bureau of Mines. Present at the Conference were Col. C. Harris, Jr., of the Ordnance Department of the Army, Lieut. Commdr. T. S. Wilkinson of the Bureau of Ordnance of the Navy, Dr. C. E. Munroe, Chairman of the Committee on Explosives Investigations, and the Acting Chairman of the Division of Chemistry and Chemical Technology. Mr. Van H. Manning of the Bureau of Mines was unable to be present owing to another engagement, but his views were known to the other members of the conference. Mr. Washburn submitted a recommendation from the conference that letters be addressed to the Bureau of Ordnance of the Navy, to the Ordnance Department of the Army, and to the Bureau of Mines, suggesting that these organizations coöperate in making arrangements for the permanent continuance of the Committee on Explosives Investigations, such coöperation to involve financial support to the extent of $2000 per annum by each of these three organizations.

Moved: That the suggestions contained in the report of Mr. Washburn be approved. (*Adopted.*)

The President of the National Academy of Sciences presented the following report on the proposed lease of the Navy League Building: An offer has been made to the agent of the building of $6500 a year for a two year lease, with the privilege of extending the lease for another year and with the understanding that the building shall be turned over to the National Research Council in thoroughly good order. It is expected that a definite report will be received by June 3.

The Acting Chairman announced that he had appointed an informal committee consisting of Messrs. Walcott, Leuschner, and Yerkes, to report on the representation of the United States at the international meetings to be held at Brussels beginning July 16. This committee reported that nominations were being received from the proper sources, that the traveling expenses of some of the nominees could be provided by educational and research institutions, but that it had become evident that the United States would not be properly represented unless additional provision for traveling expenses could be made; that in particular government bureaus were not allowed, under the law, to provide traveling expenses to these meetings for their members. Under these circumstances the President of the National Academy of Sciences had addressed a letter to the President of the United States, emphasizing the importance of adequate representation at the Brussels meeting on the part of the United States, and recommending that the Government provide the necessary traveling expenses of delegates.

The Committee further recommended that in case traveling expenses should be appropriated as requested, those delegates whose expenses are to be paid from government funds should be distributed approximately as follows:

For the International Research Council and the Unions which are already in process of formation and will be able to begin active work at Brussels, namely the Astronomical, Geophysical, Chemical, and Medical Unions, approximately 5 each.

For the conference to be held in connection with the proposed formation of Unions in Physics, Geology, Biology, Mathematics, and Bibliography 1 to 2 each, with the understanding that this distribution is only tentative. A provision of the International Research Council and of the Scientific Unions operating provisionally is that the number of delegates is not limited, and it seems desirable to the Committee that additional delegates be accredited if necessary, particularly in astronomy, if their traveling expenses can be provided for from private sources.

Moved: That the foregoing action and recommendations of the Committee, Messrs. Walcott, Leuschner, and Yerkes, on Representation of the United States, at the International meetings to be held at Brussels commencing July 16th, be approved and that the Committee be constituted a Committee on Selection of Delegates with power to nominate delegates in accordance with the plan presented. (*Adopted.*)

The Acting Chairman reported that Mr. Albert L. Barrows had accepted the appointment made on April 30 as Secretary for the Divisions of Educational and States Relations.

On recommendation of the Division of Educational Relations, it was

Moved: That Mr. Vernon Kellogg be elected a member-at-large of the Division of Educational Relations, with the recommendation to the President of the National Academy of Sciences that he be appointed a member of the National Research Council and assigned to that Division. (*Adopted.*)

Mr. Yerkes, as Chairman, presented the following nominations of members of the Research Information Service:

Mr. Joseph Stewart, Special Assistant to the Attorney General, Post Office Department, as official representative of the Department.

Dr. Edwin F. Gay, Central Bureau of Planning and Statistics, member at large.

Dr. Raymond Pearl, School of Hygiene and Public Health, Johns Hopkins University, Baltimore, member at large.

Moved: That the nominations of Messrs Joseph Stewart, Dr. Edwin F. Gay, and Dr. Raymond Pearl as members of the Research Information Service be approved, with the recommendation to the President of the National Academy of Sciences that they be appointed members of the National Research Council and assigned to that Division. (*Adopted.*)

In behalf of the Committee appointed on May 13 with power to arrange with the State Department for the continuation of the Foreign Research Information Service, Mr. Yerkes, as Chairman, presented the following report:

In accordance with motion adopted at Executive Board meeting May 13 the Acting Chairman of the Council and the Chairman of the Research Information Service with the assistance of Messrs. Bancroft, Dunn, and Mathews, as members of a committee appointed to take action with reference to the future of the foreign information service, have prepared the following resolutions.

WHEREAS the Research Information Service of the National Research Council, through its scientific attachés in London, Paris, and Rome, has rendered important service to the Departments of War and the Navy and has effectively coöperated with other Departments, notably that of State:

And whereas the importance of this scientific assistance has been recognized and appreciated by the Departments especially concerned and is deemed worthy of continuance;

And whereas the possibility and prospect of closer commercial and social relations between nations renders especially desirable prompt interchange of ideas and results of research to the end that duplication of effort may be avoided, plans coördinated, and the progress of civilization furthered by effective scientific and technical coöperation:

And whereas during the coming months and from time to time thereafter numerous international organizations (some of them based on treaties between nations) will meet in Europe for the consideration of political, social, industrial, and educational problems of the utmost importance with present varied scientific aspects;

Be It Resolved: That in the judgment of the Executive Board of the National Research Council the continuance of the Research Information Service in London, Paris, and Rome, is of vital importance;

That in each of these centers a scientific attaché should be maintained by the United States Government as an official representative of the scientific and technical interests of the Government and of the National Research Council.

On May 23 the Acting Chairman of the Council presented to the Third Assistant Secretary of State the substance of these resolutions with an estimate of the expense of continuing the service of scientific attaches in London and Paris. The Third Assistant Secretary of State advised that a statement of the need for this service for the year beginning July 1, 1919, should be transmitted immediately to the President of the United States.

In accordance with this advice a letter was prepared by your committee and transmitted to the State Department for approval and comment.

Moved: That the offices of the Research Information Service in London, Paris, and Rome be closed June 30, 1919, unless Government funds become available for the continuation of the Foreign Research Information Service. (*Adopted.*)

In behalf of the Division of Physical Sciences, Mr. Millikan as retiring Chairman, recommended that a communication be sent to the Rockefeller Foundation requesting an annual appropriation of $20,000 for two or three years for traveling expenses in connection with the plan of stimulating and organizing research in physical subjects through the formation of groups of research men in these subjects.

Moved: That the Executive Board approve the foregoing recommendations of the Division of Physical Sciences and that the Chairman of the Council be authorized to address a letter to the Rockefeller Foundation requesting an annual appropriation of $20,000 for two or three years in support of these plans. (*Adopted.*)

Moved: That the following recommendations from the Division of Biology and Agriculture be approved: It is the sense of the Division of Biology and Agriculture that it is desirable to ask support for a number of National Research Fellowships in the Division of Biology and Agriculture, comparable in general type to those offered in physics and chemistry. (*Adopted.*)

Moved: That the Division of Geology and Geography be authorized to pay from the General Maintenance allotment of the Division the expenses of railroad fare and Pullman for the members attending a meeting of the Division to be held June 5 and 6 1919. *Adopted.*

Moved: That the following Committee on Paleobotany of the Division of Geology and Geography be approved:

David White (Chairman), U. S. Geological Survey
R. S. Bassler, U. S. National Museum
E. W. Berry, Johns Hopkins University
J. M. Clarke, State Geologist of New York
F. H. Knowlton, U. S. Geological Survey
J. C. Merriam, University of California. (*Adopted.*)

Mr. Leuschner, the Acting Chairman of the Division of Physical Sciences presented a recommendation from the Provisional Executive Committee of the American Section of the proposed International Geophysical Union to the effect that the railroad fare and Pullman expenses of the members of the Section attending the organizing meeting of the Section to be held on June 24 and 25, 1919, be met from the General Maintenance fund of the Executive Board allotment.

Moved: That the railroad and Pullman expense of members attending the organizing meeting on June 24 and 25, 1919, of the American Section of the proposed International Geophysical Union, not to exceed $100 in the case of any member, be approved, such expenses to be met from the General Maintenance fund of the Executive Board allotment. (*Adopted.*)

In behalf of the Division of Chemistry and Chemical Technology Mr. Washburn, as Acting Chairman, submitted the following recommendations:

That the membership of the committee on Synthetic Drugs be constituted as follows:

Moses Gomberg, Ph.D., Prof. of Organic Chemistry, Univ. of Michigan, Ann Arbor, Mich.

G. W. McCoy, M.D., Director of the Hygiene Laboratory, U. S. Pubic Health Service, Washington, D. C.

Francis M. Phelps, Lawyer, Pacific Building, Washington, D. C.

W. A. Puckner, Phar.D., Director of Chemical Laboratory of the American Medical Association and Sec'y of the Council of Pharmacy and Chemistry of the American Medical Association.

Edw. S. Rogers, Attorney, Peoples Gas Bldg., Chicago, Ill.

Julius Stieglitz, Ph.D., Sc.D., Chem.D., Prof. of Chemistry, University of Chicago, Chicago, Ill. *(Approved.)*

That the Executive Board approve the preparation of monographs in accordance with the plan outlined in Section 3, of the report of the Committee on Publication of Compendia of Chemical Literature (see minutes of the Executive Board of April 30), and that it accept the invitation contained in the report to appoint one member of a Board of Trustees which shall have power to organize and control the undertaking. *(Approved.)*

Moved: That the following plan of organization of the Division of Anthropology and Psychology, together with the recommendations presented, be approved:

Organization of the Division of Anthropology and Psychology

It is recommended that the Division of Anthropology and Psychology of the National Research Council be organized as follows:

(1) The membership of the Division shall be equally divided between psychology and anthropology, and for the present shall be limited to nine members representing psychology and nine representing anthropology.

(2) The leading organization in each of the two subjects represented shall be asked to nominate six representatives for membership in the Division, the six persons thus nominated, in conference with the Executive Board of the Research Council to nominate the remaining three for each subject. It is understood that the American Psychological Association will nominate the six members for psychology, and the American Anthropological Association the six members for anthropology.

(3) The eighteen members selected shall elect from this number a Chairman for the Division of Anthropology and Psychology, and the nine representatives of each subject shall elect a chairman for a section, to be organized for each subject.

(4) For the present the work of the Division shall be divided so that general matters will be cared for by the whole Division under the direction of the Chairman of the Division, the sections holding rather frequent meetings under the direction of the Section Chairman.

(5) The executive business of the Division including general matters for the Division as a whole and those covering the work of the two sections shall be handled by a Secretary at a salary of $3000 per year, the Secretary to be resident in Washington. In view of this organization it seems desirable that the sum of $3000 being the difference between the salary of the Secretary and the normal salary of the head of a division be made available for expenses of the Chairman of the Division and the Chairmen of the two Sections, in connection with journeys to Washington with such residence at headquarters as may be necessary in conducting the work of the Division. (*Adopted.*)

The meeting adjourned at 1.05 p.m.

PAUL BROCKETT, *Assistant Secretary.*

MINUTES OF THE MEETING OF THE INTERIM COMMITTEE

AT THE NATIONAL RESEARCH COUNCIL BUILDING, JUNE 3, 1919, AT 9.30 A.M.

Present: Messrs. Angell, Bancroft, Clevenger, Leuschner, Mathews, Merriam, Washburn, Yerkes and, by invitation, H. E. Howe.

Mr. Merriam in the chair.

The Secretary reported that in view of the fact that the Government disbursing office in London, through which disbursements were made to the Foreign Research Information Service at London and Paris, had probably been closed on June 1, it would be necessary to advance a sufficient sum of money from the funds appropriated to the Executive Board for the continuance of the offices of the Scientific Attachés in London and Paris during the month of June and for the traveling expenses incident to the return to the United States of the staffs of these offices.

Moved: That, owing to the closing of the disbursing office in London, an advance be made from the funds of the National Research Council, allotted to the Executive Board, unless other means can be found, to cover the emergency in the disbursement of government funds for the offices of the Scientific Attachés in London and Paris during the month of June, with the understanding that the sum so advanced will be returned to the National Research Council from the President's allotment for the expenses of the Council through the Council of National Defense; that the amount to be advanced be left to the discretion of the Chairman and Secretary, provided it shall not exceed $6000 in all; $3000 for the London and $3000 for the Paris office. (*Adopted.*)

Moved: That the expenses of Scientific Attachés Mendenhall, Howe, and Washington as delegates of the National Research Council to the meetings in Brussels be referred to the Committee on Delegates (Messrs. Walcott, Leuschner, Yerkes) with power, with the understanding that the total expenses involved will not exceed $1500. (*Adopted.*)

The Chairman reported on the repairs and alterations required to prepare the Navy League Building for the use of the Council and stated that the owners had asked for a week's delay in completing the lease on account of a prospective sale of the property, and that under these circumstances an option to June 15, 1919 on the removal of the lease of the present quarters had been secured.

The Chairman reported that in accordance with the resolution passed by the Executive Board on May 27, Mr. Gellert Alleman, Swarthmore College, Swarthmore, Pa., had been appointed a member of the Board of Trustees which shall have power to organize and control the undertaking in regard to the preparation of monographs proposed in the report of the Committee on Publication of Compendia of Chemical Literature, etc. (see minutes of meeting of Executive Board of April 30).

The Chairman recommended that the record be completed by reporting the appointment of Mr. John Johnston, Chairman of the Section on Industrial Relations under the old organization, as Acting Chairman, without salary, of the Division of Industrial Relations under the new organization, to date from April 15, 1919. *(Approved.)*

The Chairman announced the membership of the joint Committee on the Publication of Critical Tables of Physical and Chemical Constants, authorized on April 15, to be as follows:

Messrs. Bumstead and Millikan, Division of Physical Sciences.
Messrs. Bancroft and Washburn, Division of Chemistry and Chemical Technology.
Mr. S. W. Stratton, Bureau of Standards.
Mr. F. E. Fowle, Smithsonian Institution.

The Acting Chairman of the Division of Chemistry and Chemical Technology presented a report of the joint Committee on the Publication of Critical Tables of Physical and Chemical Constants, which was adopted in the following amended form:

Your committee appointed to consider the plans proposed by the American Chemical Society with reference to the preparation of Critical Tables of Physical and Chemical Constants, submits the following report:

(1) It is recommended that the Council approve of preparing critical volumes of physical and chemical constants and related numerical data, as described in that report, but with the changes indicated by the italicized portions of the following extract from the report: Section II, 3 (confer minutes of meeting of April 22).

(1) The work *shall* not be a mere compilation, but *shall* represent a critical digest of physico-chemical constants.

(2) The compilation *shall* be published in separate parts. This *will* greatly facilitate the editing, greatly accelerate the publication of certain most important parts, and greatly facilitate the problem of keeping the work up-to-date without incurring the necessity of reprinting a large volume.

(3) The business control of the publication *shall* be placed in the hands of three Trustees of whom one (the Chairman) *shall* be nominated by the *Chairman* of the National Research Council and one each by the President of the American Chemical Society and the President of the American Physical Society. They shall hold office for one year and be eligible for re-election. *They shall have the power to add to their number, subject to the approval of the National Research Council.* The Trustees shall have the power to interest scientific bodies (such as the National Research Council, the Smithsonian Institution, Government Bureaus, etc.) or some publishing house in the preparation and publication either of the whole compilation (subject to the scientific control discussed below) or of individual parts of the compilation, or to recommend any plan of financing the undertaking that they may deem wise. The undertaking is to be considered primarily in charge of the National Research

Council with the support of the American Chemical Society, the American Physical Society and other scientific societies.

(4) The scientific control of the preparation of the volume on physico-chemical constants shall be vested in a committee of seven to be appointed by the Chairman of the National Research Council: three members on nomination by the Division of Chemistry and Chemical Technology and three members on nomination by the Division of Physical Sciences. The Chairman of the Committee will be the paid Editor-in-Chief of the work and will be selected by the *Chairman* of the National Research Council, the President of the American Chemical Society and the President of the American Physical Society. (Interested Scientific Societies including Electrochemical Society, the Institute of Chemical Engineers, etc., are represented in the Divisions of Chemistry and Chemical Technology and Physical Sciences of the National Research Council.) The members of the Committee are to be appointed for one year *or for such other term as the appointing power may see fit* and are to be eligible for reappointment.

(5) International coöperation is desired in carrying out this project and the International Chemical Union shall be requested to make suggestions as to such international representation on the Board of Scientific Control as it deems desirable.

Moved: That the delegates to the meetings of the International Chemical Union to be held in July at London and Brussels be authorized to present the foregoing report as the American plan for the preparation of critical tables of physical and chemical constants.
(*Adopted.*)

The Secretary reported that the Research Information Service had offered to abstract and index the minutes of the meetings of the Executive Board and of the Divisions.

Moved: That the Interim Committee approve the offer of the Research Information Service to abstract and index the minutes of the Executive Board and of the Divisions, and that the details involved in this plan be left to the Secretary of the Council and the Chairman of the Research Information Service. (*Adopted.*)

Moved: That the Chairmen of Divisions be requested to invite the Vice-chairmen of their Divisions to attend meetings of the Interim Committee whenever they are present in Washington. (*Adopted.*)

Mr. Yerkes, Chairman of the Research Information Service, presented the following report:

At a meeting held in the offices of the Research Council Thursday, May 29, the division was organized with Robert M. Yerkes as Chairman and Rear Admiral Albert P. Niblack, as Vice-chairman. An executive committee consisting of five members was elected. The membership consists of the Chairman and Vice-chairman of the Division, Wesley C. Frost representng the government, C. E. Mendenhall representing the divisions of Science and Technology (Colonel W. D. Bancroft was elected to act during the absence of Mr. Mendenhall) and Edwin F. Gay from the membership at large.

The division took the following action relative to activities proposed for consideration:

(a) *Personnel Bureau.* That the matter of establishing a personnel bureau to undertake the preparation of a catalog of men and women qualified for research in science and technology. be referred to the executive committee of the Division with instructions to report back to the Division.

That it be recommended to the Executive Board that the following projects be undertaken:

(b) *Catalog of Current Investigations.* That the preparation of a catalog of current scientific and technological investigations be undertaken by the division.

(c) *Abstracts and Handbooks.* That the division further the preparation of such aids to research as systematic bibliographies, abstracts, digests and handbooks.

(d) *Publications*. That the editing and management of the proposed bulletins of the Research Council be placed under the Research Information Service.

(e) *Centralization of Scientific Periodicals*. That a plan for coöperation in the publication of scientific materials be formulated.

(f) *Catalog of Research Information*. That a general catalog of research information be acquired and maintained by the Division.

Moved: That the Interim Committee approve the organization of the Research Information Service, as outlined and the action taken relative to proposed activities of the Service. (*Adopted*.)

On recommendation of Mr. Leuschner the Acting Chairman of the Division of Physical Sciences, it was

Moved: That chairmen of committees of the American Section of the proposed International Astronomical Union attending the meeting of the Section to be held in Washington on June 23 and 24 for the purpose of discussing reports and instructing delegates, be reimbursed for railroad fare and Pullman expenses with a limit of $100 in each case and a total limit of $250. (*Adopted*.)

The Chairman of the Research Information Service presented suggestions concerning the issuance of publications on the part of the National Research Council, especially of a series of special bulletins.

Moved: That the report on issuing publications be transmitted to the members of the Executive Board and that the report be made a special order for the meeting of the Executive Board to be held on Tuesday, June 10. (*Adopted*.)

On recommendation of Mr. Yerkes it was

Moved: That an additional sum of $300 be allotted to the Committee on Psychology from the unappropriated funds of the Council for special study of data regarding men recommended for discharge or for neuro-psychiatric examination. (*Adopted*.)

On Recommendation of the Chairman of the Research Information Service it was

Moved: To reimburse members of the Research Information Service attending the organizing meeting on May 29 for traveling expenses and to provide for their entertainment at luncheon, with the understanding that the total cost be in the neighborhood of $50, the amount to be charged to the funds appropriated to the Executive Board. (*Adopted*.)

The Interim Committee adjourned at 12.50 p.m.

PAUL BROCKETT, *Assistant Secretary*.

MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

AT THE NATIONAL RESEARCH COUNCIL BUILDING, JUNE 10, 1919, AT 9.30 A.M.

Present: Messrs. Bancroft, Flinn, Hussey, Leuschner, McClung, Merriam, Ransome, Stratton, Washburn, Yerkes, and, by invitation, H. E. Howe.

Mr. Merriam, Chairman of the Council in the Chair.

The minutes of the joint meeting of the Executive Board of the National Research Council with the Council of the National Academy of Sciences of May 27 and of the meeting of the Interim Committee of June 3 were approved as printed, with certain amendments, which have been included in the minutes as printed in the PROCEEDINGS.

The Secretary, Mr. Leuschner, reported the following membership of the Pulverizing Committee, which was approved at the meeting of the Interim Committee held on June 3, but which was not transmitted to him by the Division of Engineering in time to be included in the minutes of that meeting:

Committee on Pulverizing, Division of Engineering

Mr. G. H. Clevenger, Chairman, Engineering Division, National Research Council, New York City.

Mr. Paul Avery, Esperanza Mining Company, El Ore, Mexico.

Mr. C. H. Benedict, Calumet and Hecla Mining Company, Lake Linden, Michigan.

Mr. A. L. Bloomfield, Golden Cycle Mining and Milling Company, Colorado Springs, Colorado.

Mr. D. S. Calland, Cia. Real del Monte Pachuca, Pachuca, Hidalgo, Mexico.

Mr. Allan J. Clark, Homestake Mining Company, Lead, South Dakota.

Prof. E. W. Davis, Supt. Mines Experiment Station School of Mines, University of Minnesota, Minneapolis, Minnesota.

Mr. J. V. N. Dorr, The Door Company, New York.

Mr. H. W. Fox, Westport, Connecticut.

Mr. Frank Janney, Utah Copper Company, Garfield, Utah.

Mr. A. H. Jones, Tonopah-Belmont Development Company, Tonopah, Nevada.

Mr. B. F. Morrow, Anaconda Copper Mining Company, Butte, Montana.

Mr. Luther C. Lennox, Colorado Springs, Colorado.

Mr. W. M. Rossberg, Timber-Butte Mining Company, Butte, Montana.

Mr. W. G. Swart, 808 Sellwood Building, Duluth, Minnesota.

Mr. R. B. Yerxa, Miami Copper Company, Miami, Arizona.

Mr. Louis D. Mills, San Francisco, California.

On recommendation of the Secretary it was

Moved: That the members of the Division of Medical Sciences attending the organizing meeting of the Division on Monday, June 9, be reimbursed for railroad and Pullman fare from the General Maintenance fund of the Executive Board. (*Adopted.*)

Moved: That the members of the Division of Industrial Relations attending the organizing meeting of the Division on Wednesday, June 11, be reimbursed for railroad and Pullman fare from General Maintenance funds of the Division. (*Adopted.*)

Moved: That Mr. E. W. Washburn be added to the Committee on Organization of Administration. (*Adopted.*)

The Chairman stated that Mr. Clevenger has been invited by the American Society of Mechanical Engineers to represent the National Research Council and present a paper at its Detroit meeting June 17.

Moved: That the expenses of Mr. Clevenger incident to his attendance on the meeting of the American Society of Mechanical Engineers at Detroit be paid from the General Maintenance fund of the Division of Engineering. (*Adopted.*)

Moved: That the dues of $100 of the National Research Council to the American Council on Education for the current year be paid from the General Maintenance Fund of the Executive Board. (*Adopted.*)

Moved: That the salary of Mr. H. E. Howe of the Division of Industrial Relations begin May, 1, 1919. (*Adopted.*)

Moved: That in accordance with the recommendation of the Bureau of Ordnance, Lieutenant-Commander O. M. Hustvedt represent the Bureau of Ordnance of the Navy on the Committee on Explosives Investigations in place of Lieutenant-Commander T. S. Wilkinson. (*Adopted.*)

The Chairman reported that Messrs. John Johnston and H. E. Howe had represented the National Research Council at the meeting of The World Cotton Conference at Atlantic City, N. J., May 28, 1919.

The Chairman reported that it was necessary to finance the work of the Intelligence Scales Committee, consisting of Messrs. Haggerty, Terman, Thorndike, Whipple, and Yerkes, Chairman, from the appropriation of $25,000 from the General Education Board.

Moved: That the experts in psychology employed by the Intelligence Scales Committee be paid at the rate of $5 an hour. (*Adopted.*)

Mr. Yerkes, Chairman of the Research Information Service, presented a report on publications which was adopted in the following form:

A joint committee of the National Academy of Sciences and of the National Research Council has agreed that material of permanent scientific value offered to the Research Council for publication shall be placed in the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES and reprinted for distribution by the Research Council. All other material of the Research Council presumably would be published directly or reprinted as seemed desirable from other periodicals.

The arrangement recommended by this joint committee renders it highly desirable that the form of publication adopted by the Research Council be the same as that of the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES.

There are obvious reasons why the Research Council should seriously consider the publication of propaganda as contrasted with other types of scientific material, but it is not clear that re-publication of material of scientific value is desirable.

For consideration by the Executive Board of the National Research Council the following proposals are submitted:

(1) That a publication to be known as the *Bulletin of the National Research Council* be established.

(2) That this series shall include the proceedings of the National Research Council and its divisions, publicity and propaganda material, reports of committees, and other special reports.

(3) That the *Bulletin* shall appear irregularly and that within reasonable limits it may vary in form in accordance with the requirements of the material.

(4) That the several bulletins shall be numbered consecutively and shall be so arranged that approximately 500 pages shall constitute a volume.

(5) That the regular subscription price shall be fixed at $5.00 per volume.

(6) That the bulletins shall be sold separately, except in case of strictly publicity material, at a minimum price of ten cents for numbers of not more than ten pages, and at the rate of one-half cent for each page in excess of ten.

(7) That the size of type page be 27 by 45 picas, as in the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES, and the type number 31 monotype, 11 point, with 2 point leads.

(8) That a distinctive cover be selected for the *Bulletin*, which shall bear on its title page the name "Bulletin of the National Research Council" the volume number, bulletin number, serial number, title and author of the paper, and the date and place of publication.

Moved: That the following bulletins be published by the Council in the manner and edition indicated, and that the Committee be given power to determine the issue of the regular series of full sets and increase the number of copies up to 50%, according to demand.

1. The National Importance of Scientific and Industrial Research by Hale and others, to be printed as *number one* of the *Bulletin of the National Research Council* in an edition of *two thousand copies.*

2. Report of the Patent Committee of the National Research Council, to be printed as *number two* of the *Bulletin* in an edition of *two thousand copies.*

3. Report of the Psychology Committee of the National Research Council, to be published as *number three* of the *Bulletin* in an edition of *two thousand copies.*

4. Some Problems of Sidereal Astronomy by Henry Norris Russell, to be published in the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES and reprinted as *number four* of the *Bulletin* in an edition of *one thousand copies.*

5. Refractory Materials as a Field for Research by Edward W. Washburn, to be printed as *number five* of the *Bulletin* in an edition of *one thousand copies.*

6. Industrial Research by F. B. Jewett, to be printed as *number six* of the *Bulletin* in an edition of *two thousand copies.* (*Adopted.*)

Moved: That all material prepared under the auspices of the National Research Council and printed in other places than the *Bulletin* be collected in the form of reprints and included in a numbered series of papers to be issued by the National Research Council under such title as the Committee on Publication may decide. (*Adopted.*)

The Secretary called attention to the resolution passed some time ago that all manuscripts for publication should be submitted to the Publication Committee of the National Research Council before being published in any journal.

The Secretary presented a report of the Committee on Administration of Organization on bonus to employes as additional pay to salaries for the past year, providing for an allotment of $1320 from the unappropriated funds of the Council.

Moved: That the report of the Committee on Administration of Organization providing a bonus for certain employees be approved and that the sum of $1320 be appropriated for the purpose from the unallotted funds of the National Research Council. (*Adopted.*)

The report of the Treasurer for the month of May was received and placed on file.

On recommendation of Mr. Washburn, Acting Chairman of the Division of Chemistry and Chemical Technology, it was

Moved: That the report by the Committee on Publication of Compendia of Chemical Literature, etc., to the President and Council of the American Chemical Society, and the recommendations of the Division of Chemistry and Chemical Technology with respect to the Formation of an International Chemical Council be printed in an edition of 100 copies, for use at the meetings in London and that the expenses of the printing be paid from the Maintenance fund of the Division of Chemistry and Chemical Technology. (*Adopted.*)

The Acting Chairman of the Division of Geology and Geography presented the following report:

At a meeting of the Division held on June 5 and 6, the following temporary committees were appointed.

Funds: Ralph Arnold, Chairman.
Rocky Mountain and Geology Conference: Eliot Blackwelder, Chairman.
Committee on Certain Seismological Investigations: H. F. Reid, Chairman.
Conference Committee on Survey of the Sea: J. Russell Smith, Chairman.
List of Scientific Travelers: Isaiah Bowman, Chairman.
Development of Geographical Sciences: W. M. Davis, Chairman.

The Division passed the following resolution on the coördination in field methods of map-making agencies of the Government which it has referred to the Division of Government Relations.

WHEREAS the several mapping agencies of the United States are making actual surveys by different methods on different scales and with wide variation in accuracy of field work.

WHEREAS the results of actual field surveys are not mutually available to the different map-making organizations because of these differences and a lack of means by which the relative accuracy of the different parts of the field work can be readily ascertained.

WHEREAS such lack of coördination in field methods and degree of accuracy leads to surveys and avoidable expense, the Division of Geology and Geography.

Recommends to the Division of Government Relations that a Committee be appointed representing the several map-making agencies of the Government to arrange a plan of operation by which organizations conducting surveys secure and record their field data in such manner that they may be fully usable by other map-making agencies of the Government interested in the same areas.

To this end their Committee should define

1. The degree of accuracy or "tolerance" in field work.
2. Standard methods of expression which shall discriminate the relative accuracy of determination of the several horizontal and vertical elements.
3. One or more standard scales and projections for field mapping.

It is further suggested that this work may be done by a new committee or entrusted to the recently organized Committee of Aeronautic Mapping if its scope may be broadened.

The Division requests the appointment of a Conference Committee by the Division of Biology on Survey of the Sea, also the appointment of a member of the Division of Physical Sciences on its Committee on certain proposals for seismological investigations offered by Dr. Wood.

The Division requests the National Research Council to prepare for publication a record of the work of scientific men in connection with the war.

In response to the inquiry from the Secretary of the National Research Council, the Division

Voted that the Division of Geology and Geography approve the formation of international associations of geologists and of geographers under the auspices of the International Research Council and recommend that two delegates—a geologist and a geographer—be sent to the conference to be held in Brussels, in July 1919.

It also passed the following resolutions:

That this Division recognizing the high value of the scientific work of "The Inquiry" (preparatory to The Peace Congress) recommends to the Executive Board of the National Research Council the appointment of a committee of the Council to confer at an appropriate time with the Department of State with reference to the publication of the scientific results of "The Inquiry."

Moved: That the request of the Division of Geology and Geography that the National Research Council prepare for publication a record of the work of scientific men in connection with the war be referred to the Research Information Service for investigation and report. (*Adopted.*)

Moved: That the formation of international associations of geologists and geographers under the auspices of the International Research Council and the recommendation that two delegates, a geologist and a geographer, be sent to the conference to be held in Brussels in July be approved. (*Adopted.*)

Moved: That the recommendation of the Division of Geology and Geography that a Committee of the Council be appointed to confer at an appropriate time with the Department of State with reference to the publication of the scientific results of "The Inquiry" (preparatory to The Peace Congress) be approved. (*Adopted.*)

Mr. Hussey, Acting Chairman of the Division of Medical Sciences, presented the following report:

The organization meeting of the Division of Medical Sciences was held in Philadelphia, June 9, 1919. All elected members except five were present. There are three Society nominations which remain to be made, namely those of The American Association of Anatomists, The American Society of Biological Chemists, and the American Neurological Society. Doctors W. W. Keen and F. S. Lee were present by special invitation.

The following officers were elected: Dr. H. A. Christian, Chairman; Dr. R. G. Hussey, Vice Chairman; Dr. Hussey to serve as Acting Chairman until Dr. Christian can make arrangements to take office.

The following members were elected to serve on the Executive Committee: Doctors Simon Flexner, Reid Hunt, W. H. Howell, R. M. Pearce and Colonel F. F. Russell.

The terms of office for the members determined by lot at the meeting, are as follows:

One year term	*2 year term*	*3 year term*
George W. Crile	G. C. Huber	H. A. Christian
E. R. Stitt	W. S. Thayer	Wm. H. Welch
Victor C. Vaughan	T. B. Hartzell	Simon Flexner
A. W. Crane	Reid Hunt	Peyton Rous
H. Gideon Wells	F. P. Gay	Howard T. Karsner
David L. Edsall	Joseph Erlanger	Richard N. Pearce
W. H. Howell	Anatomical Society	F. F. Russell
Neurological Society	American Society of Biological Chemists.	

Consideration was given to a communication from the Secretary regarding the International Research Council. The following action was taken

regarding medical interests and representation in international scientific organization: that this Division go on record as being in accord with the sentiments of the resolutions passed at the London meeting regarding scientific relations with men of the Central Powers:

That this Division go on record as stating without qualifications, their approval of the admittance of Neutral Countries to any international organization that may be formed;

That this Division will serve as the American Section of any International Medical Organization proposed. Five delegates were nominated to attend the Brussels meeting.

A committee was appointed to take charge of questions regarding international organization, this committee to be known as the Committee on International Coöperation in Medical Sciences and to be constituted as follows: Doctors Christian, Flexner, Keen (Chairman), Lee, Welch, Colonel Russell and the Acting Chairman of the Division (ex officio); together with the delegates. A meeting of this Committee has been called to be held at Atlantic City, Wednesday, June 11.

The Division meeting adjourned until some date in late September or early October subject to the call of the Chairman.

Immediately following the meeting the Executive Committee met in a short session. It was voted that all existing committees be continued as constituted in the War Organization subject to later action by the Executive Committee.

On recommendation of Mr. McClung, Chairman of the Division of Biology and Agriculture, it was

Moved: That an additional sum of $500 be appropriated to the General Maintenance fund of the Division of Biology and Agriculture to meet the traveling expenses of members of the Executive Committee in attendance on the meeting held May 31–June 1—and on meeting to be held during the summer, and to provide for incidental expenses of the Division. (*Adopted.*)

Moved: That the unexpended sum of $1000 allotted to the Division of Biology and Agriculture for the rodent pest investigation revert to the general fund. (*Adopted.*)

Moved: That the sum of $1000 be appropriated for the use of the Committee on Plant Pathology of the Division of Biology and Agriculture to complete its work for the year, this amount to be used in payment of railroad and Pullman fares of committee members to conferences. (*Adopted.*)

Moved: That the following Committee on Food and Nutrition of the Division of Biology and Agriculture be approved:

J. R. Murlin, Chairman	E. V. McCollum
H. P. Armsby	H. C. Sherman
E. B. Forbes	A. E. Taylor
C. F. Langworthy	A. F. Woods
Graham Lusk	

(*Adopted.*)

The Chairman announced the following initial organization of the Committee on Pacific Exploration:

Wm. Morris Davis, Harvard University, Cambridge, Mass.
Herbert E. Gregory, Yale University, New Haven, Conn.

W. E. Ritter, Scripps Institute, La Jolla, Calif.
G. F. MacEwen, Scripps Institute, La Jolla, Calif.
R. A. Daly, Harvard University, Cambridge, Mass.
Alfred G. Mayor, Director, Department Marine Biology, Carnegie Institution, Washington, D. C. (*Approved.*)

Mr. Mathews, Acting Chairman of the Division of Geology and Geography

Moved: That Mr. J. C. Merriam be asked to serve as a member and Chairman of the Committee on Pacific Exploration. (*Adopted.*)

The Executive Board adjourned at 1.00 p.m.

PAUL BROCKETT, *Assistant Secretary.*

MINUTES OF THE MEETING OF THE INTERIM COMMITTEE

AT THE NATIONAL RESEARCH COUNCIL BUILDING, JUNE 17, 1919, AT 9.30 A.M.

Present: Messrs. Leuschner, McClung, Merriam, and Washburn.
Mr. Merriam, Chairman of the Council, in the Chair.

Moved: That an allotment of $250 be made from unappropriated government funds for the expense of the Washington branch of the Research Information Service to cover salaries and incidental expenses to June 30, 1919. (*Adopted.*)

Moved: That the Division of Engineering be authorized to apply a sum of about $50 from its maintenance fund in reimbursing members of the Committee on Investigation of Neumann Bands attending a meeting to be held in the near future, for railway and Pullman expenses. (*Adopted.*)

Moved: That a petty cash allowance of $100 be set aside from its general maintenance fund for the New York office of the Division of Engineering for June, 1919, and thereafter a monthly petty cash allowance of $30 including the balance from the previous month, with the provision that a detailed statement of expenditure be submitted to the Treasurer of the Council at the end of each month. (*Adopted.*)

The Committee on the Apportionment of Space in the Navy League Building consisting of Messrs. Mathews (Chairman), Leuschner, and Yerkes, presented a report on the assignment of rooms, repairs, and alterations, and made the following recommendation: That, on the assumption that the furniture bought from government funds will be available for the use of the Council after June 30, an offer be made to the Navy League for the purchase of the following equipment at the prices stated:

Awnings	$150.00
Fifteen metal filing cabinets $30 each	450.00
Furnishings in Women's Rest Room	40.00
Coal	75.00
Shelving	40.00
Two double pedestal typewriting desks $25 each	50.00
	$805.00

Moved: That the Committee on the Apportionment of Space in the Navy League Building confer with the chairmen of the Divisions as to the proposed assignment of space and report at the meeting of the Executive Board to be held on June 24, 1919. (*Adopted.*)

Moved: That the offer recommended by the Committee on the Apportionment of Space in the Navy League Building for the purchase of furniture and fixtures specified above be approved. (*Adopted.*)

Moved: That authority be given to the Chief Clerk to move the offices of the National Research Council from 1023 16th Street to the Navy League Building at 1201 16th Street, and that the expenses of moving be paid from government funds. (*Adopted.*)

Moved: That the Chief Clerk, be authorized to buy the paper and order the printing of the necessary letterheads for the use of the Council, using his discretion, with the approval of the Secretary, as to the use of government funds in this expenditure. (*Adopted.*)

The Secretary, Mr. Leuschner, rendered a report on the existing committees of the Executive Board.

Moved: That the following committees be discharged:

Reorganization of Council, authorized and appointed November 12, 1918, Messrs. Merriam (Chairman), H. M. Howe, Millikan, Pupin, Johnston; reorganized Dec. 31, 1918, with Mr. Noyes in place of Mr. Merriam and with Mr. Hale as Chairman.

Preparation and Publication of History of Research Activities in Science in this country during the war, authorized and appointed November 20, 1918, Messrs, Yerkes, (Chairman), Teggert, Gilbert.

Smith-Howard Bill, authorized and appointed December 4, 1918, Messrs. Leuschner, (Chairman), H. M. Howe, Johnston, Pupin.

Proposed Chicago Meeting with Research Representatives of Middle West Institutions, authorized and appointed December 31, 1918, Messrs. Hale (ex-officio), Chairman, Leuschner, Merriam, Noyes.

To prepare Revised Scheme for the Promotion of Research in Physics and Chemistry, authorized and appointed January 14, 1919, Messrs. Noyes (Chairman), Flexner, Johnston, Millikan, Nichols.

Organization of Division of Engineering, authorized and appointed January 21, 1919, Chairman of the Council, Messrs. Dunn and Howe.

Program of the National Research Council for Annual Meeting of the National Academy of Sciences, appointed April 4, 1919, confirmed April 8, 1919, Messrs. Leuschner (Chairman), Cross, H. M. Howe, Yerkes.

Organization of Government Division, appointed April 4, 1919, confirmed April 8, 1919, Messrs. Yerkes (Chairman), Bancroft, Nichols.

Organization of Division of Industrial Relations, appointed April 4, 1919, confirmed April 8, 1919, Messrs. H. M. Howe (Chairman), Bancroft, Clevenger, Nichols.

Nominations of Officers and Members-at-Large of Executive Board, authorized and appointed April 15, 1919, Messrs. Hale (Chairman), Noyes, Millikan. Discharged April 30, 1919. Reappointment reported May 27, 1919, for nomination of three Vice-chairmen.

Organization of Research Information Service, authorized and appointed May 13, 1919, Messrs. Yerkes (Chairman), Bancroft, Dunn, Merriam, Mathews. (*Adopted.*)

Moved: That the following special and standing committees be continued:

Reconstruction Problems, authorized August 13, 1918, and appointed August 20, 1918, Messrs. Vernon Kellogg (Chairman), F. H. Newell (Vice-chairman), Raphael Zon, Walton H. Hamilton.

Organization of Administration, authorized and appointed April 1, 1919, Messrs. Cross (Chairman), Clevenger, Hussey, Leuschner, Yerkes; reorganized April 22, 1919, with Secretary Leuschner as Chairman ex-officio. Mr. Washburn added June 10, 1919.

Research Council Budget for 1919–20, authorized and appointed April 1, 1919, Messrs. Leuschner (Chairman), Cross, Clevenger.

To express to Rockefeller Foundation the appreciation of the National Research Council for the support of National Research Fellowships in Physics and Chemistry, authorized and appointed April 15, 1919, Messrs. Hale (Chairman), Noyes, Millikan.

Budget—Expenses National Academy of Sciences and National Research Council, authorized and appointed April 30, 1919, President of National Academy (Chairman), Chairman of National Research Council, Treasurer of National Academy and National Research Council; May 27, 1919, function enlarged to include securing of funds to meet the needs of the National Academy and of the National Research Council, and the preparation of a budget apportioning such funds and other available funds. (Academy Committee.)

Removal of Offices of National Research Council, authorized and appointed May 8, 1919, Messrs. Walcott (Chairman), Cross, Dunn, Merriam.

To express to Retiring Treasurer Whitman Cross the appreciation of the National Research Council for his services, authorized and appointed May 8, 1919, Messrs. Woodward (Chairman), Leuschner, Washburn.

Scientific Men as Reserve Officers in the Reorganized Army, authorized March 18, 1919, appointed May 8, 1919, Messrs. Walcott (Chairman), Mathews, Leuschner.

Nominations of members at large of Executive Board (Joint Committee of Council of National Academy and Executive Board of Research Council), authorized April 30, 1919, appointed May 13, 1919, by Dr. Walcott; Messrs. Dunn (Chairman), Merriam, Yerkes; May 27, 1919, function enlarged to consider membership of Advisory Board.

Apportionment of space in Navy League Building, authorized and appointed May 20, 1919, Messrs. Mathews (Chairman), Leuschner, Yerkes.

Federal grants for research, authorized Jan. 14, 1919, appointed May 27, Messrs. Carty (Chairman), Dunn, Leuschner, Manning, Mees, Stratton, Walcott, Yerkes.

Representation of the United States at the International meetings at London and Brussels, July, 1919, authorized and appointed May 27, 1919, Messrs. Walcott (Chairman), Leuschner, Yerkes.

Committee on Patent Office, authorized and appointed July 31, 1917, Professor W. F. Durand (Chairman), Messrs. R. A. Millikan, S. W. Stratton. Reorganized December 14, 1917 as follows: Messrs. L. H. Baekeland, W. F. Durand (Chairman), Thos. Ewing, Frederick P. Fish, R. A. Millikan, E. J. Prindle, M. I. Pupin, S. W. Stratton. March 1918, Mr. Baekeland appointed Acting Chairman, approved at meeting of April 30, 1918. April 19, 1918, Mr. Reid Hunt added to the Committee. March 11, 1918, Mr. C. P. Townsend added to the Committee. Present Organization, Messrs, W. F. Durand (Chairman), L. H. Baekeland (Acting Chairman), Thos. Ewing, F. P. Fish, R. Hunt, R. A. Millikan, E. J. Prindle, M. I. Pupin, S. W. Stratton, C. P. Townsend. (*Adopted.*)

Moved: That the matter of preparation and publication of History of Research Activities in Science in this country during the war be referred to the Research Information Service. (*Adopted.*)

The Committee on Organization of Administration, Mr. Leuschner, chairman, submitted the following office organization for the National Research Council, which was approved as presented.

Office Organization of National Research Council

I. Executive Staff of the Board:
   1. Chairman of the Council
   2. Secretary of the Council
   3. Treasurer of the Council
   4. Chairman and other executive officers of Divisions

II. Technical Staff:
   1. Scientific Associates
   2. Assistants to Officers
   3. Division Secretaries

III. Business Staff:
   1. Bursar
   2. Chief Clerk

IV. Clerical Staff:
   1. Secretaries to Chairmen
   2. Division Clerks
   3. Stenographers
   4. General Clerks
      a. Draughtsmen
      b. Special or Head of c, d, e.
      c. Bookkeeper
      d. Typists
      e. Filing

V. Miscellaneous Employees:
   1. Superintendent of Building
   2. Telephone Operators
   3. Messengers
   4. Watchmen or Guards
   5. Police workers

Moved: That in the absence of a superior executive officer the Secretary of the Division, exercising executive functions, be designated as "Executive Secretary." (*Adopted.*)

The Secretary presented a joint memorandum from the Committee on Organization of Administration and from the Budget Committee, regarding the salary list and bonuses for the year commencing July 1. The matter was left over for further consideration and the Secretary was requested to send a copy of the proposed salary and bonus schedules to the heads of Divisions and to present a further report at the next meeting of the Interim Committee.

Considering the question of expenses of members in attendance at meetings of the Executive Board and Executive Committees of the Divisions, it was

Moved: That members of the Executive Board residing out of the city of Washington shall be allowed their transportation expenses and in addition, expenses, not to exceed $6 per day, for maintenance during their attendance at meetings of the Executive Board, and that members of the Executive Committees of the Divisions shall be allowed transportation and maintenance expenses on the same basis as members of the Executive Board or only transportation expenses, at the discretion of the Division concerned.* (*Adopted.*)

In behalf of the Committee on Representation of the United States at the International Meetings at London and Brussels, July, 1919, Messrs, Walcott (Chairman), Leuschner, and Yerkes, Mr. Leuschner reported that a cablegram had been received from Mr. Mendenhall, Scientific Attaché at London, containing a provisional program of meetings; that in accordance with this schedule, meetings of the International Research Council, of the proposed International Astronomical, Chemical and Geophysical Unions, and conferences to discuss the question of internationl organizations in Medicine, Mathematics, Geology, Biology, and Physics, and conferences on International Patents, and

* This motion was subsequently amended. It will appear in the amended form in the printed minutes of a later meeting.

North Atlantic Fisheries, would begin at Brussels on July 18, 1919, and continue until July 30, 1919, and that a preliminary meeting of the International Chemical Union would be held at London beginning July 15, 1919. The Committee, which was given power to act, reported, further, that it had received nominations of delegates to all of these meetings and conferences from the proper sources, and submitted the names of representatives of the National Academy of Sciences and of the National Research Council to the various meetings and conferences, with the recommendation that proper credentials be issued to them by the President of the National Academy of Sciences, subject to the concurrence of the Foreign Secretary of the National Academy.

Meetings of International Research Council and of Affiliated Organizations at Brussels, Beginning July 18, 1919

PRECEDED BY MEETING OF INTERNATIONAL CHEMICAL UNION, LONDON, BEGINNING JULY 15

DELEGATES FROM THE UNITED STATES OF AMERICA

International Research Council

W. W. Campbell, *Chairman*

W. W. Campbell, Chairman of Amer. Del., International Astronomical Union
H. M. Howe, Scientific Attaché at Paris, also representing Engineering
C. E. Mendenhall, Scientific Attaché at London, also representing Physics
H. S. Washington, Scientific Attaché at Rome, also representing Geology
Major Wm. Bowie, Chairman of Amer. Del., International Geophysical Union
E. W. Washburn, Chairman of Amer. Del., International Chemical Union
W. S. Thayer, Representing Medicine
John C. Penny, Representing Patents
.... Representing Mathematics
Maj. Doug. W. Johnson, Representing Geography
H. F. Moore, Representing Biology and Fisheries

Astronomical Union

W. W. Campbell, *Chairman*

W. S. Adams
S. I. Bailey
Benjamin Boss
W. S. Eichelberger
Major Philip Fox
W. J. Humphreys
S. A. Mitchell
F. R. Moulton
H. N. Russell
Frank Schlesinger
C. E. St. John
F. H. Seares
Joel Stebbins

Geophysical Union

Major Wm. Bowie, *Chairman*

L. A. Bauer
H. C. Graves
A. O. Leuschner
G. W. Littlehales
C. F. Marvin
H. F. Reid
Capt. Edward Simpson
J. T. Watkins

Alternates: W. J. Humphreys, J. F. Hayford, W. J. Peters.

Chemical Union

E. W. Washburn, *Chairman*

Lt. Col. Edward Bartow
F. G. Cottrell
Chas. L. Parsons
Julius Stieglitz
H. S. Washington

Alternates: Lt. Col. A. B. Lamb, Lt. Col. Jas. F. Norris

Medicine

W. S. Thayer, *Chairman*, Geo. W. Crile, F. P. Gay, C. L. Gibson, W. H. Howell, Victor C. Vaughn

Mathematics

J. L. Coolidge

Geology

H. S. Washington

Geography

Major Douglas W. Johnson

Biology

H. F. Moore

Alternates: R. P. Bigelow, R. E. Coker

Physics

C. E. Mendenhall, S. W. Stratton

Patents

John C. Penny

Fisheries

H. F. Moore

Moved: That the report of the Committee on Representation of the United States at the International Meetings at London and Brussels, July, 1919, be approved and that the appointments of the delegates named be confirmed. (*Adopted.*)

The Chairman presented the following letter from the Adjutant General of the Army:

Chairman, National Research Council, New York, N. Y.

DEAR SIR: The report of Lieut. Colonel R. A. Millikan, Chairman, Physical Science Division of your Council, dated April 18, 1919, in which he states that the fund of fifteen thousand dollars originally allotted for research and development of a centrifugal gun is about exhausted and recommends that the Army take over further development of this type of ordnance has been given due consideration.

As a result of this investigation, the Secretary of War has directed that further development in connection with this subject be placed in the hands of the Inventions Section; War Plans Division, General Staff, and requests that the models, drawings, specifications and all data you may have in connection with the centrifugal gun be turned over to that section.

Arrangements will be made by the Inventions Section for the transfer of such skilled mechanics now employed by you at Pittsburgh as may be desirable for duty at the Bureau of Standards where future development of the centrifugal gun will take place.

Very truly yours,

[Signed] P. C. HARRIS,
The Adjutant General.
Per: J. C.

Moved: That the transfer of the models, drawings, etc., related to the development of a centrifugal gun be referred to the Chairman of the Division of Physical Science with request that he carry out the order of the Adjutant General of the Army. (*Adopted.*)

Moved: That the Council subscribe to the publication *Science* for the use of its members. (*Adopted.*)

Moved: That ten copies of the minutes and reports accompanying the minutes of the Divisions be sent to the Research Information Service, five for the use of the Washington office and five for the foreign offices. (*Adopted.*)

The Chairman presented the following nominations from the President of the American Psychological Association for membership in the Division of Anthropology and Psychology:

W. D. Scott, Scott Company, Drexel Building, Philadelphia, Pa.

E. L. Thorndike, Professor of Educational Psychology, Teachers' College, Columbia University, New York City.

Raymond Dodge, Professor of Psychology, Wesleyan University, Middletown, Conn.

James R. Angell, Chairman, National Research Council.

G. M. Whipple, Carnegie Institute of Technology, Pittsburgh, Pa.

C. E. Seashore, Head of Dept. of Philosophy and Psychology, University of Iowa, Iowa City, Ia.

Moved: That the names of Messrs. W. D. Scott, E. L. Thorndike, Raymond Dodge, James R. Angell, G. M. Whipple and C. E. Seashore be recommended to the President of the National Academy of Sciences for appointment as members of the National Research Council, with the request that they be assigned to the Division of Anthropology and Psychology. (*Adopted.*)

Moved: That as soon as, in accordance with the plan of organization of the Division of Anthropology and Psychology adopted May 27, 1919, the six members nominated by the American Psychological Association shall have nominated the remaining three psychologists, in conference with the Executive Board, the nine members representing Psychology may organize as a Section in Psychology of the Division and elect a Chairman of the Section and proceed with its work, pending the complete organization of the Division of Anthropology and Psychology. (*Adopted.*)

After consideration of a request of the Engineering Foundation that the National Research Council suggest means of furthering the study of problems of industrial employment, particularly the mental problems of industry, it was

Moved: That a committee consisting of representatives of each of the following divisions (Anthropology and Psychology, Educational Relations, Engineering, Industrial Relations, and Medicine) and the Chairman of the National Research Council who shall serve as a chairman of the committee be appointed by the Chairman of the Council to consider means of furthering the study of problems of Industrial employment in accordance with the request of the Engineering Foundation and to formulate a plan for presentation to the Foundation. (*Adopted.*)

The Chairman reported that Major R. G. Hussey, Acting Chairman of the Division of Medical Sciences, would be able to give only part of his time to the Nationl Research Council after his discharge from the Army, June 9.

The Secretary reported that the organizing meeting of the Division of Industrial Relations was held on Wednesday, June 11, 1919, at the Engi-

neering Society's Building, New York, and that the following actions were taken:

1. By lot, the terms of the 12 elected members of the Division were determined as follows:
One year, Carty Dunn, Washburn, and Whitney.
Two years, Baekeland, Johnston, Mees, and Townsend.
Three years, Flinn, Howe, Rautenstrauch, and the twelfth member, not yet elected.

2. The following officers were elected: Chairman, John Johnston, Vice-Chairman, Harrison E. Howe; Executive Committee, the Chairman and the Vice-Chairman ex officio, C. E. K. Mees, G. K. Burgess and Alfred D. Flinn.

3. It was voted to promote the establishment of an alloys research association, confirming action previously taken by the Industrial Research Section.

4. It was voted that when an alloys research association shall have been incorporated, it shall undertake the conduct of the research.

5. It was voted that four members of the Division of Industrial Relations, Howe, Rautenstrauch, Washburn, and Burgess, be appointed a committee to formulate a plan for an alloys research, and to visit users and others interested in alloys and confer with them concerning the formation of an association for conducting research.

6. It was the sense of the meeting that the Alloys Research Committee, while promoting this research, should investigate also the feasibility of promoting researches in refractories and in metal cutting. (*Adopted.*)

The Acting Chairman of the Division of Chemistry and Chemical Technology presented for approval the following personnel of the Committee on Ceramic Research:

A. V. Bleininger, Chairman ex officio, Bureau of Standards, Pittsburgh, Pa.
R. B. Sosman, Geophysical Laboratory, Washington, D. C..
Homer F. Staley, Bureau of Standards, Washington, D. C.
Arthur L. Day, Corning Glass Works, Corning, N. Y.
E. W. Washburn, Department of Ceramic Engineering, Univ. of Illinois. (*Approved.*)

In behalf of the Division of Chemistry and Chemical Technology, the Acting Chairman of that Division presented the following request:

In accordance with resolutions adopted at the meeting of the Executive Committee of the Division of Chemistry and Chemical Technology on June 3, 1919, the Division asks for an appropriation of $7000, to be used in carrying out a program for stimulating and organizing coöperative research in Chemistry. The plan approved by the Executive Committee of the Division includes the calling of conferences of investigators working along similar lines. The program of work for each such conference will be laid out carefully in advance by the Chairman of the Division, after correspondence or personal conference with the interested investigators. It is hoped that approximately five such conferences can be planned for and held during the coming year and it is estimated that the cost of holding these five conferences will be approximately $7000.

Moved: That the request of the Division of Chemistry and Chemical Technology for $7000 to carry out the plans for stimulating and organizing cooperative research in Chemistry be referred to the Budget Committee to be considered with other plans of a similar nature. (*Adopted.*)

On recommendation of the Acting Chairman of the Division of Chemistry and Chemical Technology, it was

Moved: That authority be given to the Division of Chemistry and Chemical Technology to have printed, at the expense of their maintenance fund, stationery for the Committee on Explosives, the Committee on Sewage Disposal, the Committee on Ceramic Research, and the Committee on Synthetic Drugs. (*Adopted.*)

With reference to the expenses of delegates to the international meetings to be held in London and Brussels in July, 1919 who are now in Europe, it was

Moved: That the Budget Committee be asked to consider and make financial arrangements to cover the expenses of delegates to the international meetings to be held in London and Brussels in July, 1919, who are now in Europe. (*Adopted.*)

The meeting adjourned at 12.30 p.m. to meet again at 9.15 a.m., Wednesday, June 18.

The Adjourned meeting of the Interim Committee was called to order at 9.15 a.m. Wednesday, June 18, at the office of the National Research Council.

Present: Messrs. Leuschner, McClung, Merriam, Ransome, and Washburn.

The Secretary submitted the following recommendation of the Acting Chairman of the Division of Geology and Geography:

The Division of Geology and Geography request authority for the publication of a small four to six page folder indicating the organization of the Division to be sent to all members of societies represented on the Division. The manner of publication should be in accord with the system of publication of publicity leaflets to be issued by the Council.

Moved: That the printing of a folder indicating the organization of the Division of Geology and Geography and the principle of the publication of folders of uniform size and content be approved. (*Adopted.*)

On recommendation of the Chairman of the Division of Biology and Agriculture it was

Moved: That the Committee on Instruction in Universities, of the Division of Biology and Agriculture, consisting of Messrs. Bradley M. Davis and C. E. McClung, be continued, and that this committee be included in the membership of the Committee of that Division on Educational Relations as a sub-committee. (*Adopted.*)

Moved: That the question of a liaison member on the Division of Educational Relations from the sub-committee on Instruction in Universities of the Committee on Educational Relations of the Division of Biology and Agriculture be referred to the Division of Educational Relations for consideration and report. (*Adopted.*)

Moved; That the whole question of coordination of the Divisions of Science and Technology with the Division of Educational Relations be referred to the Division of Educational Relations for consideration and report, with particular reference to the Committee on Educational Relations of the Division of Biology and Agriculture. (*Adopted.*)

Moved: That the Committee on Salting Fish in Southern Climates, of the Division of Biology and Agriculture, consisting of Messrs. H. F. Moore, and B. E. Livingston, be discontinued. (*Adopted.*)

Moved: That the Advisory Board of American Plant Pathologists, consisting of Messrs. G. R. Lyman, H. W. Barre, H. P. Barss, H. S. Jackson, P. A. Murphy, and C. R. Orton, be recognized as a Committee on Plant Pathology representing the Division of Biology and Agriculture, and that it be continued with Mr. G. R. Lyman as Chairman. (*Adopted.*)

Moved: That a Committee of the Division of Biology and Agriculture on Cooperation and Coordination be appointed; three members to be appointed by the Chairman of the

Division with authority to enlarge the committee so that it will include not more than seven members. (*Adopted.*)

The Chairman of the Division of Biology and Agriculture presented the names of Messrs. F. R. Lillie (Chairman), G. R. Lyman and J. R. Murlin to act on this committee.

Moved: That a Special Committee of three of the Division of Biology and Agriculture be appointed to consider ways and means of securing and publishing current problems in biology, and to make recommendations to the Executive Committee. (*Adopted.*)

The Chairman of the Division of Biology and Agriculture presented the names of Messrs. G. N. Collins (Chairman), Barrington Moore and P. J. Parrott to serve on this committee. (*Approved.*)

Moved: That a committee of at least five, of the Division of Biology and Agriculture, be appointed to consider the general question of Biological Investigations in Tropical America and report back to the Division. (*Adopted.*)

The Chairman of the Division of Biology and Agriculture presented the names of Messrs. A. S. Hitchcock (Chairman), L. R. Jones, Barrington Moore, Wilfred Osgood and H. N. Whitford to serve on this committee. (*Approved.*)

Moved: That Mr. Vernon L. Kellogg be nominated as tenth member-at-large of the Division of Biology and Agriculture for a period of one year, with recommendation to the President of the National Academy of Sciences that he be appointed a member of the National Research Council and assigned to that Division. (*Adopted.*)

Moved: That the recommendation of the Division af Biology and Agriculture for the appointment of a Committee of that Division on the Study of Oceanography be approved. (*Adopted*)

The Chairman of the Division of Biology and Agriculture stated that in his opinion it was desirable that the Chairmen of Divisions be present at the meetings of other Divisions, and recommended further that the minutes of the Divisions be exchanged.

Moved: That the suggestion of the Chairman of the Division of Biology and Agriculture that Chairmen of Divisions be present whenever possible at the meetings of other Divisions be specifically approved, and that the minutes of the Divisions be exchanged. (*Adopted.*)

Moved: That the matter of acceptance of alterations made by the Navy League on the building recently leased by the National Research Council at 201 Sixteenth Street be referred to the Committee on Space, with power to act. (*Adopted.*)

The Chairman of the Council brought up the question of the organization of the Division of Government Relations. It was

Moved: That the President of the National Academy of Sciences be requested to confer with the heads of those departments of the Government which, by action of the Executive Board of the National Research Council and the Council of the National Academy of Sciences, are represented on the Government Division, with a view to selecting representatives of these Departments to be included in the Government Division; that the President of the National

Academy of Sciences be requested to invite the Secretary of the Smithsonian Institution to serve as Chairman of the Government Division; and further that the President of the Academy be requested to present to the President of the United States the personnel of the Government Division to be designated as members of the National Research Council in the Government Division. (*Adopted.*)

The Secretary presented a provisional draft of a bulletin of the National Research Council containing a full description of its organization including the membership of its Divisions and Committees.

Moved: That the draft of the organization of the National Research Council, including the membership of its Divisions and Committees, be printed, subject to the regulations relating to the printing of bulletins adopted at the meeting of the Executive Board on June 10, 1919. (*Adopted.*)

Moved: That Mr. S. L. G. Knox be appointed Assistant to the Scientific Attachés at Paris and Rome, without salary, from April 15 to June 30, 1919, and to Sept. 30, 1919, if Attachés are continued. (*Adopted.*)

Moved: That Mr. Donald J. Cowling, President of Carleton College, be nominated to represent the Association of American Colleges in the Division of Educational Relations in the National Research Council, with recommendation to the President of the National Academy of Sciences that Mr. Cowling be appointed a member of the National Research Council and assigned to the Division of Educational Relations. (*Adopted.*)

The meeting adjourned at 10.45 a.m.

PAUL BROCKETT, *Assistant Secretary.*

REPORT OF THE ANNUAL MEETING OF THE ACADEMY

PREPARED BY THE HOME SECRETARY

The Annual Meeting of the National Academy of Sciences was held at the Smithsonian Institution, Washington, D. C., April 28, 29, and 30, 1919, with President Walcott presiding.

Seventy members were present as follows: C. G. Abbot, Aitken, Ames, Benedict, Birkhoff, Boas, Bumstead, Cannon, Cattell, F. W. Clarke, J. M. Clarke, Conklin, Cross, Cushing, Dall, Davenport, Davis, Day, Dickson, Fewkes, Flexner, Forbes, Hale, E. H. Hall, Hayford, Hillebrand, Holmes, Howard, Howe, Iddings, Jennings, Jewett, Kasner, Langmuir, Leuschner, Loeb, Lusk, Mayor, Mendel, C. H. Merriam, J. C. Merriam, Millikan, Morgan, Moulton, E. L. Nichols, E. F. Nichols, A. A. Noyes, W. A. Noyes, H. F. Osborn, T. B. Osborne, Pearl, Ransome, Reid, Rosa, Russell, Schlesinger, Schuchert, Erwin F. Smith, Stieglitz, Stratton, Thorndike, Ulrich, Van Vleck, Vaughan, Walcott, Webster, Wheeler, D. White, H. S. White, Woodward.

BUSINESS SESSIONS

The Annual Report of the President, containing that of the Treasurer for 1918 in printed form was presented, accepted and distributed to the members.

The following assignments of Biographical Memoirs were announced: Memoirs of Geo. Francis Atkinson to N. L. Britton, of George F. Becker, to A. G. Webster, of Maxime Bôcher to Wm. E. Story, of Edward C. Pickering to H. N. Russell, of Wallace C. Sabine to Edwin H. Hall, and of Charles R. Van Hise to T. C. Chamberlin.

Changes in personnel of sections and committees were announced as follows:

Local Committee: F. L. Ransome, chairman; M. T. Bogert, A. O. Leuschner, C. H. Merriam, E. F. Nichols, E. B. Rosa.

Henry Draper Fund: W. W. Campbell to succeed himself, term expiring in 1924.

J. Lawrence Smith Fund: E. W. Morley, to succeed himself, term expiring in 1924. A. O. Leuschner to fill the unexpired term of E. C. Pickering, expiring in 1920.

Comstock Fund: W. R. Whitney to succeed A. A. Noyes, term expiring in 1924.

Marsh Fund: J. C. Merriam to succeed E. H. Moore as chairman and member, term expiring in 1922.

Murray Fund: W. M. Davis, to succeed himself, term expiring in 1922.

Marcellus Hartley Fund: A. A. Noyes, chairman and member, to succeed G. F. Becker, term expiring in 1922. S. W. Stratton to succeed W. F. Hillebrand, term expiring in 1922.

REPORTS FROM OFFICERS OF THE ACADEMY

The Home Secretary presented the following report, which was accepted.

April 28, 1919.

The President of the National Academy of Sciences,
Washington, D. C.

Sir: I have the honor to present the following report of the Acting Home Secretary on the publications and membership of the National Academy of Sciences for the year ending April 30, 1919.

The exigencies of war have retarded printing in Washington and for that reason no scientific memoirs have been published during the year. Publication has been taken up again, however, and four memoirs are now in preparation:

"The Complete Classification of the Triad Systems in Fifteen Elements," by H. S. White, F. N. Cole, and Miss L. D. Cummings, received in manuscript on June 20, 1917 and sent to the Public Printer to be published as Volume XIV, Second Memoir.

"Tables of the Minor Planets—Discoveries by James C. Watson, Part 2: On Von Zeipel's Theory of the Perturbations of the Minor Planets of the Hecuba Group," by A. O. Leuschner, A. E. Glancy, and S. H. Levy, received in manuscript on January 7, 1919, and sent to the Public Printer to be published as Volume XIV, Third Memoir.

"Minor Constituents of Meteorites," by George P. Merrill, received in manuscript on May 11, 1918 and sent to the Public Printer to be published as Volume XIV, Fourth Memoir.

"Tables of the Exponential Function and of the Circular Sine and Cosine to Radian Argument," by C. E. Van Orstrand, received in manuscript May 24, 1918 and sent to the Public Printer to be published as Volume XIV, Fifth Memoir.

No Biographical Memoirs have been published, owing to pressure of war work, but the following biographies have been sent to the printer and are near completion:

Cleveland Abbe, by W. J. Humphreys, received in manuscript November 7, 1917.

Lewis Boss, by Benjamin Boss, received in manuscript March 20, 1917.

William Bullock Clark, by John M. Clarke, received in manuscript January 2, 1918.

James Mason Craft, by Charles R. Cross, manuscript received November 10, 1918.

James Dwight Dana, by L. V. Pirsson, received in manuscript November 12, 1917.

Arnold Hague, by J. P. Iddings, received in manuscript August 2, 1918.

Eugene Woldemar Hilgard, by F. Slate, Jr., received in manuscript October 11, 1918.

Alpheus Spring Packard, by T. D. A. Cockrell, received in manuscript October 16, 1918.

Benjamn Osgood Peirce, by E. H. Hall, received in manuscript November 20, 1917.

With the publication of these memoirs Volume VIII of the Biographical Memoirs will have been published and ready for distribution, while two thousand copies of the Third Annual Report of the National Research Council will be issued in a few days. The PROCEEDINGS of the Academy have been published regularly and have reached the third number of the fifth volume.

Six members have died since the last meeting:

George Francis Atkinson, elected 1918, died November 14, 1918.

George F. Becker, elected 1901, died April 20, 1919.

Maxime Bôcher, elected 1909, died September 12, 1918.

Edward C. Pickering, elected 1873, died February 3, 1919.

Wallace C. Sabine, elected 1917, died January 10, 1919.

Charles R. Van Hise, elected 1902, died November 19, 1918.

One foreign associate, Sir William Crookes, elected in 1913, died April 4, 1919.

There are now on the membership list 164 active members, 1 honorary member, and 35 foreign associates.

C. G. ABBOT, *Acting Home Secretary.*

The Foreign Secretary presented a report on the work of his office during the past year, stating that it largely had to do with the organization of the International Research Council, and that this was fully reported on in the work of the National Research Council.

Moved: That the Report of the Foreign Secretary be accepted, and the action reported therein of the delegates to the International Research Council be approved. (*Adopted.*)

REPORTS FROM COMMITTEES ON TRUST FUNDS

A report was received from the Directors of the Bache Fund, signed by Edwin B. Frost (Chairman) stating that since the annual meeting of the Academy in April, 1918, grants Nos. 210–213 as announced in the PROCEEDINGS, p. 492, below) had been made. (A provisional grant of $1000 was made for another research, but the applicant had meantime started on his trip of exploration and has not been heard from since, so that it is not certain that the money will be required.) Reports on these and previous grants have been submitted by the recipients, to wit:

The recipient of a grant made some years ago, whose work had not yielded results entirely to his satisfaction, offered to return the money which he had expended, with interest. The directors, however, unanimously declined to accept such return, regarding it as establishing a very unwise precedent which would imply that the recipient of a grant might be personally responsible if a research seriously prosecuted did not yield the results hoped for by him.

No. 202. W. C. Allee, Lake Forest, Illinois. The survey of the effect of the re-agents on reactions to light of Mayfly Nymphs. Results published, *Biol. Bul.*, 32, 93–97; and *J. Exper. Zoöl.*, 26, 423–459. Further work in progress on reversals to light; cause and effects of aggregation in certain Arthropods; effect of cyanides on Arthropods.

No. 203. Joseph P. Iddings, Brinklow, Md. The work continues and will not be finished for some time yet.

No. 205. T. H. Goodspeed, Berkeley, Cal. Experiments on Nicotiana, Publication of the final results has been delayed by military and civilian service of the collaborators.

No. 207. T. H. Gronwall. Mathematical investigations. Preliminary results published in these Proceedings 5, 22–24, January, 1919, under the title, "A Theorem of Power Series with an Application to Conformal Mapping." Three notes embodying further results are in preparation and will soon be sent to the editor of the Proceedings.

No. 208. A. F. Shull, Ann Arbor, Mich. Research is concluded on (1) Cell inconstancy in hydatina; and (2) Relative effectiveness of food, oxygen, etc. in causing or preventing male production. Results published (1) *J. Morph.*, 30, No. 2, March 1918; (2) *J. Exper. Zool.*, 26, No. 3, Aug. 1918. Research is still in progress on nuclea volume in relation to life cycle of hydatina; also on phenomena of maturation in relation to life cycle of hydatina; and on rate of metabolism in relation to life cycle of aphids.

No. 209. Cecil K. Drinker, Boston. In addition to the first paper, reported last year three others will be sent to the *J. Exper. Medicine* during the coming year, based on the study of material which has been accumulated with the aid of this grant.

No. 210. W. J. Atwell, Buffalo, N. Y. A research on the development of human hypophysis cerebri is in progress. Three models have been completed by the Bron wax-late method, showing hypophysis of 10 mm., 16 mm., 30 mm. human embryos.

No. 211. George H. Shull, Princeton, N. J. The studies on heredity in Shepherd's-purse and Evening Primroses are being carried on with a minimum of labor and would have been impossible without the grant.

The American collaborators of the Nomenclator Animalium Generum et Subgenerum have been paid the amounts due for their work.

A report was received from the Committee on the Henry Draper Fund, signed by W. W. Campbell (Chairman) as follows:

The Committee has unanimously recommended to the Council that the Henry Draper Gold Medal be awarded to Professor Charles Fabry of the University of Marseilles, in recognition of his researches in Physics and Astronomy, chiefly by means of interferometers.

There have been no applications received for grants from the Henry Draper Fund in support of research during the past year. This is probably due in large measure to the fact that many investigators have been engaged upon war problems and that investigators remaining at home have not planned for extensions of instrumental means.

The total amount of income available for the encouragement of research was $2234.94 on April 1, 1919. Of this sum $382.44 was cash on hand and $1952.50 was invested in securities.

A report was received from the Trustees of the Watson Fund, signed by A. O. Leuschner (Chairman), G. C. Comstock and W. L. Elkins, stating that

grants Nos. 18–20 (as announced below, p. 492) were recommended and that reports of progress on previous grants were as follows:

No. 15. A grant of $300 was made in April, 1917, to Professor HERBERT C. WILSON of Carleton College, Northfield, Minn,, for the continuation of the photographic determination of the positions of minor planets. During the past year about 60 plates of minor planets have been secured, but measurement of the positions has been interrupted by loss of assistants. The results obtained under previous grants have been published in Publications of the Goodsell Observatory of Carleton College, Numbers 5 and 6, 1917–18 under the title, "Photographic Observations of Asteroids," by H. C. Wilson, C. H. Gingrich,and Julia M. Hawkes (No. 6).

No. 16. A grant of $500 was made to Professor JOHN A. MILLER, Director of the Sproull Observatory of Swarthmore College, Pennsylvania, for the employment of an assistant in measuring and reducing plates for the determination of parallaxes. This fund is being paid to Miss Carolyn H. Smedley, Research Assistant in the Observatory. In the fall she also aided in studying the corona of the eclipse of 1918. The sum of $500 is only part of her salary. The observatory has recently put into press fifty new parallaxes. A great number of series are in process of measurement. Between twenty and twenty-five additional parallaxes are ready for publication.

A report was received from the Committee on the J. Lawrence Smith Fund signed by E. W. Morley (Chairman) stating that reports on previous grants were as follows:

No. 4. Professor C. C. TROWBRIDGE, of the Department of Physics, Columbia University, New York, has received grants amounting to $1400 to aid in the study of luminous trains of meteors. His lamented and untimely death in June last has put an end to this investigation.

An unfinished paper on the spectra of luminous trains has been completed from his notes by a research assistant familiar with the whole investigation, and is now ready for submission to the Academy. Two other papers are to be completed in the same way: one contains a summary of theories hitherto advanced in explanation of meteor trains, and the other discusses certain auroral phenomena and their relation to theories concerning meteor trains.

Two other parts of Professor Trowbridge's research have probably not gone so far that they can be completed without further investigation. One concerns the drift and diffusion of meteor trains as bearing on the phosphorescent gas theory; the other concerns the height of meteor trains and its relation to the height of the atmosphere. A large amount of classified information, arranged for convenient reference, has been collected by Professor Trowbridge, which it is hoped will be utilized by some one who may interest himself in these or cognate matters.

The department of physics in Columbia University has inquired what disposition shall be made of this material, and the committee suggest that it be forwarded to the Secretary of the Academy for preservation till it may become useful. We have also requested the research assistant of Professor Trowbridge to prepare a paper stating the nature and extent of the information contained in this material, which will soon be ready.

No. 9. Professor S. A. MITCHELL, University of Virginia, University, Va., has received grants amounting to $1500 to aid in securing observations of meteor paths and radiants, and in computing orbits where the observations suffice. During the last year, war-time activities have interrupted this valuable work. It is now resumed, and the unexpended balance of the grant may well be sufficient for a time.

There is now $850.39 cash in hand, as well as $2032.50 invested income, making $2882.89 available for grants.

A report was received from the Directors of the Benjamin Apthorp Gould Fund, signed by F. R. Moulton, E. E. Barnard and R. S. Woodward, stating that on April 1, 1919, the total income balance of the Gould Fund was $7028.46, of which $1773.46 was in cash and $5255 in interest-bearing securities; and that since April 1, 1919, a grant of $500 had been made to Benjamin Boss for the support of the *Astronomical Journal*.

A report was received from the Directors of the Wolcott Gibbs Fund, signed by C. L. Jackson, Edgar F. Smith, and T. W. Richards stating that the unexpended income of the fund amounted to $622.76, of which $500 had been invested temporarily in Liberty Bonds; that no award had been made; and that reports on previous awards were as follows:

No. 8. Professor R. L. Datta, University College of Sciences, Calcutta, reports that in spite of hindrance from the war he has procured organic chemicals to the value of three-quarters of the grant, and that these have enabled him to carry on work described in two papers now ready for publication in the *Journal of the American Chemical Society*—"Iodination by Means of Nitrogen Iodide and Replacement of Iodine by Nitro Groups," by R. L. Datta and J. Lahire. This paper contains a careful study of the two reactions including the description of seven new compounds. "The Replacement of Sulphuric Groups of Chlorine and the Preparation of Organic Chloro-derivatives," by R. L. Datta and H. K. Miller. This paper contains a careful study of this reaction.

No reports were received from Professor W. B. Harkins, (Nos. 4 and 7), or from Professor G. P. Baxter, (No. 6).

(No report was presented from the Committee on the Marcellus Hartley Fund, owing to the recent death of the Chairman, Dr. George F. Becker.)

A report was received from the Committee on the Marsh Fund, signed by E. H. Moore (Chairman), stating that the condition of the Fund was

Total capital April 1,1919	$19,500.00
In addition, cash	375.47
Two Liberty bonds	150.00
Accrued income to be added May 1, 1919, about	227.00

By the addition of $500 from balance and income the capital is to be raised to $20,000 February 1, 1920. Reports on previous grants are as follows

No. 1. Dr. John M. Clarke, Albany, New York: "Mutualism, Symbiosis and Dependent Life among Animals of Geologic Time." In view of the difficulty of completing this investigation under war conditions Dr. Clarke was authorized to make use of part of this Grant for clerical work in the preparation for the Academy of the James Hall biography. Dr. Clarke reports—First—the original investigation is progressing favorably. "Study of geological evidences of parasitism and dependence has proceeded as opportunity permitted. Surveys and selections have been made of the material in several of the larger museums. Special collecting has been done, a good degree of careful preparation made of the material and a considerable number of enlarged stereo photographs made." Second—"The Hall Memoir, which has involved the reading and extraction of 10,000 letters and many other records, is now approaching conclusion, having been brought up to the year 1880."

No. 2. M. Ferdinand Canu, Versailles, France, "Early Tertiary Bryozoa of North America." This grant was to further the completion of investigations in progress in coöperation with Dr. R. S. Bassler of the United States National Museum, who reports: "This work is

now in press as Bulletin 106 of the United States National Museum, consisting of 162 quarto plates and a correspondingly large number of text figures and pages of text. With this assistance we have also completed a study on 'Early Tertiary Cyclostomatous Bryozoa,' which when printed, will amount to about 100 pages of text and 30 to 40 plates."

A report was received from the Committee on the Murray Fund, signed by Wm. H. Dall (Chairman), stating that the total amount of income available from this fund according to the Treasurer, is $916.91; of which $516.91 was in cash and $400 in Liberty Bonds; and that the Committee duly reported their recommendation in regard to the award of the medal which was approved by the Council, and would be carried out at the dinner on April 29, 1919.

GENERAL BUSINESS

A report by the Auditing Committee, C. G. Abbot, L. O. Howard, and David White, stating that the regular financial statement of the Treasurer and the special account in support of the Division of Medicine of the National Research Councils had been duly audited and found correct.

The following report of the Editorial Board of the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES was received:

The Editorial Board reports as follows regarding its activities during the past year: Volume 4 of the PROCEEDINGS has been completed, and the numbers of Volume 5 are being regularly issued each month.

The articles in Volume 4 of the PROCEEDINGS may be summarized as follows: Mathematics, 9; Astronomy, 11; Physics and Engineering, 25; Chemistry, 5; Geology and Paleontology, including Mineralogy and Petrology, 9; Botany, 3; (see also Genetics); Zoology, including General Biology, 12, (see also Genetics); Genetics, 6; Physiology and Pathology, 10; Anthropology and Psychology, 1; a total of 91 articles. The division of these articles between members of the Academy and non-members is 39 and 52 respectively.

The list of institutions which have contributed three or more articles is as follows: Carnegie Institution, 15, divided as follows: Solar Observatory, 7; Nutrition Laboratory, 4; Geophysical Laboratory, 1; Marine Biology, 2; Station for Experimental Evolution, 1; Harvard University, 15; Brown University, 7; University of Illinois, 5; Bermuda Biological Station for Research, 4; University of California, 4; University of Chicago, 4; University of Pennsylvania, 4.

On April 28, 1919 the Editorial Board held its annual meeting, the following members being present: Messrs. Pearl, Wilson, Day, Hale, Clarke, Jennings, Lusk, Mayor, Millikan, A. A. Noyes, W. A. Noyes, Thorndike, and Wheeler. In regard to the general editorial policy of the PROCEEDINGS, it was held to be desirable to make the PROCEEDINGS, in so far as possible, a publication of permanent original, scientific value, in which duplication of publication should be avoided so far as was consistent with the general policy which has prevailed in the PROCEEDINGS throughout its existence of publishing short articles embodying a brief statement of the results of important researches which were eventually to be published in detail elsewhere. It was agreed that the rule of the PROCEEDINGS requiring prior publication should be waived in the case of summaries of an extended series of papers, some (if not all) of which had already appeared elsewhere, provided that the summary was so written as to be of practical use in enforcing the points made in individual papers and in attracting the attention of workers in related fields, thus making the summary in the PROCEEDINGS a real contribution to the subject.

It was agreed in general to adhere to the present policy of the PROCEEDINGS in retard to having the bulk of its contents made up of numerous short articles of first-class quality, fall ing in general under the 6-page limit. It was thought desirable, however, to continue the policy of accepting longer articles under special circumstances, 12 to 15 pages being regarded as a proper maximum length for such articles. It was agreed that the Managing Editor and the Chairman of the Board should have all reasonable freedom in exercising their judgment on the administration of policy as to length of articles in particular cases.

A special joint committee of the Editorial Board of the PROCEEDINGS on the one hand, and the National Research Council on the other hand, met and considered the general question of the future relations of the National Research Council and the PROCEEDINGS. The special committee was constituted as follows: For the Editorial Board: the Chairman, the Managing Editor, and Dr. Day; for the National Research Council: Doctors Hale, Merriam and Yerkes. The general policy worked out by this joint committee was agreed to by the Editorial Board. It was in effect this, that the PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES should become in a broader sense than had been true in the past, representative of the activities of the National Research Council, as well as of the National Academy. It was understood that the National Research Council should offer to the PROCEEDINGS as official communications of the Research Council of the National Academy such material (intended for publication as bulletins of the Council) as the Council considered to be of permanent scientific value in the PROCEEDINGS. It was felt that in view of the proposed enlargement of the scope of the PROCEEDINGS to represent the activities of the National Research Council that some changes in the typography of the PROCEEDINGS might be desirable and it was agreed that changes looking to the improvement of the PROCEEDINGS in this regard would be inaugurated as soon as agreement on the matter had been reached between the conferees from the two interested bodies.

Although no formal report can be made on the matter at this time, the Editorial Board is in possession of information which leads it to hope that the financial support of the PROCEEDINGS will be more adequately provided for in the near future. In this event, the Editorial Board expects to inaugurate, with the approval of the Council of the Academy, certain changes in details of policy which will be in the direction of more liberal arrangements between the PROCEEDINGS and its contributors, particularly in the matter of providing a certain number of free reprints to authors. The wisdom of such policy has always been clear to the Editorial Board, but the simple fact of lack of available funds in the past has made it impossible for the Board to carry out its wishes in this matter.

Respectfully submitted,
RAYMOND PEARL, *Chairman.*

The following organization of the National Research Council was presented and, upon motion, was adopted as a whole, with power of amendment vested in the Council of the National Academy of Sciences.

Organization of the National Research Council

Preamble

The National Academy of Sciences, under the authority conferred upon it by its charter enacted by Congress, and approved by President Lincoln on March 3, 1863, and pursuant to the request expressed in an Executive Order made by President Wilson on May 11, 1918, hereto appended, adopts the following permanent organization for the National Research Council, to replace the temporary organization under which it has operated heretofore.

Article I.—Purpose

It shall be the purpose of the National Research Council to promote research in the mathematical, physical, and biological sciences, and in the application ot these sciences to engineer-

ing, agriculture, medicine, and other useful arts, with the object of increasing knowledge, of strengthening the national defense, and of contributing in other ways to the public welfare, as expressed in the Executive Order of May 11, 1918.

Article II.—Membership

Section 1. The membership of the National Research Council shall be chosen with the view of rendering the Council an effective federation of the principal research agencies in the United States concerned with the fields of science and technology named in Article I.

Section 2. The Council shall consist of

1. Representatives of national scientific and technical societies;
2. Representatives of the Government, as provided in the Executive Order;
3. Representatives of other research organizations and other persons whose aid may advance the objects of the Council.

Article III.—Divisions

Section 1. The Council shall be organized in Divisions of two classes:

A. Divisions dealing with the more general relations and activities of the Council;
B. Divisions dealing with related branches of science and technology.

Section 2. The initial constitution of the Divisions of the Council shall be as follows:

A. Divisions of General Relations:
- I. Government Division.
- II. Division of Foreign Relations.
- III. Division of States Relations.
- IV. Division of Educational Relations.
- V. Division of Industrial Relations.
- VI. Research Information Service.

B. Divisions of Science and Technology:
- VII. Division of Physical Sciences.
- VIII. Division of Engineering.
- IX. Division of Chemistry and Chemical Technology.
- X. Division of Geology and Geography.
- XI. Division of Medical Sciences.
- XII. Division of Biology and Agriculture.
- XIII. Division of Anthropology and Psychology.

Section 3. The number of divisions and the grouping of subjects in Article III, section 2, may be modified by the Executive Board of the National Research Council.

Section 4. The Divisions of General Relations shall be organized by the Executive Board of the National Research Council (Article IV, section 2).

Section 5. To secure the effective federation of the principal research agencies in the United States, provided for in Article II, a majority of the members of each of the Divisions of Science and Technology shall consist of representatives of scientific and technical societies, chosen as provided for in Article V, section 2. The other members of the Division shall be nominated by the Executive Committee of the Division, approved by the Executive Board of the National Research Council, and appointed in accordance with Article V, section 4.

Section 6. The Divisions of the Council, with the approval of the Executive Board, may establish sections and committees, any of which may include members chosen outside the membership of the Council.

Article IV.—Administration

Section 1. The affairs of each Division shall be administered by a Chairman, a Vice-Chairman, and an Executive Committee, of which the Chairman and the Vice-Chairman shall be ex-officio members; all of whom shall be elected annually by the Division and confirmed by the Executive Board.

Section 2. The affairs of the National Research Council shall be administered by an Executive Board, of which the officers of the Council, the President and Home Secretary of the National Academy of Sciences, the President of the American Association for the Advancement of Science, the Chairmen and Vice-Chairmen of the Divisions of Science and Technology and the Chairmen of the Divisions of General Relations shall be ex-officio members. The Executive Board may elect additional members, not to exceed ten in number, who, if not already members of the National Research Council, shall be appointed thereto, in accordance with Article V, section 4.

Section 3. The officers of the National Research Council shall consist of a Chairman, one or more Vice-Chairmen, a Secretary, and a Treasurer, who shall also serve as officers of the Executive Board of the Council.

Section 4. The officers of the National Research Council, excepting the Treasurer, shall be elected annually by the Executive Board. The Treasurer of the National Academy of Sciences shall be ex-officio Treasurer of the National Research Council.

Section 5. The duties of the officers of the Council and of the Divisions shall be fixed by the Executive Board.

Article V.—Nominations and Appointments

Section 1. The Government bureaus, civil and military, to be represented in the Government Division, and the scientific and technical societies, to be represented in the Divisions of Science and Technology of the National Research Council, shall be determined by joint action of the Council of the National Academy of Sciences and the Executive Board of the National Research Council.

Section 2. Representatives of scientific and technical societies shall be nominated by the societies, at the request of the Executive Board, and appointed by the President of the National Academy of Sciences to membership in the Council and assigned to one of its Divisions.

Section 3. The representatives of the Government shall be nominated by the President of the National Academy of Sciences after conference with the Secretaries of the Departments concerned, and the names of those nominated shall be presented to the President of the United States for designation by him for service with the National Research Council.

Section 4. Other members of the Council shall be nominated by the Executive Committees of the Divisions, approved by the Executive Board, and appointed by the President of the National Academy of Sciences to membership and assigned to one of the Divisions.

Section 5. Prior to the first annual meeting of the Council following January 1, 1919, all Divisions shall be organized by appointment of their members in accordance with Article II and Article V, sections 1 to 4.

Section 6. As far as practicable one-third of the original representatives of each scientific and technical society and approximately one-third of the other original members of each of the Divisions of Science and Technology shall serve for a term of three years; one-third for a term of two years, and one-third for a term of one year, their respective terms to be determined by lot. Each year thereafter, as the terms of members expire, their successors shall be appointed for a period of three years.

Section 7. The Government representatives shall serve for periods of three years, unless they previously retire from the Government office which they represent, in which case their successors shall be appointed for the unexpired term.

Section 8. As far as practicable a similar rotation shall be observed in the appointment of the members of the Divisions of General Relations.

Article VI.—Meetings

Section 1. The Council shall hold one stated meeting, called the annual meeting, in April of each year, in the city of Washington, on a date to be fixed by the Executive Board. Other meetings of the Council shall be held on call of the Executive Board.

Section 2. The Executive Board and each of the Divisions shall hold an annual meeting, at which officers shall be elected, at the time and place of the annual meeting of the Council, unless otherwise determined by the Executive Board, and such other meetings as may be required for the transaction of business.

Section 3. Joint meetings of the Executive Board of the National Research Council and the Council of the National Academy of Sciences shall be held from time to time, to consider special requests from the Government, the selection of organizations to be represented in the National Research Council, and other matters which, in the judgment of the President of the National Academy, require the attention of both bodies.

Article VII.—Publications and Reports

Section 1. An annual report of the work of the National Research Council shall be presented by the Chairman to the National Academy of Sciences, for submission to Congress in connection with the annual report of the President of the Academy.

Section 2. Other publications of the National Research Council may include papers, bulletins, reports, and memoirs, which may appear in the Proceedings or Memoirs of the National Academy of Sciences, in the publications of other societies, in scientific and technical journals, or in a separate series of the Research Council.

The following communication from Mr. Whitman Cross, resigning the position of Treasurer, which he had held for eight years, was presented and accepted with regret, in view of the valuable services which Mr. Cross had rendered to the Academy.

April 21, 1919.

My dear Dr. Walcott:

I hereby respectfully tender my resignation as Treasurer of the National Academy of Sciences, to take effect on May first, or as soon thereafter as my successor can qualify. It has been a great pleasure to render to the Academy such service as I could as Treasurer during the last eight years. It is now necessary that I should be able to devote myself exclusively to the completion of work which has been in progress for many years in connection with the United States Geological Survey.

With high appreciation of the opportunity for connection with the work of the Academy for so many years, I am, very sincerely yours,

WHITMAN CROSS.

The Home Secretary was requested to transmit the thanks of the Academy to the Signal Corps of the United States Army, the Smithsonian Institution, the Bureau of Standards, and to the Cosmos Club, for the courtesies extended during the meeting.

ELECTION OF OFFICERS AND MEMBERS

C. G. ABBOT was unanimously elected Home Secretary.

F. L. RANSOME was unanimously elected Treasurer.

J. J. CARTY, H. H. DONALDSON, and RAYMOND PEARL were elected members of the Council.

The following persons were elected as new members of the Academy.

JOSEPH BARRELL, Yale University, New Haven, Conn.
GARY NATHAN CALKINS, Columbia University, New York.
HEBER DOUST CURTIS, Lick Observatory, California.
GANO DUNN, 43 Exchange Place, New York.
LAWRENCE JOSEPH HENDERSON, Harvard University, Cambridge, Mass.
REID HUNT, Harvard Medical School, Boston, Mass.
TREAT BALDWIN JOHNSON, Yale University, New Haven, Conn.
WINTHROP JOHN VANLEUVEN OSTERHOUT, Harvard University, Cambridge, Mass.
FREDERICK HANLEY SEARES, Mt. Wilson Observatory, Pasadena, Cal.
WILLIAM ALBERT SETCHELL, University of California, Berkeley, Cal.
GEORGE OWEN SQUIER, Major General U. S. A., Washington, D. C.
AUGUSTUS TROWBRIDGE, Princeton University, Princeton, N. J.
OSWALD VEBLEN, Princeton University, Princeton, N. J.
ERNEST JULIUS WILCZYNSKI, University of Chicago, Chicago, Ill.
EDWIN BIDWELL WILSON, Massachusetts Institute of Technology, Cambridge, Mass.

SCIENTIFIC SESSIONS

Two public lectures on the WILLIAM ELLERY HALE Foundation were given on April 28 and 29 by JAMES HENRY BREASTED, of the University of Chicago, on the Origin of Civilization.

Four public scientific sessions were held on April 28 and 29 at which the following papers were presented:

(NOTE: A dagger † indicates that the paper has been or shortly will be printed in these PROCEEDINGS; the numbers following the dagger are page references to this volume.)

ALFRED G. MAYOR: The age of the fringing reef of Tutuila, American Samoa.

CHARLES D. WALCOTT: Seaweeds and sponges of the Middle Cambrian.

ROBERT G. AITKEN: The spectra of the visual binary stars.

GEORGE E. HALE, F. ELLERMAN, S. B. NICHOLSON and A. H. JOY: The magnetic polarity of sun spots.

WALTER S. ADAMS and A. H. JOY: The motions in space of some stars of high radial velocity.† 239–241.

WALTER S. ADAMS and G. STRÖMBERG: The use of the spectroscopic method for determining the parallaxes of the brighter stars.† 228–232.

ADRIAAN VAN MAANEN (introduced by George E. Hale): Evidence of stream-motion afforded by the faint stars in the Orion Nebula.† 225–228.

GRAHAM LUSK and H. V. ATKINSON: The production of fat from protein after giving meat in large quantity to a dog.† 246–248.

WILLIAM S. HALSTED: End-to-end anastomosis of the intestine—experimental study.

ROBERT M. YERKES (introduced by George E. Hale): Psychological examining in the United States Army.

R. S. LULL (By title): Biographical Memoir of SAMUEL WENDELL WILLISTON.

FREDERICK H. SEARES (introduced by George E. Hale): Relation between color and luminosity for stars of the same spectral type.† 232–238.

FREDERICK H. SEARES, A. VAN MAANEN, and F. ELLERMAN (introduced by George E. Hale): Deviations of the sun's general magnetic field from that of a uniformly magnetized sphere.† 242–246.

W. W. CAMPBELL: The solar corona.

HERBERT E. GREGORY (introduced by W. M. Davis): Plans for exploration of the Pacific.

FRANCIS G. BENEDICT, W. R. MILES, and ALICE JOHNSON: The temperature of the human skin.† 218–222.

S. J. Meltzer and M. Wollstein: The influenze of degeneration of a vagus nerve upon the development of pneumonia.†

Demonstration of war research problems at the National Bureau of Standards.

Edwin H. Hall: The effect of great pressure on the electric conductivity and thermo-electric properties of metals.

Edwin H. Hall: Comments on the results of Bridgman's experiment.

Charles Lane Poor (introduced by J. S. Ames): Line of position computer.

Irving Langmuir: The arrangement of electrons in atoms and molecules.† 252–259.

Henry F. Osborn: Palaeomastodon, the ancestor of the long-jawed mastodons only.† 265–266.

Henry F. Osborn: Seventeen skeletons of Moropus: probable habits of this animal.† 250–252.

Thomas B. Osborne and Alfred J. Wakeman: The preparation of vitamine-green proteins.

Arthur G. Webster: Quantitative results in interior ballistics.† 259–263.

Arthur G. Webster: Quantitative results in elastic hysteresis.

Edwin H. Hall: Thermal conduction in metals, from the standpoint of dual electric conduction.

Edwin H. Hall: The thermo-electric equation $P = T\,dV/dT$ once more.

A. O. Leuschner and Sophia H. Levy: Perturbations of minor plants discovered by James C. Watson: (104) Clymene, (106) Dione, (168) Sibylla, (175) Andromache. (By title.)

Arthur G. Webster: The most perfect tuning fork.

Arthur G. Webster: Angle of repose of wet sand.† 263–265.

Arthur G. Webster (By title): On the equation of state of powder gases whose specific heats satisfy the law of Mallard and Le Chatelier.† 286–288.

C. C. Trowbridge: Meteor train spectra.

Edward Kasner: Geometry of the wave equation.

C. G. Abbot: Rotating projectiles from smooth-bore guns (illustrated).† 386–388.

C. G. Abbot: Means for measuring the speed of projectiles in flight (illustrated).† 388–389.

C. G. Abbot: Recent simultaneous measurements of the solar constant of radiation at Mount Wilson, California, and Calama, Chile (illustrated).† 383–386.

John C. Merriam: Human remains from the Pleistocene of Rancho La Brea (illustrated).

One public session of the Academy with the National Research Council was held on April 30 at which the following papers were presented:

George E. Hale: The past work and future plans of the National Research Council.

John C. Merriam: The Division of General Relations, Section on Relations with Educational Institutions and State Committees.

R. A. Millikan: The Division of Physics, Mathematics, Astronomy and Geophysics.

Dayton C. Miller: Pressures and velocities, internal and external, due to the discharge of large guns.

E. W. Washburn: The Division of Chemistry and Chemical Technology.

A. A. Noyes: Nitrate investigations.

Whitman Cross: The Division of Geology and Geography.

R. G. Hussey: The Division of Medicine and Related Sciences.

R. M. Yerkes: Psychology in relation to the war.

C. E. McClung: The Division of Agriculture, Botany, Forestry, Zoology and Fisheries.

G. H. Clevenger: The Division of Engineering.

AWARD OF MEDALS

The Academy assembled for dinner at the Wardman Park Inn on the evening of April 29, when medals were presented as follows:

The Agassiz Medal, for research in oceanography to Albert I, Prince of Monaco.

The Henry Draper Medal, to Charles Fabry, of the University of Marseilles, in recognition of his researches in Physics and Astronomy, chiefly by means of interferometers.

RESEARCH GRANTS FROM THE TRUST FUNDS OF THE ACADEMY

During the twelve months preceding the Annual Meeting of the Academy the following grants for the promotion of research were made from the Trust Funds of the Academy.

GRANTS FROM THE BACHE FUND

No. 210. Wayne J. Atwell, University of Buffalo, Medical Department, $210. For investigation on the development of the human hypophysis cerebri.

No. 211. George H. Shull, Princeton, N. J. $360. For continuation of studies in linkage and other genetical phenomena of Oenothera, and duplication of factors, multiple allelomorphism and other genetical phenomena in Shepherd's-purse.

No. 212. Arthur G. Webster, Clark University, Worcester, Mass., $1500. For study of the air-wave accompanying a projectile.

No. 213. Duncan S. Johnson, Johns Hopkins University. $300. For a study of the behavior and character of the chromosomes at spermatogenesis, etc., in certain tropical liverworts.

GRANTS FROM THE WATSON FUND

No. 18. John A. Miller, Sproul Observatory, $500. For the employment of assistance in measuring and reducing plates for the determination of parallaxes for the year beginning July 1, 1919. (Supplementary to Grant No. 17.)

No. 19. Herbert C. Wilson, Goodsell Observatory, $300. For the employment of assistance in securing, measuring, and reducing photographic positions of minor planets for the year beginning July 1, 1919. (Supplementary to Grant No. 16.)

No. 20. J. A. Parkhurst, $500. For the employment of assistance in measuring and reducing plates for photographic and photovisual magnitudes in the North Polar Sequence and in the parallax field on which the Yerkes Observatory is working; the grant to be available during the year beginning June 1, 1919.

GRANTS FROM THE MARSH FUND

No. 3. Roy L. Moodie, University of Illinois, College of Medicine, Chicago, $200. For "Studies in Paleopathology" for the purpose of contributing to the History of Medicine and attempting to determine what part, if any, disease has played in the extinction of races.

PROCEEDINGS

OF THE

NATIONAL ACADEMY OF SCIENCES

Volume 5 NOVEMBER 15, 1919 Number 11

THE INFLUENCE OF DEGENERATION OF ONE VAGUS NERVE UPON THE DEVELOPMENT OF PNEUMONIA

By S. J. Meltzer and Martha Wollstein

Rockefeller Institute for Medical Research, New York

Read before the Academy, April 28, 1919

Since Galen's time experiments have been made upon the vagus nerve and it has been generally known that section of both nerves leads to an early death of the animal. It was however in the seventeenth century that it first became known, through the observations of Valsalva and Morgagni, that the section of the vagi causes lesions in the lungs, lesions which were designated by Vieussens as inflammation. Since this period numerous studies have been made regarding the nature of the influence of cutting of the vagi which leads to inflammation of the lungs, to pneumonia. Many investigators were of the opinion that the section of the nerves leads directly to changes in the pulmonary tissue. The last representative of this group of investigators was Schiff who in the forties of the last century described the lesion as being due to a *neuro-paralytic* action. At the same period, however, Traube carried out many series of well conducted experiments by which he seemed to prove that the section of both vagi causes lesions in the lung by paralysis of the oesophagus, which prevents the transportation of food into the stomach, and by paralysis of the nerves of the larynx, a paralysis which facilitates the entrance of food, saliva, and other foreign bodies into the trachea and the lungs. In other words, the pneumonia following section of both vagi was not due to direct changes in the lung tissue but indirectly to the entrance of foreign bodies through the trachea into the lung; the inflammation following section of both vagi was of the nature of 'aspiration

pneumonia.' In the last six or seven decades many more experiments have been made on the section of both vagi; but all investigators have accepted Traube's interpretation, namely, that the section of the vagi does not cause directly any changes in the tissues. but that the section of the nerves leads to a 'foreign body pneumonia.'

In the above mentioned numerous experiments, as a rule, both vagi were cut; not very many experiments were made by cutting only one vagus. But when such an experiment was made, it was found practically invariably that cutting of one vagus produced neither any clinical effect during life nor were there pathological lesions of the lung at autopsies. I may add that our own experimental experience supports this finding: Cutting of one vagus of a normal dog never leads to a pulmonary disease or lesion.

In the course of the last eight years we carried out in this laboratory numerous experiments on the production of pneumonia by direct injection through the unoperated larynx into the bronchi of normal dogs of various micro-organisms capable of producing various forms of pneumonia. Most of our experiments were recently made with intrabronchial injections of culture of pneumococcus type I, and it was established that the degree of the disease which it produces or the length of time required for the fatal outcome depend upon the virulence of the organism as well as the quantity of the culture injected. Recently a long series of such experiments were carried out with a view of studying the *direct action of vagus nerves upon the development of pneumonia.* Since in these experiments the nerves of the oesophagus and of the larynx remained intact, any acceleration in the development of the disease or the pulmonary lesion could be ascribed only to the interruption of the nervous impulse normally transmitted through the intact vagus nerve to the lung tissue. We shall not enter here into the details of the experiments; but merely state briefly that: In one series of experiments one vagus was cut but no organisms insufflated; these animals, as mentioned above, remained normal. In another series of experiments a definite quantity of the culture was insufflated in normal dogs. These animals either remained alive or died with some pulmonary lesions many days after the injection. In a third series the same culture and the same quantity of it was insufflated on the same day and under the same condition as in the last mentioned series; but in these animals one vagus was cut either four days or ten or more days before the culture was insufflated. These experiments brought to light a striking result. *In the series in which one vagus was cut ten or more days before the insufflation of the culture nearly all*

animals died in less than twenty-four hours after receiving a certain (minimum) amount of the culture, while in the animals in which one vagus was cut four days before the injection or not cut at all, not a single animal died in such a short period, and most of them remained alive. In other words, a certain minimum quantity of culture which was injected ten or more days after one vagus was cut proved to be rapidly fatal; while the injection of the same quantity remained ineffective if no vagus was cut or was cut only a few days before the injection of the culture.

We shall confine ourselves here to the communication of these facts and to only one interpretation of them. First, these experiments show unmistakably that the integrity of a certain form of nerve impulses is indispensable for the normal resistance of the lung tissue to infection or intoxication; if one vagus nerve was cut ten days before the injection of a pneumococcus culture a certain minimum of the culture was nearly invariably rapidly fatal, while in a normal animal or in an animal in which one vagus was cut only a few days before the injection or not cut at all, the same minimum of culture has little or no effect.

Second, for the vessels of most of the tissues it is known that they are innervated by vasoconstrictors and vasodilators. It is further known that the vasoconstrictors degenerate about four days after the section of the nerves, while the vasodilators degenerate only about ten days or later after the section. If we assume that the lungs are also provided with vasoconstrictors and vasodilators and that these vasomotor fibres are carried to the lungs in the vagus nerves, we would have a plausible explanation for the phenomena with which we were confronted in our experiments, namely, cutting of the vagi four days previous to the culture injection is capable of causing degeneration of only the vasoconstrictor nerve fibres, which may not be indispensable to the upholding of the normal activity of the lung tissue. If, however, the injection is made about ten days or longer after the cutting of one vagus, also the vasodilators are degenerated and their integrity may be indispensable for upholding the circulation and the normal resistance of the lung tissue to infection or intoxication by a virulent culture of pneumococcus type I.

At any rate, our experiments brought out the facts that nerve impulses are running through the vagus nerves which are important to the upholding of the normal resistance of the lung tissue itself. That even one vagus nerve is an important factor in this process; and that these activities of the vagus nerves can be recognized only after an intrabronchial or intratracheal injection of a virulent culture ten days after the cutting of one vagus.

Furthermore, the fact that the intrabronchial injection of a minimum culture proved fatal only when the injection was made ten days after the section of one vagus and the further consideration of the fact that it takes about ten days for the degeneration of vasodilators lend support to the assumption that it is the degeneration of the vasodilators which is responsible for the fatal results observed in our experiments.

ON THE ETHOLOGY OF CHITON TUBERCULATUS[1]

By W. J. Crozier and L. B. Arey

University of Illinois and Northwestern University

Communicated by E. L. Mark, September 5, 1919

Chitons of the species *C. tuberculatus* Linn. are an important element in the shore-fauna of the Bermuda Islands. Their large size, their abundance, and the diversity of the habitats which they occupy within the tidal zone, make them animals appropriate for a variety of investigational purposes—notably for study of the relations found among the features of local habitats, on the one hand, and, on the other, certain definite sensory and other modifications of these animals which develop with the advancing age of a chiton. The organization of the Placophora, probably primitive with reference to that of other molluscs, makes it important also to obtain physiological evidence as to the characteristics of the nervous system in these animals. On the structural side it is well known that the chiton central nervous system is in the form of strands, containing perikarya throughout their length, but with no concentrations of these cells into distinct ganglionic enlargements.

We find in *Chiton tuberculatus* decided indications of but a relatively incipient degree of nervous centralization. The autonomy of the several portions of the body, or of the parts into which it may be artificially separated, is conspicuous. At the sides of the body, the parts (girdle, ctenidia, etc.) innervated by the pallial strands are pronouncedly homolateral in their responses. The coördinating mechanism for the production of pedal locomotor waves is locally contained. The absence of strong anterior nervous centralization is nicely indicated by the exhibition of backward creeping (especially in Ischnochiton) under proper conditions of stimulation by light.

Of the several kinds of sensory receptors which we have distinguished in *Chiton tuberculatus*, the nervous elements in the shell tegmenta, now for the first time proved to be photosensitive, are perhaps the most

important in determining the general habits of the animal. Through the mediation of the nervous elements of the shell-valves Chiton exhibits precise phototropic orientations. The younger individuals are photonegative, the older ones photopositive, to sunlight. Chitons of intermediate age are positive to weak daylight, negative to strong. The 'intermediate' age is generally in the neighborhood of six years (5 to 7 cms. length).[2] This alteration in the behavior of Chiton toward daylight as the animal becomes older is primarily responsible for the exhibition of a complex series of harmonious environmental interrelations. The progressive inversion in the sense of Chiton's phototropism is due to the elimination of the shell photoreceptors by erosion, thus conditioning in the older individuals a lower specific stimulating power of the light. Erosion of the shell is itself produced through normal growth effects, in part; more directly, by physical agencies of the environment and by the activities of various small organisms which come to live on the chiton shell.

The nature of the algal food, varying somewhat from place to place, and the presence of epiphytes, barnacles, tubiculous worms, and the like, on the dorsal surface of a chiton, determine automatically a certain degree of homochromic coloration. The established fact that a given Chiton tends to remain within a relatively small area for long periods, affords opportunity for the institution of correlations of this nature. These correlations concern practically every feature of the bionomics of Chiton: its method of feeding, its breeding habits, and other matters which have been studied; and they follow automatically in the wake of the changing phototropism of the animal, itself largely controlled by environmental causes. Granted the original condition that those inherited mechanisms for response present in the young post-larval chiton lead to its living under loosely piled small stones at the very upper limit of the tide, it can be shown that the 'adaptive' cycle of subsequent changes, leading ultimately to the life of the oldest individuals on brilliantly illuminated rock surfaces, is produced in automatic sequence. It is important that the environmental effects of a particular location are in general not such as to cause a chiton to become more closely adapted to the peculiarities of that situation, but on the contrary result in the animal's going somewhere else—namely, into a more brightly illuminated area, where the environment as a whole is in certain respects fundamentally different.

From the standpoint of the theory of adaptation, the most important result of our inquiry into the natural history of Chiton may be given thus:

a number of precise and intricate bionomic correlations result automatically from the animal's modes of reaction, mediated by a central nervous apparatus which is relatively diffuse, less centralized functionally than is that of other molluscs; the habits so expressed determine the nature of the environment in which the Chiton lives.

Full reports of these studies will be found in papers to appear in the *Journal of Experimental Zoölogy*, and in the *American Naturalist*.

[1] Contributions from the Bermuda Biological Station for Research, No. 112.

[2] cf. Crozier, W. J., These PROCEEDINGS, November, 1918.

THE NERVOUS ORGANIZATION OF A NUDIBRANCH[1]

BY L. B. AREY AND W. J. CROZIER,

NORTHWESTERN UNIVERSITY AND UNIVERSITY OF ILLINOIS

Communicated by E. L. Mark, September 5, 1919

Having in mind the important position of the mollusca with respect to the evolutional elaboration of central nervous organs, we have sought to obtain evidence permitting a more precise statement of the functional relations between the peripheral and the ganglionic conducting pathways in a nudibranch, *Chromodoris zebra*. The absence of a shell in these animals, and certain more specific peculiarities of their construction, afford favorable conditions for such analysis. A full account of our observations is in press, to appear in the *Journal of Experimental Zoölogy*.[2]

The dorsally placed 'crown' of gills comprises in *Chromodoris* about 12 distinct plumes arising from a ridge almost surrounding the anal aperture. Each plume may contract independently. All of the plumes may also contract simultaneously, and the whole gill-crown may be concealed within a collared pocket. During the day-time, and under certain conditions of alkalinity and temperature in the seawater,[3] the plumes are extended. If a plume be lightly touched at one point, the common orm of response is constriction at that level, resulting in a slight swaying of the plume. More vigorous stimulation likewise leads to this locally confined unsymmetrical contraction, which however is now seen to spread distally from the site of activation, as a collapse and shrivelling of the plume, and is also accompanied by the downward pulling of the entire plume through the traction of muscles situated in the basal tissue of the gill-crown. Still stronger activation induces longitudinal shortening of the plume, both distally and proximally to the point of stimulation, and in the basal tissue.[4]

The polarity evident in these reactions of the single gill plumes has certain fundamental resemblances to that seen in the tentacles of sea anemones.[5] Like the latter, it pertains to other than tactile forms of activation (e.g., shading), is obliterated by magnesium sulphate anaesthesia, and persists in all its aspects in the plumes of an excised gill-crown (tied off basally to preserve internal pressure). It is therefore a local matter, conditioned by a self-contained nervous structure which conducts impulses more easily distalward than basally. The nature of this autonomous nervous equipment is defined by the fact that strychnine, in concentrations amply sufficient to affect certain other responses of the nudibranch known to be mediated by the central nervous ganglia, is without influence upon the responses of the gill plumes. The type of nonsynaptic conducting mechanism (nerve net) which is thus indicated for the gill crown, we attribute, on similar evidence, to all the peripheral parts of *Chromodoris*.

If the oral tentacle of one side of a *Chromodoris* be stimulated, the homolateral dorsal tentacle ('rhinophore') contracts together with the activated tentacle; the opposite oral tentacle does not respond. If however a 'rhinophore' be stimulated, the tentacle of the same side does not react with it, nor does the other 'rhinophore' respond. The same homolateral nature and irreciprocal character of nervous transmission between the several reacting parts is further found in a detailed study of the activities of the mouth and protrusable pharynx, tentacles, 'rhinophores,' and the anterior edge of the foot. Conduction such that a 'rhinophore' responds when its homolateral tentacle is stimulated disappears when the supra- and suboesophagal ganglia have been removed.

Under the influence of injected strychnine solution, we find that these reactions involving central, inter-organ transmission are profoundly modified. In general, the threshold for such reactions (now followed by a relatively long 'refractory interval') is lowered, the responses themselves enhanced. The irreciprocal character of the conduction, as between tentacle and 'rhinophore,' is abolished; so likewise is the normally pronounced homolateral bias of the reactions obliterated. There are other additional evidences of facilitated intraganglionic communication under strychnine. These effects are not manifest in deganglionated individuals treated with strychnine.[6, 7]

It may therefore be assumed that peripherally, in the body wall and its projecting outgrowths, there are nerve-nets concerned with local responses; that these nets are characteristically polarized; and that they are dominated by the central nervous system of the nudibranch, the latter

being essentially a synaptic system. It would appear that that primitive type of nervous organization predominantly present in such coelenterates as the sea-anemones, but preserved in vertebrates only among certain autonomous internal organs,[5] still forms in molluscs a highly important feature of the animal's action system.[8]

[1] Contributions from the Bermuda Biological Station for Research, No. 113.

[2] Crozier, W. J., and Arey, L. B., "Sensory reactions of Chromodoris zebra," *J. Exper. Zoöl., Phila.*, (in press).

[3] Crozier, W. J., 1919, *J. Gen. Physiol., Baltimore*, 1, No. 6.

[4] For more detailed treatment of the complex conditions here entering, consult papers cited in footnotes two and three.

[5] cf. Parker, G. H., 1919. *The Elementary Nervous System.* (Philadelphia.)

[6] Frölich (*Zs. Allgem. Physiol., Jena*, 11, 1910, p. 269) had already demonstrated that in *Aplysia* the site of action of strychnine is the 'cerebral' ganglion.

[7] Cushny (*Quart. J. Exp. Physiol., London*, 12, 1919, p. 153) points out that the depression of the reflex thresholds, rather than, as often held, the conversion of 'inhibition' to 'activation' (where reciprocal innervation is involved), is the essential feature of the strychnine effect.

[8] This conception of the nervous organization of *Chromodoris* agrees with the opinion held by Bethe respecting *Aplysia*, but the evidence here relied on is much more complete (cf. Bethe, *Allgem. Anat. u. Physiol. d. Nervensystems*, Leipzig, 1903).

ARE GENES LINEAR OR NON-LINEAR IN ARRANGEMENT?

By W. E. Castle

Bussey Institution, Harvard University

Communicated, August 13, 1919

As to the question whether the genes in a linkage system are linear or non-linear in arrangement, Morgan and his associates[1] still maintain their former view that the arrangement is strictly linear. I have questioned the validity of this view on the following grounds.[2] (1) The forces which link the genes together are possibly molecular rather than mechanical. If so, it is doubtful whether the entire system consists of a simple thread-like chain. (2) Construction of a model in three dimensions of the relations of the genes in the sex chromosome of *Drosphila ampelophila* as shown by the data of Morgan and Bridges,[3] and on their own assumption that distances are proportional to cross-over values, proves that the arrangement can not be linear. A similar reconstruction for the sex-linked genes of *D. virilis* shows the same thing for that species even more emphatically. (3) The linear hypothesis makes necessary the further assumption that cross-overs greater than 50% occur within the linkage system.

Such values have not been observed and are logically impossible. (4) The hypothesis of non-linear arrangement is simpler because it eliminates secondary hypotheses needed on the linear hypothesis to harmonize greater with lesser cross-over values, particularly the hypothesis of double crossing-over.

Point (1) is dismissed by Morgan and his associates with the remark that it is "probably not intended to be taken seriously." I, therefore, renew the suggestion and invite "serious" consideration of it. They argue that chromosomes are thread-like and that accordingly linkage-systems must be thread-like. This seems to me unconvincing, because we do not know how much of the visible chromosomes is composed of genes and how much is something else. Further it may be that the genes are not *in* the chromosomes at all but merely *attached* to them, especially if the attachments are molecular rather than mechanical. All the observational facts concerning linked genes would be equally well satisfied by such a view. It should be borne in mind that we have no evidence whatever that the chromosomes *are* genes but only that the genes are in some way connected with them. Accordingly the spatial relations of genes can not be legitimately inferred from the shape of the chromosomes.

Against point (2) the chief attack of Morgan and his associates is directed. No objection is made to the method employed of reconstruction in three dimensions, but the reliability of the data used is repeatedly assailed. Now the data consisted of Table 65 of Morgan and Bridges on which the authors based their Diagram I showing the arrangement of the genes as linear. Surely this same data could legitimately be used in testing the hypothesis that the arrangement is non-linear. Moreover the Table 65 is, so far as I know, the *only* table in which the authors have ever given a comprehensive summary of their data. At the head of the table, they say "In Table 65 all data so far secured upon the sex-linked characters are summarized." Now my model, based on actual measurements of distances indicated by Table 65, showed conclusively that the arrangement of the genes can not possibly be linear. No alternative is therefore left to Morgan and his associates except either to repudiate their Diagram I or to repudiate the data on which that diagram was ostensibly based. They choose the latter alternative. In discussing the relation to each other of any three genes, they now reject all data except such as are based on simultaneous observation of all three loci. This is done for the seemingly good reason that in this way all disturbing

agencies are eliminated, such as differences of temperature, age, or associated genes. But by so doing, they introduce other and much more serious disturbing agencies.

Take the case of the three genes, yellow, white and bifid, which both they and I have singled out as well illustrating the merits of our respective methods. Morgan and Bridges[3] in describing their method of constructing 'Diagram I' say. "Thus if the experiments give a cross-over value of 5 per cent for white and bifid, we say that white and bifid lie 5 units apart in the X chromosome. Other experiments show that yellow and white are about 1 unit apart, and that yellow and bifid are about 6 units apart. We can therefore construct a diagram with yellow as the zero, with white at 1, and with bifid at 6." But the Table 65 of this same publication, on the basis of which, it is explicitely stated, Diagram I was constructed, gives these distances more exactly as yellow-white, 1.1; white-bifid, 5.3; yellow-bifid, 5.5. The authors in the passage quoted ignore a considerable

TABLE 1

	FULL DATA		SELECTED DATA	
	Cross-over values	Cases	Cross-over values	Cases
Yellow-white	1.1	81,299	1.2	1,218
White-bifid	5.3	23,595	3.5	1,218
Yellow-bifid	5.5	3,681	4.7	1,218

discrepancy with these figures, since it is obvious that if the three genes are, as supposed, in a straight line, the distance yellow-bifid should equal the sum of yellow-white plus white-bifid, or 6.4 (instead of 5.5). Commenting on the actual observations in the case, I said, "Therefore bifid can lie neither above nor below yellow and white, in the line which joins them, but must lie laterally about equidistant from both." Returning to the case, Sturtevant, Bridges and Morgan now reject all but a small part of their previous data, retaining only two experiments in which all three loci were simultaneously under observation. The changed character of the data is shown in Table 1. Yellow-white is not materially affected but white-bifid is now 3.5, instead of 5.3, and yellow-bifid is 4.7, instead of 5.5. Are the new values more reliable than the old ones? So far as amount of data is concerned, they are much more liable to error through random sampling. If disturbing agencies are at work in isolated experiments, their effect is less likely to be felt in larger totals based on experi-

ments made under varied conditions. On this general ground the full data would seem to be preferable to the selected data. But let us, for the sake of argument, accept the smaller amount of selected data. What are the observational facts? In a total of 1218 flies, the gene yellow was observed to separate from the other two genes in 15 cases, and bifid was observed to separate from the other two genes in 43 cases, but white was not observed to separate from the other two genes. The 15 cases, in which yellow separates, make up 1.2% of the total, which is put down as the cross-over value between yellow and white, but in reality it is the cross-over value between yellow on one hand and the group white-bifid on the other hand. Likewise the 43 cases, which form 3.5% of the total, are put down as the cross-over value between white and bifid, but in reality they represent the cross-overs between bifid on one hand and the group yellow-white on the other. It is not certain that white with yellow attached would show the same percentage of cross-overs as without yellow attached. Indeed, in their doctrine of *interference*, the authors have long held that it would not. According to that doctrine, in case a break occurs between yellow and white, there is no chance whatever that a break will occur simultaneously in the near-by region between white and bifid, and *vice versa*, if a break occurs between bifid and white, there is no chance that a break will occur also between yellow and white. Accordingly in calculating the percentage of cross-overs for yellow-white, 15 should be deducted from the total number of cases, and in calculating the percentage of cross-overs for white-bifid, 43 should be deducted from the total, *if the principle of interference is valid.* Such correction would increase slightly the cross-over values for yellow-white and yellow-bifid, whose sum would now exceed the cross-over value given for yellow-bifid, 4.7, and would again put the three genes out of line.

It appears, then, that the authors have failed in two different attempts to establish their linear theory in the case of the three genes yellow, white and bifid. Morgan and Bridges in their equation, 'about 1' + 5 equals 'about 6,' seriously distort facts which should be very exactly stated if the linear hypothesis is to have a basis in fact as well as in fancy. The real relations, as given in their Table 65, are $1.1 + 5.3 > 5.5$. The new equation, which they now offer as a substitute for the old one, is $1.2 + 3.5 = 4.7$, which looks all right. The two halves of the equation balance, but the balance is a forced one. To obtain it two very questionable things are done. First, nearly 99% of the authoritative observations are suppressed;

secondly, the remaining observations are treated in utter disregard of the principle of 'interference' which has long been recognized by the authors and has never been repudiated by them.

But we are not without other and independent evidence of the relations between the three genes, yellow, white and bifid. Each of these has known linkage relations with other genes of the system. Pertinent facts from Table 65 of Morgan and Bridges are reproduced herewith in Table 2. They show that yellow is farther removed from vermilion, miniature and rudimentary than is either white or bifid, since the cross-over values are in every case higher. This supports Morgan's view that yellow is at the distal end of the system. If white and bifid lie in succession below yellow, white should be *farther* removed than bifid from genes lower in the system. Data are given

TABLE 2

	CROSS-OVER VALUES	CASES
Yellow-vermilion	34.5	13,271
Yellow-miniature	34.3	21,686
Yellow-rudimentary	42.9	2,563
Bifid-vermilion	31.1	2,724
Bifid-miniature	30.6	219
Bifid-rudimentary	42.7	899
Bifid-forked	42.5	306
White-vermilion	30.5	27,962
White-miniature	33.2	110,701
White-rudimentary	42.4	6,461
White-forked	45.7	3,664

for the distances to four such genes. In two cases (vermilion and rudimentary) white is nearer than bifid, in two other cases (miniature and forked) bifid is nearer than white. The more reliable cases, based on larger numbers, are those which place ***white below bifid***, minimal observations 2724 and 899 respectively. The numbers which place bifid below white are 306 and 219, too small to carry much weight. We may conclude that so far as evidence given by table 65, Morgan and Bridges, is concerned, bifid is certainly as remote as white and probably more remote than white from the lower parts of the linkage system. If so, bifid cannot lie in line with yellow and white, below them in the linkage chain, as maintained by Morgan and his associates. This conclusion agrees with the relations shown in my model, but not with those shown in their Diagram I.

With reference to the reconstruction for *D. virilis*, which is very clearly three-dimensional, the authors have nothing whatever to say. It would not be known that they had seen the paper[4] in which that reconstruction is described except for a footnote at the very end of their paper, which reads "Castle concluded that the white-forked value of 45.7 used by him, is somewhat too high, which is true; but there were available to him more than 40,000 flies (Bridges, *Genetics*, 1, 1916 and Weinstein, *Ibid*, 3, 1918) giving the lower value, in contrast to the less than 4000 flies on which the high value was based." There seems to be here a veiled rebuke that I did not look outside Table 65 of Morgan and Bridges for additional data. I saw no reason for doing so since Morgan and Bridges had based their linear diagram exclusively on "the data summarized in Table 65," and it seemed to me fair to test an alternative hypothesis by the same data. Moreover I was not at that time aware that additional cases had been recorded bearing on the white-forked linkage relation. In comparing the linkage system of *D. ampelophila* with that of *D. virilis*, I mentioned quite incidentally the longer distance recorded for the former species but that my model indicated this value to be probably too high, "since the wire joining white with forked in the model is too long to harmonize fully with other linkage values given by Morgan and Bridges, the wire being curved." If my model, merely by a curved wire, was able to detect an error which would otherwise have been overlooked by me, and which it required 40,000 additional flies to prove, the reconstruction method certainly has utility. Now I have made certain predictions as to what some undetermined linkage relationships in *D. virilis* will be found to be, based on the reconstruction figured in my former paper. An experimental test of their correctness is invited. Let us taste this pudding to prove or disprove its edibility.

With reference to point (3), Morgan and his associates consider cross-over values greater than 50 possible, though not as yet observed. They are led to this conclusion by the consideration that values greater than 50 are obtained by summation of short distances, on the hypothesis of linear arrangement. Forced to a choice between non-linear arrangement and map distances greater than 50, they choose the latter, and then contend that distances greater than 50 are possible and would actually be observed except for double crossing-over. For my part, I can not conceive of a mechanism which would tie two genes *together* in such a way that they will subsequently *separate* from each other oftener than they will remain together, yet this is what the idea of cross-overs in excess of 50 per cent amounts to.

With reference to point (4), it is admitted by Morgan and his associates that "double crossing-over has no meaning, if three genes are imagined as not lying in a straight line." It is accordingly a secondary hypothesis needed to help out the hypothesis of linear arrangement, but it can not be cited as proof of that hypothesis, which must stand or fall on its own merits. I can see no reason *a priori* why two or more breaks should not occur simultaneously in different parts of a linkage system, whether linear or non-linear, but this is no evidence that, every time a particular gene separates from two others, it has done so by two independent breaks, the view necessitated by the linear hypothesis. The relation of certain observational facts to the idea of double crossing-over is correctly stated by Plough[5] in relation to the alternative hypotheses. On a non-linear hypothesis, temperature affects "long chromosomal distances" (high cross-over values) less than small ones; on the linear hypothesis, temperature acts by changing the frequency of double crossing-over. This is no proof of either hypothesis, but a statement of fact in terms of each.

[1] Sturtevant, A. H., Bridges, C. B., and Morgan, T. H., these PROCEEDINGS, **5**, 1919, (168).
[2] Castle, W. E., these PROCEEDINGS, **5**, 1919, (25).
[3] Morgan, T. H., and Bridges, C. B., "Sex-linked inheritance in Drosophila," *Carnegie Inst., Washington Publ.*, No. 237, May 8, 1916.
[4] Castle, W. E., these PROCEEDINGS, **5**, 1919, (32).
[5] Plough, H. H., these PROCEEDINGS, **5**, 1919, (167).

ON VARIATION IN TARTARY BUCKWHEAT, FAGOPYRUM TATARICUM (L.) GAERTN.

By JACOB ZINN

MAINE AGRICULTURAL EXPERIMENT STATION, ORONO

Communicated by R. Pearl, September 5, 1919

Introductory.—From the morpho-genetic point of view the manifestation of dimorphism in certain races of plants—the so called ever-sporting varieties—presents a very interesting problem. The remarkable feature of these races is the constancy with which the two diverging forms of the same organ are transmitted in ever-sporting fashion: no breeding method has, as yet, been conceived by which, for instance, certain variegated types of plants or certain strains of Matthiola, could be induced to breed true. These races appear as compound forms ever-transmitting the potentialities of the two component types.

Dimorphism manifests itself in two externally different forms. The different characteristics may appear simultaneously distributed in the organs of the same individual as in *Trifolium pratense quinquefolium* De Vries or in *Veronica agrestis;* in another group of plants each individual of the race may display only one of the dimorphic characters as in the case of certain strains of Matthiola, Antirrhinum, *Dipsacus sylvestris torsus*, etc.

In interpreting these phenomena, De Vries whose investigations involved a great abundance of material, assumes the peculiar behavior of these races to be due to the interaction of two "antagonistic, mutually exclusive characters." The operation of these two contending characters within the individual leads to the formation of two distinct groups of plants, the half-races and the middle races or ever-sporting varieties. Opposed to this interpretation is the view held by certain writers who consider the ever-sporting nature of many of these races as mere somatic variations and relegate them into the group of non-heritable modifications.

More recently, however, some of the ever-sporting types in plants as well as in animals have been subjected to a genetic analysis and their peculiar mode of inheritance has been explained on Mendelian grounds.

The purpose of the present communication is to record the results of a study on a highly variable, ever-sporting race which I have discovered in *Fagopyrum tataricum* Gaertn. (*Polygonum tataricum* Linn.). In the course of observations on this race my attention was chiefly devoted to the study of variation and transmission of the external characters in an endeavor first to establish by direct experiment the behavior of this race under different conditions before attempting an analysis of the underlying genetic causes.

The full account of this investigation will be published in *Genetics*.

Material and experimental methods.—The race with which the present account is concerned originated from commercial fruits of Fagopyrum tataricum, Tartary Buckwheat, which had grown in Maine. In a population of several hundreds of plants, one plant was found to be distinguished by a particularly high degree of variability in the structure of its flowers. This plant was selected as a starting point of a strict pedigree culture, and since its isolation in 1916 five generations have been grown. The study of floral variations of this race involved the examination of more than 57,000 flowers and fruits.

The manifestation of variations of this race was studied under different conditions of environment. The cultures grew in pots under greenhouse

conditions and in the garden. Two greenhouses were used whose conditions differed greatly, notably with respect to humidity and temperature. In one of the greenhouses prevailed what might be called a moist and hot condition, the temperature varying only slightly, from 75°F. during the day to 70°F. at night. In the other greenhouse where the cultures grew in the summer time no artificial heat was used, the temperature following the natural daily amplitude. The air in this greenhouse was quite dry.

In connection with the study of the effect of nutrition and starvation upon the teratological development of this race, the cultures were grown in different nutritional media comprising rich composted or fertilized soil, ordinary soil, sand, and gravel.

Observations and results.—The variations here considered occur in the gynoecium, the perigone, and the vegetative organs of this race. Most of these variations have hitherto not been recorded for Fagopyrum tataricum.

The variations in the gynoecium are characterized by the production of supernumerary carpels. The number of carpels per pistil was found to vary from 3 up as high as 25. Under ordinary conditions of growth the number of flowers with normal gynoecia predominates over or equals the number of flowers with abnormal gynoecia. Under conditions favoring the development of abnormal flowers the variation is bilateral, and can be represented by a curve the apex of which is formed by the abnormal four-carpelled flowers. The frequency distribution of flowers with respect to number of carpels is given in table 1.

From table 1 it will be noted that the frequency distribution of flowers with abnormal gynoecium decreases as the number of aberrant carpels per pistil increases.

Associated with the abnormal gynoecia are abnormal perigones with a varying number of segments ranging from the normal number of 5 as high as 18. The favorable conditions capable of transforming the unilateral variation of the gynoecia into a bilateral one, failed to affect the perigones in the same manner. The variation in the number of perigone leaves remained unilateral with the frequency of the normal, five-parted perigone forming the apex of the skew curve (table 2).

The frequency of the normal, five-parted perigones decreases as the number of carpels per pistil increases. The relationship between the number of carpels and perigone leaves is illustrated in table 3.

All descendants of the ever-sporting race were found to reproduce the ever-sporting type of the mother plant regardless of whether they originated from normal or abnormal fruits of the parent.

TABLE 1

ACTUAL AND PERCENTAGE FREQUENCIES OF NUMBER OF FLOWERS WITH REGARD TO NUMBER OF CARPELS

PLANT NUMBER	3		4		5		6		7		8		9		10		11		12		13		14		15		16		17		18		19		SYNANTHIES		TOTAL
	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	
Pot 36 1	78	17.69	320	72.56	27	6.12	5	1.13	2	0.45	1	0.23	1	0.23	2	0.45	1	0.23					1	0.23											3	0.68	441
Pot 36 2	49	14.41	249	73.24	17	5.00	10	2.94	1	0.29	3	0.82			1	0.29							1	0.29											9	2.65	340
Pot 36 3	119	19.97	389	65.27	46	7.72	10	1.69	3	0.50	10	1.68	3	0.50	5	0.84			1	0.17															10	1.68	596
Pot 37 4	130	16.62	520	67.01	66	8.51	14	1.80	5	0.64	10	1.29	10	1.29	5	0.64			2	0.26							1	0.13							13	1.68	776
Pot 37 5	136	20.80	430	65.60	51	7.80	8	1.22	6	0.92	7	1.07	1	0.15	3	0.46																			12	1.83	654
Pot 37 6	150	21.08	488	68.54	49	6.88	7	0.98	3	0.42	2	0.28	2	0.28	4	0.56																			7	0.98	712
Pot 39 7	127	15.05	572	67.77	67	7.94	24	2.83	9	1.07	7	0.83	4	0.47	8	0.95			8	0.95	2	0.24											1	0.12	15	1.78	844
Pot 39 8	190	17.48	719	66.51	101	9.34	15	1.39	6	0.56	7	0.65	6	0.56	13	1.20	3	0.28	5	0.46	1	0.09	1	0.09							1	0.09			13	1.20	1081
Total..	979		3687		424		93		35		47		27		41		4		16		3		3				1				1		1		82		5444
Percentage..	17.98		67.73		7.79		1.71		0.64		0.86		0.49		0.75		0.07		0.29		0.06		0.06				0.02				0.02		0.02		1.50		

TABLE 2

ACTUAL AND PERCENTAGE FREQUENCIES OF NUMBER OF FLOWERS WITH RESPECT TO NUMBER OF PERIGONE LEAVES

PLANT NUMBER		NUMBER OF PERIGONE LEAVES																TOTAL
		5		6		7		8		9		10		11		12		
		Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	
Pot 36	1	308	70.32	105	23.97	18	4.11	4	0.91	2	0.46	1	0.23					438
	2	214	64.65	89	26.89	21	6.34	5	1.51	1	0.30	1	0.30					331
	3	410	69.97	141	24.06	23	3.92	7	1.19	3	0.51	2	0.34					586
Pot 37	4	541	70.90	162	21.23	37	4.85	14	1.83	1	0.13	7	0.92			1	0.13	763
	5	447	69.63	150	23.86	29	4.52	11	1.71			5	0.78					642
	6	538	76.31	131	18.58	28	3.98	5	0.71	2	0.28	1	0.14					705
Pot 39	7	604	72.86	169	20.39	31	3.74	13	1.57	7	0.84	5	0.60					829
	8	784	73.41	212	19.85	44	4.12	10	0.94	4	0.37	10	0.94	1	0.09	3	0.28	1068
Total		3846		1159		231		69		20		32		1		4		5362
Percentage		71.72		21.62		4.31		1.29		0.37		0.59		0.02		0.08		

TABLE 3

PERCENTAGE FREQUENCIES SHOWING RELATION BETWEEN NUMBER OF CARPELS AND NUMBER OF PERIGONE LEAVES

NUMBER OF PERIGONE LEAVES	NUMBER OF CARPELS																
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
5	89.68	71.30	58.96	35.48	54.29	42.35	48.15	36.59	25.00	56.25	33.33						
6	8.78	25.33	24.76	26.88	2.85	14.89	3.70										
7	1.33	3.09	14.15	23.66	11.43					6.25							
8	0.20	0.27	1.88	10.75	14.29	31.91	3.70	24.39	25.00		33.33	33.33					
9			0.24	3.23	17.14	2.13	18.52	7.32	25.00								100
10						8.51	25.93	26.83	25.00	31.25	33.33	33.33					
11								2.44									
12								2.44		6.25		33.33				100	
Total...	99.99	99.99	99.99	100.00	100.00	99.99	100.00	100.00	100.00	100.00	99.99	99.99				100	100

TABLE 2

ACTUAL AND PERCENTAGE FREQUENCIES OF NUMBER OF FLOWERS WITH RESPECT TO NUMBER OF PERIGONE LEAVES

PLANT NUMBER		NUMBER OF PERIGONE LEAVES																TOTAL
		5		6		7		8		9		10		11		12		
		Actual	Per-centage	Actual	Per-centage	Actual	Per-centage	Actual	Per-centage	Actual	Per-centage	Actual	Per-centage	Actual	Per-centage	Actual	Per-centage	
Pot 36	1	308	70.32	105	23.97	18	4.11	4	0.91	2	0.46	1	0.23					438
	2	214	64.65	89	26.89	21	6.34	5	1.51	1	0.30	1	0.30					331
	3	410	69.97	141	24.06	23	3.92	7	1.19	3	0.51	2	0.34					586
Pot 37	4	541	70.90	162	21.23	37	4.85	14	1.83	1	0.13	7	0.92			1	0.13	763
	5	447	69.63	150	23.86	29	4.52	11	1.71			5	0.78					642
	6	538	76.31	131	18.58	28	3.98	5	0.71	2	0.28	1	0.14					705
Pot 39	7	604	72.86	169	20.39	31	3.74	13	1.57	7	0.84	5	0.60					829
	8	784	73.41	212	19.85	44	4.12	10	0.94	4	0.37	10	0.94	1	0.09	3	0.28	1068
Total		3846		1159		231		69		20		32		1		4		5362
Percentage		71.72		21.62		4.31		1.29		0.37		0.59		0.02		0.08		

TABLE 3

PERCENTAGE FREQUENCIES SHOWING RELATION BETWEEN NUMBER OF CARPELS AND NUMBER OF PERIGONE LEAVES

NUMBER OF PERIGONE LEAVES	NUMBER OF CARPELS																
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
5	89.68	71.30	58.96	35.48	54.29	42.35	48.15	36.59	25.00	56.25	33.33						
6	8.78	25.33	24.76	26.88	2.85	14.89	3.70										
7	1.33	3.09	14.15	23.66	11.43					6.25							
8	0.20	0.27	1.88	10.75	14.29	31.91	3.70	24.39	25.00		33.33	33.33					
9			0.24	3.23	17.14	2.13	18.52	7.32	25.00								100
10						8.51	25.93	26.83	25.00	31.25	33.33	33.33					
11								2.44									
12								2.44		6.25		33.33				100	
Total...	99.99	99.99	99.99	100.00	100.00	99.99	100.00	100.00	100.00	100.00	99.99	99.99				100	100

The ratio between the normal and abnormal flowers was found to be a function of the environment. Under a given set of environmental conditions this ratio as well as the relationship between the different forms of abnormal flowers *inter se* is constant to a very marked degree.

Selection carried out for five years had no visible effect upon the type and range of floral variations of this race. The ever-sporting strain after isolation at once displayed the highest degree of abnormality ever reached in the subsequent generations under similar conditions of environment.

Under conditions controlling the intensity of abnormal development, optimum nutrition or starvation, while affecting the habit of the plant, appeared to have no effect upon the degree of manifestation of floral abnormalities. The evidence from the study of this race under different conditions of environment points to high humidity and temperature as the factors favoring the expression of abnormality. Under conditions void of optimum humidity and temperature, the influence of starvation and lack of water upon the degree of abnormal development was noticeable.

The results of a study of the frequency distribution of the different types of flowers upon the plant point to the existence of a definite region on the plant in which the tendency to vary and proliferate is most pronounced. Considering the plant as a whole, this region is confined to the basal, differentiated parts of the plant. The frequency distribution given in table 4 shows that the first three branches on the main stem from below, especially the second one, mark the seat of greatest abnormal development while the racemes in the axils of the 4th, 5th, and 6th branch show a low degree of variability as well as the lowest absolute number of flowers.

Similar but more marked differences prevail in the individual branches of the second and third order. Here it is again the buds in the axils of the second leaf and in the basal region of the terminal raceme that show the greatest relative number of abnormal flowers as well as the greatest range of variability as measured by the frequency occurrence of the most aberrant variants.

Relative to the frequency occurrence of the different types of flowers at different periods of the flowering season, under the conditions prevailing in the greenhouse the first and second week of the flowering season mark the lowest relative production of abnormal flowers, after which a marked increase in the output of abnormalities follows when the secondary and tertiary branches begin to develop their flowers. Towards the end of the flowering season the upper regions of the plants produced only

TABLE 4

ACTUAL AND PERCENTAGE FREQUENCY DISTRIBUTIONS OF FLOWERS BORNE BY EACH ORGAN OF EACH PLANT, WITH RESPECT TO NUMBER OF CARPELS

POSITION OF FLOWERS	NUMBER OF CARPELS																																			Total
	3		4		5		6		7		8		9		10		11		12		13		14		15		16		17		18		19		Synanthies		
	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	Actual	Percentage	
In axils of Cotyledons........	97	16.33	389	65.71	60	10.14	15	2.53	3	0.51	5	0.84	3	0.58	4	0.68	1	0.17	5	0.84							1	0.17							9	1.52	592
On Branch 1....	234	18.75	829	66.43	105	8.41	21	1.68	8	0.64	19	1.52	6	0.48	12	0.96															1	0.08			13	1.04	1248
On Branch 2....	245	18.39	880	66.07	98	7.36	25	1.88	14	1.05	8	0.60	4	0.30	15	1.13	2	0.15	5	0.38	2	0.15	2	0.15									1	0.15	31	2.33	1332
On Branch 3....	242	17.60	927	67.42	111	8.00	25	1.82	8	0.58	13	0.95	13	0.95	7	0.51	1	0.07	6	0.44	1	0.07													21	1.53	1375
On raceme in axil of leaf 4...	37	16.30	167	73.57	15	6.61	2	0.88	1	0.44					2	0.88							1	0.44											2	0.88	227
On raceme in axil of leaf 5...	20	15.75	93	73.23	11	8.66	1	0.78																											2	1.57	127
On raceme in axil of leaf 6...	38	19.10	144	72.36	11	5.93	2	1.01	1	0.50					1	0.50																			2	1.01	199
Terminal raceme	66	19.19	258	75.00	13	3.72	2	0.58			2	0.58	1	0.29																					2	0.58	344
Total........	979		3687		424		93		35		47		27		41		4		16		3		3				1				1		1		82		5444

very few flowers while the lower differentiated parts of the plants sustained their flower production to the end of the flowering season.

Floral prolifications in the form of various types of synanthous flowers, often giving rise to syncarpous fruits, were found to be transmitted from generation to generation in fairly constant proportions under given conditions of environment.

The teratological development of the vegetative organs appeared in the form of more or less developed fasciations. Fasciated branches were first discovered on the plants of the fourth generation grown under crowded conditions, in pots. In the next generation, under favorable conditions of nutrition, the fasciated character asserted itself in a manner typical of the ever-sporting races the fasciations being reproduced by half of the progeny.

THE EFFECT OF MILLING ON THE DIGESTIBILITY OF GRAHAM FLOUR

By C. F. Langworthy and H. J. Deuel

Office of Home Economics, U. S. Department of Agriculture

Communicated by W. A. Noyes, October 14, 1919

The bulk of wheat used for flour in this country is made into patent flour which contains about 72% of the wheat kernel. Entire or whole-wheat flour which contains 85% of the wheat and true Graham flour which contains 100% are also we l-known commodities.

The digestibility of patent flour is considerably higher than that of entire-wheat or Graham flours. An average[1] of 31 tests by other investigators with patent flour shows that the coefficient of digestibility for the protein is 88.1% and for carbohydrate 95.7%, while an average of 43 as yet unpublished tests made in this laboratory[2] on patent flour gave the coefficient 89.5% for the digestibility of protein and 99.9% for that of carbohydrate. An average[1] of 23 tests of the digestibility of entire-wheat flour (85% extraction) gave the coefficient 81.9% for the protein and 94.0% for the carbohydrate while an average[2] of 16 tests on similar flour by this office[2] gave the coefficient 87.1% for the protein and 98.3% for the carbohydrate. The average[1] of 24 tests on true Graham flour was 76.9% for protein and 90.1% for carbohydrate and an average of 33 experiments on the same flour by this office[2] gave the value 84.2% for protein and 94.4% for carbohydrate.

It has been a question as to how the milling of Graham flour effects its digestibility. Wheat milled by different processes gives bran particles varying in size from the very small ones obtained with a burr-stone mill to very large ones with a roller mill. The method of milling also effects the extent to which the walls of the aleurone cells are broken or weakened. These, if intact, prevent the digestion of their contents, and so the more they are broken the more completely are the nutrients of the flour digested. Lapicque and Liacre[3] found that kneading the bread broke the aleurone cell walls at points weakened by the milling process. Obviously, the method of milling would affect the extent to which the walls of the aleurone cells would be weakened. The experiments here reported were undertaken to determine how different methods of milling effected the digestibility of Graham flour.

The flours were all made from a single lot of Minnesota spring wheat secured through the courtesy of the Plant Chemical Laboratory of the Bureau of Chemistry. Portions of the wheat were ground by the following methods: (1) Small laboratory roller mill, (2) commercial roller mill, (3) burr-stone mill, (4) steel-burr mill, (5) steel attrition mill.

The portions of flour milled on the laboratory roller-mill, the burr-stone mill, and the steel-burr mill were prepared on the m lls of the Plant Chemical Laboratory, Bureau of Chemistry. The commercial roller-mill flour and the attrition-mill flour were prepared by two commercial concerns.

As was the case in many other tests in this laboratory, the flour was fed in the form of a simple 'quick bread,' which was baked each day. A little ginger not only added to the palatability but masked any differences between the breads in the different tests. The following recipe was used:

Experimental bread

15 cups flour	3¾ teaspoons salt
3¾ teaspoons soda	5 teaspoons ginger
1⅞ cups molasses	1 scant cup lard

1⅞ quarts hot water

The lard was added to the hot water, this mixture was added to the other ingredients. This was thoroughly mixed and baked for 1½ hours.

With a generous portion of the bread a simple basal ration of fruit (oranges), butter, and sugar, with coffee or tea without cream, if desired, was eaten. The tests were of three days or nine meals duration. The separation of the feces, analyses, etc., were those usually followed.

The subjects were young men, twenty to thirty years old, students in a local university, in good physical condition, familiar with this type of work, and entirely trustworthy.

Granulation tests made with the different flours showed the following percentages remaining on each sieve.

Results of granulation tests with different Graham flours

KIND OF MILL USED	PER CENT ON 20 SIEVE	PER CENT ON 40 SIEVE	PER CENT ON 70 SIEVE	PER CENT ON 90 SIEVE	PER CENT ON 100 SIEVE	PER CENT THROUGH 100 SIEVE
Laboratory roller mill	18.0	36.6	21.8	9.0	4.6	8.2
Commercial roller mill	10.6	20.6	23.8	14.2	7.0	22.8
Steel-burr mill	4.2	23.8	18.0	13.2	8.4	32.4
Attrition mill	0.8	23.0	17.6	13.4	8.8	36.6
Stone-burr mill	3.2	13.4	12.2	19.4	11.6	39.2

It will be noted that the flour from the laboratory roller mill was the coarsest, while the stone-burr mill gave the flour of the greatest fineness. In this table and those that follow the flours are given in the order of their size from the coarsest to the finest.

The condensed results of the experiments appear in the tables which follow:

Average amount of Graham bread and total food eaten per man per day

	WEIGHT	CONSTITUENTS OF FOODS				
		Water	Protein	Fat	Carbohydrate	Ash
	grams	*grams*	*grams*	*grams*	*grams*	*grams*
Laboratory roller-mill flour						
Bread	475.9	178.2	32.4	31.1	224.5	9.7
Total food	978.4	450.2	35.5	88.8	390.7	13.2
Commercial roller-mill flour						
Bread	487.9	150.8	37.2	28.3	260.8	10.9
Total food	1,024.0	465.9	40.7	89.6	412.8	14.8
Steel-burr mill flour						
Bread	527.0	164.3	44.5	31.4	274.4	12.3
Total food	1,037.0	440.2	47.8	104.9	427.6	16.5
Attrition mill flour						
Bread	457.3	143.9	35.7	29.0	238.3	10.4
Total food	907.2	399.4	38.7	84.6	370.7	13.9
Stone-burr mill flour						
Bread	534.8	179.7	39.5	27.5	275.8	12.2
Total food	1,046.6	459.2	42.8	99.2	429.1	16.4

Average digestibility of Graham flours milled by different methods

KIND OF FLOUR	NUMBER OF EXPERIMENTS	DIGESTIBILITY OF ENTIRE RATION				ESTIMATED DIGESTIBILITY OF PROTEIN ALONE	ESTIMATED DIGESTIBILITY OF CARBOHYDRATE ALONE
		Protein	Fat	Carbohydrate	Ash		
		per cent	*per cent*	*per cent*	*per cent*	*per cent*	*per cent*
Laboratory roller mill flour	5	72.1	94.4	96.0	70.1	70.7	95.3
Commercial roller mill flour	5	72.1	95.6	95.4	67.6	70.4	93.8
Steel-burr mill flour	4	79.2	95.0	96.1	73.8	78.5	95.3
Attrition mill flour	3	75.5	92.9	96.1	69.7	74.5	95.4
Stone-burr mill flour	4	79.2	96.5	96.6	78.4	78.2	96.8

The subjects in reports of their condition mentioned no discomfort. They did, however, speak of a somewhat laxative effect but noted no difference in this respect between the flours in which the bran was finely ground and those in which it was coarse.

The amount of protein digested varied from about 70% in the roller mill flours in which the particles of bran were the largest to 78% in the stone-burr mill and steel-burr mill flours in which the bran particles were much finer. This is what we should expect for it has been shown that the bran protein[1] is about 44% digested in the case of fine bran while it is only 28% digested in coarse bran.

The coefficient of digestibility for carbohydrate varied from 93.4% in the coarse roller mill flour to 96.8% in the stone-burr mill flour. The estimated digestibility of the protein and carbohydrate represent the digestibility of the protein and carbohydrate of the flour alone after allowance has been made for the undigested residues of the accessory food. Possibly because of the kind of wheat used, the figures for protein are somewhat lower than corresponding figures in other digestion experiments on 100% flour. Nevertheless, since those here studied are all from the same lot of wheat they are directly comparable with one another. The finer the particles, the more completely the protein was absorbed while the absorption of carbohydrate varied only slightly.

In conclusion, it is fairly safe to say that the finer a bran-containing flour is ground, the more completely it is utilized by the human body.

[1] *Washington, U. S. Dept. Agric. Bull.*, No. 751, 1919, (pp. 20).
[2] Unpublished experiments.
[3] Lapicque, L., and Liacre, A., *C. R. Soc. Biol., Paris*, 81, 1918, No. 5, (pp. 217–220).

THE OZONE FORM OF HYDROGEN

By Gerald L. Wendt

Department of Chemistry, University of Chicago

Communicated by W. A. Noyes, September 22, 1919

In spite of its having only a single valence bond, hydrogen seems to form an active variety bearing the same relation to ordinary hydrogen that ozone bears to oxygen. The chemical activity of this form of hydrogen is not attributable to single free atoms, since the characteristics of the atomic form are well known, and different from those of this ozone form.

Professor Sir J. J. Thomson,[1] in working with positive rays in 1912, showed the existence in a hydrogen discharge tube of particles with a molecular weight of 3. He gave the tentative symbol of X_3 to this material, and investigated its chemical properties in some detail. He found that it combines with oxygen under the action of light. It is also destroyed by sparking with oxygen, as well as by being heated in a quartz tube with copper oxide. In the absence of oxygen the gas could be heated to a high temperature without destruction. Attempts to obtain spectroscopic evidence of the new form of hydrogen failed, only the usual spectrum being evident.

An investigation begun in 1914 with the co-operation of Professor William Duane[2] showed that hydrogen becomes chemically active under the influence of alpha rays from radium emanation, and that when this active variety is formed there is a distinct contraction in volume. The active hydrogen attacks sulphur, forming hydrogen sulphide. This reaction is the simplest test for the presence of the active modification: The hydrogen is activated by the rays; it is allowed to pass over sulfur, and then passes over a strip of filter paper moistened with lead acetate which it blackens, due to the hydrogen sulphide present. Other reactions are equally simple and characteristic. It reduces neutral permanganate solution to give a precipitate of manganese dioxide; acid permanganate is entirely decolorized, giving a manganous salt. Arsenic is reduced to arsine; phosphorus gives phosphine. There is some action even on mercury, giving a lustrous yellow compound, presumably a hydride, which decomposes on heating. This activity is not due to the presence of charged molecules or ions, as these can be completely removed by an electrostatic field of a thousand volts per centimeter without destroying the chemical activity of the gas.

The active modification is unstable, and must be allowed to react with the oxidizing substance within three or four minutes of the time of its formation. Here again its properties are those to be expected from an ozone form. Passage of the gas through a tube immersed in liquid air destroys its activity either by condensing the active variety which thus has a boiling point very much higher than that of ordinary hydrogen, or by causing its disintegration into ordinary H_2.

A molecule larger than H_2 is indicated by various facts. The purest obtainable hydrogen contracts when under the influence of alpha rays. This has recently been confirmed by Lind,[3] working with large quantities of radium emanation. Usher,[4] working in Ramsay's Laboratory in 1909, observed the same effect, but attributed the loss of volume to the projection of hydrogen molecules into the glass walls of the retaining vessel when bombarded by alpha rays, although the volume of hydrogen lost was larger than the number of alpha rays would permit, and although he was also unable to regain an adequate volume of hydrogen when the glass walls were pulverized and heated.

The second line of evidence results from Langmuir's[5] preparation of an active form of hydrogen which is probably monoatomic. This hydrogen is present in an electric light bulb when the filament is heated to a high temperature, and is undoubtedly a consequence of the dissociation of hydrogen molecules at the high temperature of the filament. When the pressure in the bulb is very low this atomic form of hydrogen forms a layer on the inner surface of the glass wall, and although thus attached, is chemically active. It reacts rapidly with phosphorus vapor, for instance. It has two marked characteristics which distinguish it from the ozone form; namely, that it will not pass through a plug of glass wool, being adsorbed strongly by this substance; and secondly, that it does not exist except at the very highest vacua. When more hydrogen is admitted into the bulb than is necessary to make a layer one atom deep on the glass wall, the amount of active hydrogen is much reduced, the excess molecules seeming to react with the active atoms. The ozone form of hydrogen, on the other hand, will pass through long layers of glass wool and exists for some time at atmospheric pressure.

The third and direct evidence that this hydrogen is polyatomic is found in the work of Sir J. J. Thomson and of Dempster,[6] both of whom made use of the method of positive ray analysis to determine the constituents of a gaseous mixture in the presence of an electric discharge at high vacuum. The photographic method of Thomson and the electrical measurement method of Dempster both show the presence of

considerable quantities of H_3 in an H_2 tube. At pressures of 0.05 mm., indeed, Dempster has shown that H_3 predominates.

Very recent work has shown that this active hydrogen is present in large quantities in any ordinary hydrogen discharge tube. A Geissler tube under a pressure as high as 6 cm. contains the ozone hydrogen. If hydrogen is passed continuously through such a tube and is then allowed to react with sulfur a lead acetate paper is decidedly blackened in the course of two or three minutes. Potassium permanganate is decolorized with equal speed. This is an effective lecture experiment. No ions are present in this hydrogen, as can readily be shown by passing the gas through a sensitive emanation electroscope. When this instrument is sensitive enough to detect 10-12 Curie of radium emanation with ease, the electroscope leaf remains perfectly steady though the active hydrogen is passed through for hours. The activity is therefore due to molecules.

The mechanism by which these larger and unstable molecules are formed is still obscure. In the case of their formation under the action of alpha rays they may be the product of the bombardment of ordinary hydrogen molecules by means of the swift and very energetic alpha particles. A hydrogen atom thus projected by bombardment into an ordinary hydrogen molecule may remain there in the metastable condition of a triatomic molecule. The formation of this variety of hydrogen, however, in the cathode ray discharge tube, where the only bombardment is by a stream of electrons, indicates that probably nothing more than ionization is necessary. The most plausible hypothesis is that the ions collect about them a cluster of molecules, forming a so-called large ion, and that the valence bonds in such a group are so disintegrated that the cluster breaks down with the formation of various types of fragments, among them H_3 as well as H_2. Cluster ions are undoubtedly present in such a tube, though their quantity is hard to estimate. Dempster has shown, however, that at low pressures both H_3 and H_2 are present in large quantities, with H_3 predominating in certain ranges of pressure. Still more information on the mechanism of the formation of this type of molecule would be obtainable from a study of the action of very short wave-length light on hydrogen. The Schumann region of the spectrum is absorbed by hydrogen, and the ionization or molecule shattering that probably then occurs may also give rise to this ozone form. This phase of the question is now being investigated.

The existence of such a molecule as H_3 does not accord with the present theories of valence. The univalence of hydrogen is the basis of the

present theory, and no diagram of H_3 can be made without in one manner or another splitting the single unitary valence bonds into fractions, as for instance is possible with the conception of J. Stark. H_3 has been shown to be consistent with Bohr's model of the hydrogen atom. But the chemist will require a more definite picture to show the structure of this compound and to reconcile its very ordinary existence and properties with his theoretical conceptions. Hydrogen seems no different from oxygen in this regard. Indeed, it is probably only the injunction due to the valence theory which has prevented the observation of this variety of hydrogen at a much earlier time than the present.

[1] *Rays of Positive Electricity*, Longmans, 1913.
[2] Duane and Wendt, *Ithaca, Physic. Rev.* **10**, 1917, (116–128).
[3] S. C. Lind, *Easton, Pa., J. Amer. Chem. Soc.*, **41**, 1919, (545).
[4] F. L. Usher, *London, J. Chem. Soc.*, **97**, 1909, (400).
[5] Irving Langmuir, *J. Amer. Chem. Soc.*, **34**, 1912, (860, 1310); **36**, 1914, (1708); **37**, 1915, (417).
[6] A. J. Dempster, *London, Phil. Mag.*, **31**, 1916, (438–43).

A VAPOR-FREE VACUUM SEAL

By A. H. Pfund

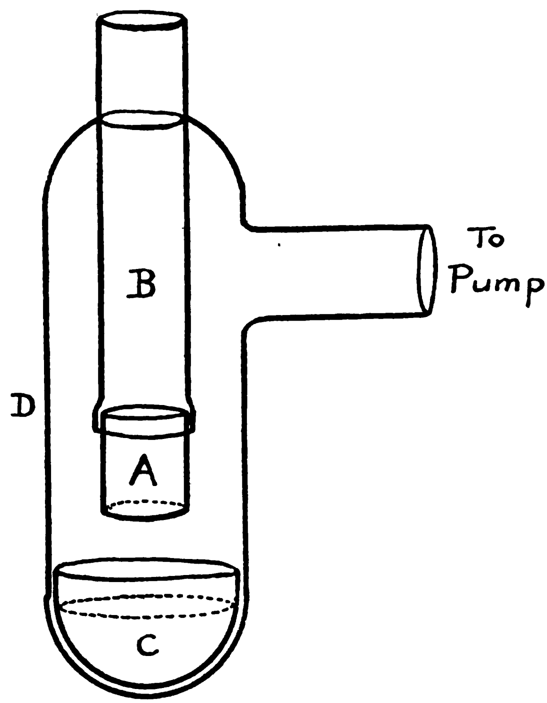
Department of Physics, Johns Hopkins University

Communicated by J. S. Ames

The behavior of stop-cocks in vacuum manipulations is, as a rule, quite satisfactory so long as the degree of exhaustion is not very high. However, when the utmost vacuum must be reached and maintained, the ordinary stop-cock is useless. Quite apart from its tendency either to leak or to fill up with grease, its main defects are: 1. Evaporation of volatile constituents of the grease into the evacuated chamber. 2. Comparatively small diameter of opening. Another element has been brought to light by Shrader who has recently shown (*Physical Review, Ithaca*, June, 1919, p. 434) that the gases liberated from the seal, when a glass bulb is sealed off the pump, increase the pressure several hundred per cent over that which had existed previously. An attempt has been made to overcome these difficulties by designing a vacuum seal of the type shown in sketch.

The apparatus takes the form of the well-known trap, slightly modified. A hollow cylinder of platinum (A), either seamless or made

of foil soldered with gold, is fused to the lower end of the central glass tube (B). The outer surface of the platinum is first tinned and is then coated with a thin layer of low melting point solder, such as Wood's metal or equal parts of ordinary solder and bismuth. An iron cup (C), which fits into the tube D rather snugly, is filled to a depth of about 5 mm. with this solder. This system must receive the usual baking treatment to drive off absorbed gases. In order to effect a seal, the lower half of the bulb (D) is immersed in hot water or is gently heated



with a bunsen burner until the solder melts. Then the cup is raised by means of an electro-magnet until the platinum tube is imbedded in the solder. The seal is tight as soon as solidification has taken place. In order to re-open the seal, the solder is melted and the cup lowered. It is obvious that this device may be used to seal off a piece of apparatus either temporarily or permanently.

Experience has shown that a seal of the above type possesses the following advantages:

1. It is tight and vapor free. 2. The openings may be made large. 3. Apparatus which has become gassy may be re-exhausted without admitting air—as is unavoidable in present practice.

NATIONAL RESEARCH COUNCIL

MINUTES OF JOINT MEETING OF THE EXECUTIVE BOARD OF THE NATIONAL RESEARCH COUNCIL WITH THE COUNCIL OF THE NATIONAL ACADEMY OF SCIENCES

AT THE NATIONAL RESEARCH COUNCIL BUILDING, JUNE 24, 1919, AT 9.30 A.M.

Present: Messrs. Bancroft, Clevenger, Dunn, Hussey, Leuschner, Mathews, McClung, Merriam, Ransome,* Stratton, Walcott* Washburn, Woodward.

Mr. Walcott in the chair during the transaction of the business of the Council of the Academy.

Mr. Merriam in the chair during the transaction of the business of the National Research Council.

The minutes of the joint meeting of the Executive Board of the National Research Council with the Council of the National Academy of Sciences of May 27 and minutes of the meeting of the Executive Board of the National Reseach Council of June 10 were approved as circulated.

The President of the National Academy of Sciences presented the following resolution which was passed by the Carnegie Corporation of New York on June 3, 1919, making provision to cover expenses of the National Research Council during the coming year:

RESOLVED, That, pursuant to Paragraph 3 of the resolution recording action taken at the special meeting of the Board of Trustees held March 28, 1919, the sum of One Hundred Thousand Dollars ($100,000) be and it hereby is appropriated to the National Academy of Sciences for the use of the National Research Council for the year beginning July 1, 1919; and that the Treasurer be and he hereby is authorized to make payments as needed to the extent of $100,000 on certificates of the Chairman of the National Academy of Sciences and the Chairman of the National Research Council.

Moved: That the Executive Board of the National Research Council go on record as appreciating the recognition by the Carnegie Corporation of New York of the work which it is accomplishing by appropriating the sum of $100,000 for its use for the year beginning July 1, 1919. (*Adopted.*)

To further the plans for a new building for the National Academy of Sciences and the National Research Council, it was

Moved: That the President of the National Academy of Sciences be authorized to enlarge the functions and personnel of the present Committee on Building Plans so that the Committee shall be constituted hereafter as follows: Messrs. Geo. E. Hale (Chairman), J. R. Angell, Gano Dunn, J. C. Merriam, R. A. Millikan, A. A. Noyes, C. D. Walcott, with Mr. H. S. Pritchett in an advisory capacity. (*Adopted.*)

Messrs. Dunn and Leuschner verbally submitted amendments to the Articles of Organization of the National Research Council providing for the continua-

* Members of the Council of the National Academy of Sciences.

tion of retired Chairmen of the Council as members of the Executive Board and for the appointment of Chairmen of the Divisions of General Relations.

Moved: That Messrs. Dunn and Leuschner be requested to formulate suitable amendments to the Articles of Organization of the National Research Council providing for the continuation of retired Chairmen of the Council as members of the Executive Board, and for the appointment of Chairmen of the Divisions of General Relations. (*Adopted.*)

Mr. Leuschner, Acting Chairman of the Division of Physical Sciences, submitted the following resolution of the American Section of the International Astronomical Union:

RESOLVED, That it be the sense of the American Section of the International Astronomical Union that the Executive Board of the National Research Council be requested to guarantee a sum, not to exceed $1500, to cover the American Share of the Administrative expenses of the International Astronomical Union.

Moved: That the Executive Board of the National Research Council underwrite the share of the United States of the administrative expenses of the International Research Council and of its affiliated Unions to an amount not to exceed $5000 and that this item be provided for in the provisional budget for 1919–20. (*Adopted.*)

The Chairman of the National Research Council presented the following letter from the Rockefeller Foundation, appropriating the sum of $20,000 to meet the expenses involved in conferences of special sub-committees on research subjects of the Division of Physical Sciences.

THE ROCKEFELLER FOUNDATION

June 20, 1919.

My Dear Mr. Merriam:

I have the honor to inform you that at a meeting of the Executive Committee of the Rockefeller Foundation held June 16, 1919, the following resolution was adopted:

RESOLVED: That the sum of Twenty thousand dollars ($20,000) be, and it is hereby, appropriated to the National Research Council for the Division of Physical Sciences, of which so much as may be necessary shall be used to defray the necessary travelling and other expenses involved in conferences of the sub-committees of that division during the year 1919.

Very truly yours,

EDWIN R. EMBREE, *Secretary.*

Moved: That the Chairman of the National Research Council express in behalf of the Executive Board its appreciation of the interest which the Rockefeller Foundation has shown in the research work of the Division of Physical Sciences by appropriating the sum of $20,000 to meet the expenses involved in conferences of special sub-committees on research subjects of that Division. (*Adopted.*)

The Chairman of the National Research Council announced the following Committee on Publication of 'The Inquiry' Results; authorized June 10, 1919; Messrs. C. D. Walcott (Chairman), James R. Angell, E. B. Mathews.

The Chairman raised the question of continuing the status and titles of the scientific attachés at the American embassies in Europe from July 1 to September 30, 1919.

Moved: That the necessary steps be taken by the President of the National Academy of Sciences to continue the status and titles of the scientific attaches at the American embassies in Europe from July 1 to September 30, 1919. (*Adopted.*)

In accordance with the resolution adopted at the meeting of the Interim Committee of March 25, 1919, providing that a committee on Scientific Men as Reserve Officers in the Reorganized Army be appointed by the Chairman to formulate plans for advising the Army and Navy in the reorganization of their research activities at the close of the war, Mr. Walcott presented the following resolution in behalf of the Committee (Messrs. Walcott, Chairman, Mathews, Leuschner):

WHEREAS, the present war has shown the necessity for scientific experts trained in lines not essentially military, and that in times of peace many serviceable men occupy civil positions from which they cannot be drawn by the ordinary attractions of military service, and

WHEREAS, the development of a corps of scientific experts for military purposes in times of emergency involves great loss of time in finding and organizing suitably trained experts to apply their expert knowledge to military problems

The National Research Council respectfully suggests and recommends that in the reorganization of the Army and Navy suitable provision be made for a number of reserve officers who may become, during time of peace, familiar with the military applications of their special knowledge and in case of war serve as a nucleus for temporarily enlarged forces in their special lines. Such reserve officers should be primarily scientific experts in physics, chemistry, geography, biology, geology, psychology, mathematics, astronomy, engineering, medicine, and such other sciences as may be appropriated.

They should be relieved, if necessary, from the requiremet of previous military training, and from annual active military service, and be required to give the equivalent time to the investigation in their special fields of problems of military application.

In order that no hardship may be imposed upon regular officers through service under the direction of specialists with lower military rank such reserve officers should have a rating comparable to their work.

Moved: That the foregoing resolution presented by the Committee on Scientific Men as Reserve Officers in the Reorganized Army, be approved. (*Adopted.*)

On recommendation of the Chairman, it was

Moved: That a committee be appointed for the consideration of all requests for the financial support of the objects of the National Research Council and for the unification of plans for securing financial support, with power until the next meeting of the Executive Board. (*Adopted.*)

Appointed: Messrs. J. R. Angell (Chairman), John C. Carty, Gano Dunn, Geo. F. Hale, H. E. Howe, Vernon Kellogg, and J. C. Merriam.

Moved: That the President of the National Academy of Sciences request the Department of State to detail some official already in Europe to attend the meetings at Brussels and assist the delegates to the International Research Council with advice on diplomatic matters. (*Adopted.*)

On recommendation of the Committee on Delegates, Messrs. Walcott, Leuschner, Yerkes, to the international conferences to be held at Brussels July, 1919, it was

Moved: That the following be approved as possible delegates to the meetings of the International Research Council at Brussels July 1919; Major J. L. Coolidge, nominated by the American Mathematical Society, and Dr. C. L. Gibson, nominated by the Division of Medical Sciences. (*Adopted.*)

Mr. Leuschner, Acting Chairman of the Division of Physical Sciences, presented a request from the American Section of the International Geophysical Union that surveys of international geophysical organizations prepared by members of the Section for the use of its delegates at Brussels July, 1919, be printed in connection with similar surveys to be prepared by the various divisions.

Moved: That the Executive Board initiate a survey of the past history, present status, and future work of international organizations in science along the lines followed in the statements prepared for the benefit of the International Geophysical Union. (*Adopted.*)

Moved: That Railroad and Pullman fare of members attending the preliminary organization meeting of the Division of Educational Relations on June 25, and such other expenses pertaining thereto as the Chairman of the Council may direct, be paid from the General Maintenance Fund of the Executive Board. (*Adopted.*)

The work of the Committee on Reconstruction Problems was discussed, and it was

Moved: That the question of publication of reports of the Committee on Reconstruction Problems as a whole or of its members be referred to Mr. Vernon Kellogg, Chairman of the Committee, with power. (*Adopted.*)

The Chairman announced that Mr. A. L. Barrows, Secretary of the Division of States Relations and of Educational Relations, would arrive on June 27. After discussion of the future work of these Divisions it was

Moved: That Mr. J. C. Merriam be appointed non-resident Chairman of the Division of States Relations, without salary. (*Adopted.*)

With reference to the situation created by the resignation of Messrs. Hale and Johnston as chairman and secretary, respectively, of the Council, it was

Moved: That by unanimous consent the Executive Board of the National Research Council recognize that Mr. J. C. Merriam, elected Acting Chairman on February 11, 1919, became Chairman on April 30, 1919, on the acceptance of the resignation of Mr. Geo. E. Hale. (*Adopted.*)

Moved: That by unanimous consent the Executive Board of the National Research Council recognize that Mr. A. O. Leuschner, elected Acting Secretary on April 1, 1919, became Secretary on April 16, 1919, on the acceptance of the resignation of Mr. John Johnston. (*Adopted.*)

In behalf of the Committee on Apportionment of Space, Mr. Mathews, Chairman, presented a plan for the allotment of space in the new quarters of the Council at 1201 Sixteenth Street, to the Divisions of the Council. The report was approved as presented.

On recommendation of Mr. Dunn it was

Moved: That Mr. Mathews be added to the Committee on Removal of Offices of the National Research Council. (*Adopted.*)

The Secretary submitted a provisional budget for the National Research Council for 1919–20 based on the available funds for general maintenance and prepared by the Committee on Budget with the assistance of the Committee on Organization of Administration. The budget is to be supplemented later with estimated expenditures from funds available for special purposes. In the provisional budget is included the complete salary list of officers and employees of the Council and all appropriations to the Executive Board and to the various Divisions, of funds for general maintenance, including traveling expenses, which are subject to the rules of the Council.

Moved: That the general principles adopted in preparing the provisional budget for 1919–20, including the provisional salary list and apportionment of funds be approved with the specification that the budget is to be interpreted as operative for the first half year, from July 1, 1919, to December 31, 1919, and that the Interim Committee have power to make such changes and reductions as may be deemed advisable. (*Adopted.*)

On recommendation of the Secretary, it was

Moved: That the losses on Government funds incurred by foreign exchange be made good from and be charged to the Maintenance Fund of the Executive Board. (*Adopted.*)

Mr. Leuschner, Acting Chairman of the Division of Physical Sciences presented the following resolution of the American Section of the International Astronomical Union:

That the Executive Board of the National Research Council be requested to secure funds for the preparation of a history and bibliography of comets and asteroids, with the understanding that this work will be undertaken in the United States.

Moved: That the securing of the necessary funds for the preparation of a history and bibliography of comets and asteroids be referred to the Committee on Funds for Research Activities. (*Adopted.*)

On recommendation of the Chairman of the Division of Biology and Agriculture, it was

Moved: That the sum of $149.80 to cover the expenses of the Committee on Salt Requirements be reappropriated to the Division of Biology and Agriculture from the unappropriated funds of the National Research Council. (*Adopted.*)

On recommendation of the Chairman of the Division of Biology and Agriculture it was

Moved: That Mr. Lafayette B. Mendel be added to the Committee on Food and Nutrition of the Division of Biology and Agriculture. (*Adopted.*)

In behalf of the Committee on Nomination of Members-at-large of the Executive Board, Mr. Dunn presented for election the names of Mr. R. S. Wood-

ward, President of the Carnegie Institution, Mr. Michael I. Pupin, Professor of Electro-Mechanics, Columbia University, Mr. Raymond Pearl, Biologist, Maine Agricultural Experiment Station, Mr. Edward Dean Adams, 71 Broadway, New York City, and Mr. C. P. Townsend, Patent Lawyer, 918 F Street, Washington, D. C.

Moved: That the Secretary of the Council cast the ballot for the election of Messrs. R. S. Woodward, M. I. Pupin, Raymond Pearl, E. D. Adams, and C. P. Townsend as members-at-large of the Executive Board of the National Research Council, and that the President of the National Academy of Sciences be requested to appoint them members of the National Research Council. (*Adopted.*)

Mr. Leuschner, the Secretary, thereupon reported that he had cast the ballot for Messrs. R. S. Woodward, M. I. Pupin, Raymond Pearl, E. D. Adams, and C. P. Townsend as members-at-large of the Executive Board of the National Research Council, and the Chairman declared these nominees so elected.

It was agreed that the regular meeting of the Executive Board in July should be dispensed with.

The Executive Board adjourned at 1.35 p.m.

PAUL BROCKETT, *Assistant Secretary.*

MINUTES OF THE MEETING OF THE INTERIM COMMITTEE

AT THE NATIONAL RESEARCH COUNCIL BUILDING, 1201 SIXTEENTH STREET NORTHWEST, JULY 1, 1919, AT 9.30 A.M.

Present: Messrs. Angell, Bancroft, H. E. Howe, Leuschner, Mathews, Ransome, and by invitation, A. L. Barrows, Executive Secretary of the Divisions of States and of Educational Relations, H. O. Wood, Assistant to the Secretary of the Council, and J. H. J. Yule, Bursar.

Mr. Angell in the chair.

The reading of the minutes of the previous meeting was dispensed with.

At his suggestion the secretary was instructed to address a letter to each Division regarding the allotment of funds for its use for the current fiscal year, with the suggestion that each Division prepare a budget of estimated expenditures, based on its allotment, to cover general maintenance, traveling expenses, and committee expenses, and with the understanding that no expenditures shall be incurred by a Division for any purpose until an estimate shall have been submitted in advance to the Bursar for each proposed expenditure and until the Bursar shall have certified that a sufficient balance is available in the allotment of the Division.

Moved: That during the absence of the Secretary, Mr. H. E. Howe act as Chairman of the Committee on Budget and as Chairman of the Committee on Organization of Administration, of which the Secretary of the Council is Chairman, and that he be authorized to sign bills in behalf of the secretary. (*Adopted.*)

The Secretary directed attention to the fact that the annual report of the National Research Council to the Council of National Defense should be completed with as little delay as possible and requested that Chairman of Divisions and other executive officers place their material in the hands of Mr. Wood for general assembling during the month of July.

Moved: That the Chairmen of the Divisions and other executive officers of the National Research Council be requested to bring the material as published in the Third Annual Report of the National Research Council presented to Congress down to June 30, 1919, and that these reports be submitted to the Assistant to the Secretary by July 15, 1919. (*Adopted.*)

The Secretary read a communication from Col. L. H. Ruggles, Chief of the Technical Staff of the Ordnance Office dated June 27, in which coöperation is requested in securing experts in dynamics and mathematics. The Secretary was requested to send copies of this communication to the Chairman of the Division of Physical Sciences, the Division of Engineering, and the Division of Chemistry and Chemical Technology.

Moved: That the Interim Committee, acting for the Executive Board is in favor of a change of name from the Division of Industrial Relations to the Division of Industrial Research, if the Division of Industrial Relations submits a definite recommendation to that effect. (*Adopted.*)

The Interim Committee adjourned at 12.30 p.m., to meet again on July 15, 1919.

PAUL BROCKETT, *Assistant Secretary.*

MINUTES OF THE MEETING OF THE INTERIM COMMITTEE

AT THE NATIONAL RESEARCH COUNCIL BUILDING, JULY 15, 1919, AT 9.30 A.M.

Present: Messrs. Angell, Bancroft, Barrows, H. E. Howe, Kellogg, Mathews, Ransome, Wood, Yerkes, and, by invitation, J. H. J. Yule.

Mr. Angell in the chair.

The minutes of the Interim Committee of July 1, 1919, were approved as circulated.

The Bursar raised the question as to method to be adopted by the Council for computing annual or monthly compensation for services.

Moved: That the following rules for compensation of any person in the Service of the United States Government be approved for the computation of pay for services rendered to the Council. (*Adopted.*)

PAYMENT OF SALARIES AND COMPENSATION

1914
Department Circular No. 35
—

TREASURY DEPARTMENT
OFFICE OF COMPTROLLER OF THE TREASURY
Washington, August 18, 1914.

Comptroller of the Treasury.

The following information is given for computing annual or monthly compensation for services rendered the United States.

The Act of Congress approved June 30, 1906 (34 Stat., 763) provides as follows:

Sec. 6. Hereafter, where the compensation of any person in the service of the United States is annual or monthly the following rules for division of time and computation of pay for services rendered are hereby established:

Annual compensation shall be divided into twelve equal installments, one of which shall be the pay for each calendar month; and in making payments for a fractional part of a month one-thirtieth of one of such installments, or of a monthly compensation, shall be the daily rate of pay. For the purpose of computing such compensation and for computing time for services rendered during a fractional part of a month in connection with annual or monthly compensation, each and every month shall be held to consist of thirty days, without regard to the actual number of days in any calendar month, thus excluding the thirty-first of any calendar month from the computation and treating February as if it actually had thirty days. Any person entering the service of the United States during a thirty-one day month and serving until the end thereof shall be entitled to pay for that month from the date of entry to the thirtieth day of said month, both days inclusive; and any person entering said service during the month of February and serving until the end thereof shall be entitled to one month's pay less as many thirtieths thereof as there were days elapsed prior to date of entry: *Provided*, That for one day's unauthorized absence on the thirty-first day of any calendar month one day's pay shall be forfeited.

This act is construed as requiring that—

1. Each calendar month shall consist of thirty days, and the computation of salary shall be by each month separately, one-twelfth of an annual salary constituting the compensation of each month.

2. One-thirtieth of a monthly installment of salary is to be allowed each day of service from the first to the thirtieth, inclusive. The last day of February counts as three days of service for pay purposes (two days in leap years).

3. The thirty-first day of a month enters into the computation of salary only where there is *one day's* absence in a nonpay status on that day—that is, absence in a nonpay status did not occur also on the thirtieth. For such absence on the thirty-first one day's pay is forfeited.

Reference is made to 20 Comp. Dec., 772 and 867.

Department circulars No. 46 of 1904 and No. 67 of 1906, are superseded.

Approved:

WM. P. MALBURN, *Acting Secretary of the Treasury.*
GEO. E. DOWNEY, *Comptroller.*

Moved: That it is the interpretation of the Interim Committee of the motion of March 18, 1919 that all terms of service expire on June 30 of the appropriate year. (*Adopted.*)

Considering the question of incidental printing of the Divisions, it was the sense of the Interim Committee that the Chairman would charge such expenses to their own divisional allotments unless special approval had been received from the Interim Committee for the cost to be paid from other funds.

Moved: That the Chairman of the National Research Council appoint an Editorial Committee of three members to consider the matters of printing relating exclusively to the Council and its work, to consist of the Chairman of the Council, the Secretary of the Council, and the Chairman of the Research Information Service. (*Adopted.*)

Considering the question raised by Mr. Clevenger, Acting Chairman of the Division of Engineering as to when the term of service of the Committees of the Divisions expires,

This matter was referred to the Committee on Organization of Administration.

Mr. Barrows, Executive Secretary of the Division of Educational Relations presented the following nominations:

Representing the National Association of State Universities—Frank L. McVey, President University of Kentucky, Lexington.

Representing the Bureau of Education—S. P. Capen, Specialist in Higher Education, Bureau of Education, Washington, D. C.

Moved: That the nominations of Messrs. Frank L. McVey and S. P. Capen be approved, with recommendation to the President of the National Academy of Sciences that they be appointed members of the National Research Council, and assigned to the Division of Educational Relations. (*Adopted.*)

On recommendation of the Treasurer, Mr. Ransome, it was

Moved: That the Treasurer be authorized to send the following letter to the Chairman of Divisions and that the Chairman be requested to furnish the information sought as promptly as possible:

My dear Mr. ———

Inasmuch as it is highly important that all funds donated for the work of the National Research Council should be accounted for in a thoroughly business-like manner and to the complete satisfaction of the donors, you are requested to submit to the Treasurer a concise, carefully ordered statement concerning each fund from which expenditures under your control are made. The statement should be arranged under the following heads:

1. Source of the fund.
2. Purpose of the fund as stated by donor.
3. Amount of the fund.
4. Plan or times of payment.
5. Conditions of accounting, including items or head under which expenditures are to be classified by you, as the basis for financial statements to the donor of the fund.

The Treasurer proposes to submit these statements, with the attached form of letter, to the donors of the funds in order to ascertain whether there is complete understanding between each donor and the National Research Council. If you prefer, the letter accompanying this reference, although it should conform as closely as possible to the Treasurer's draft, can be sent over your own signature. After such submission and certification the statements will serve as the Treasurer's guide and authority in all subsequent payments from these funds and in his statements of account to the donors.

Very truly yours,

....

Treasurer.

My dear Mr. ————

It has seemed desirable in connection with the reorganization of the National Research Council on a peace basis to bring together and to place definitely on record the objects and the conditions of gift of the various funds donated for the work of the Council. The following statement concerning the fund .... has been drawn up by ...., Chairman of the Division .... who controls the expenditure of this fund. May I ask you to examine this statement and to inform me whether it is in essential agreement with your record and particularly whether the mode of accounting for expenditures from this fund is entirely satisfactory to .....

Yours very truly,

....
Treasurer.

Concerning the matter of the organization of the Division of Educational Relations it was

Moved: That the Interim Committee recommend to the Executive Board the approval of the election of Mr. Vernon Kellogg as Chairman of the Division of Educational Relations. (*Adopted.*)

The question of traveling expenses of Chairmen of Divisions when called to Washington while on leave of absence was considered. After discussion it was

Moved: That when members of the Executive Board or of the Interim Committee on leave are required to travel to Washington on official business their traveling expenses, when approved by the Chairman, shall be paid by the National Research Council. (*Adopted.*)

The Chairman reported that the Psychology Section of the Division of Anthropology and Psychology was in process of organization.

The Interim Committee adjourned at 11.10 a.m. to meet at the call of the Chair.

PAUL BROCKETT, *Assistant Secretary.*

MINUTES OF THE MEETING OF THE EXECUTIVE BOARD OF THE NATIONAL RESEARCH COUNCIL ON AUGUST 12, 1919

AT THE OFFICE OF THE NATIONAL RESEARCH COUNCIL AT WASHINGTON, D. C.

The meeting was called to order at 9.55 a.m. with Mr. Angell in the chair.

Present: Messrs. Abbot, Adams, Angell, Barrows, Flinn, H. M. Howe, H. E. Howe, Hussey, Johnston, Kellogg, Ransome, Wood, Woodward, Yerkes, and by invitation J. H. J. Yule.

The minutes of the meeting of the Executive Board of the National Research Council of June 24 were approved as circulated with certain textual corrections.

The Treasurer, Mr. Ransome, presented a financial statement for the month of June, stating that the report for July was in preparation and would be distributed shortly.

Mr. Woodward, as chairman of a committee appointed at the meeting of the Executive Board on May 8, 1919, recommended for approval the following resolution:

WHEREAS the inauguration of any important enterprise for the improvement and progress of communities or states requires that its affairs be conducted by men whose altruism and fidelity command confidence; and whereas the office of Treasurer of the National Academy of Sciences and of the National Research Council, held by Mr. Whitman Cross during the past year and four months, is a position of vital and exacting responsibilities, therefore, be it

RESOLVED, That the National Research Council hereby expresses and records its cordial appreciation of the laborious and painstaking services rendered by Mr. Cross during this formative period of the Council, when there was special need of the patriotism and the altruism he displayed in his work.

Moved: That the report of the Committee be accepted by the Executive Board as expressing its appreciation of the services rendered by the Treasurer, Mr. Whitman Cross, and that the Secretary be instructed to transmit a copy of this minute and report to Mr. Whitman Cross. (*Adopted.*)

Mr. Kellogg, Chairman of the Division of Educational Relations presented the following recommendations:

In compliance with action of the Interim Committee at its meeting on June 17, 1919, it is recommended, after discussion and agreement with the Executive Committee of the Division of Biology and Agriculture at its recent meeting at Woods Hole, that informal liaison be maintained between the Committee on Educational Relations of the Division of Biology and Agriculture and the Division of Educational Relations through the chairmen of these Divisions, instead of by the appointment of a liaison member from the former in the latter Division.

The question of coördination of Divisions of Science and Technology with the Division of Educational Relations has been considered. It is believed that the action of the Interim Committee on June 17, to the effect that chairmen of divisions be present whenever possible at the meetings of other divisions, sufficiently covers this situation, and it is recommended that this system be developed and maintained.

Moved: That the recommendations of the Division of Educational Relations be approved. (*Adopted.*)

The following nominations were presented by the Chairman of the Division of Educational Relations for membership in that Division:

Representing the Association of American Universities—President A. Ross Hill, University of Missouri, Columbia, Mo.

Representing the American Association of University Professors—Professor E. Percival Lewis, Professor of Physics, University of California, Berkeley, California.

Moved: That the nominations of President A. Ross Hill, of the University of Missouri, and Professor E. Percival Lewis, of the University of California, be approved, with recom-

mendation to the President of National Academy of Sciences that they be appointed members of the National Research Council and assigned to the Division of Educational Relations. (*Adopted.*)

Moved: That the appointment of Mr. Vernon Kellogg as Chairman of the Division of Educational Relations as of July 15, 1919, be approved. (*Adopted.*)

Mr. Yerkes, Chairman of the Research Information Service, asked further consideration of the price to be charged for certain bulletins of the National Research Council.

Moved: That the price of certain bulletins be fixed at one cent a page instead of one-half cent a page. *Adopted.*

Moved: That in each case the authors be requested to indicate if they desire separates, and that the Board authorize the supplying of fifty copies of a reprint to each of the authors gratis. (*Adopted.*)

In compliance with action of the Interim Committee at its meeting on June 17, 1919, which provided that the Research Information Service be instructed to investigate and report on the desirability of preparing for publication a record of the work of scientific men in connection with the War, Mr. Yerkes, Chairman of the Division reported a diversity of opinion in the several Divisions, and recommended as a result of his investigation that action with reference to this matter be left, at least for the present, entirely to the initiative of the Divisions.

Moved: That the report and recommendation of the Chairman of the Research Information Service be approved. (*Approved.*)

Mr. Yerkes asked consideration of the preparation of a volume in the Century Series on "The War and Science," which he has in preparation in coöperation with Mr. H. M. Howe.

Moved: That Mr. Yerkes and Mr. Howe be authorized and requested to prepare the manuscript on the volume in the Century Series, entitled "The War and Science," without expense to the Council. (*Adopted.*)

Moved: That the Research Information Service be required to establish and maintain a Personnel Bureau in connection with the service, which shall include a research personnel list in science and technology. (*Adopted.*)

Moved: That Mr. G. S. Fulcher be appointed Scientific Associate from July 1, 1919. (*Adopted.*)

Mr. H. M. Howe, Chairman of the Division of Engineering, presented the resignation of Mr. G. H. Clevenger as Vice Chairman of the Division.

Moved: That the resignation of Mr. Clevenger be accepted with regret, and that a vote of thanks of the Board be extended to him for efficient and faithful service. (*Adopted.*)

Mr. Howe presented an informal statement of the work of the Division of Engineering.

Mr. Howe presented to the Executive Board his resignation as Chairman of the Division of Engineering.

Moved: That the resignation of Mr. Henry M. Howe as Chairman of the Division of Engineering be accepted with regret, and that the National Research Council desires to express its appreciation of the valuable services which Mr. Howe has rendered to the Council in its work, both as Chairman of the Division of Engineering, and as Scientific Attaché of the United States and representative of the Research Information Service at Paris. (*Adopted.*)

Mr. Howe inquired whether there was any objection on the part of the Executive Board to the printing of the scientific papers prepared by those connected with the Division in scientific journals provided due credit was given to the Council for the work.

Moved: That on the title page of scientific papers from the Division of Engineering published in journals other than those of the National Academy of Sciences and the National Research Council, a footnote be included to read as follows: "Progress Report on Research under the Division of Engineering, National Research Council." (*Adopted.*)

Moved: That the name of the Committee on "Improvements of Metals at Blue Heat" be changed to "Physical Changes in Iron and Steel Below the Thermal Critical Range." (*Adopted.*)

Moved: That the Research Committee of the American Bureau of Welding be designated as The Welding Research Committee of the Division of Engineering of the National Research Council. (*Adopted.*)

Moved: That the Executive Board approve of the publication of a booklet on the relation of the Engineering Foundation to the National Research Council, if funds are provided. (*Adopted.*)

Mr. Hussey, Vice-Chairman of the Division of Medical Sciences presented the following nominations for membership in that Division:

Representing the American Anatomical Society—Professor Clarence M. Jackson, University of Minnesota.

Representing the American Neurological Society—Dr. E. E. Southard, 74 Fenwood Road, Boston, Mass.

Moved: That the nominations of Professor Clarence M. Jackson and Dr. E. E. Southard be approved, with recommendation to the president of the National Academy of Sciences that they be appointed members of the National Research Council and assigned to the Division of Medical Sciences. (*Adopted.*)

Moved: That the Division of Medical Sciences, in view of the special conditions presented by Mr. Hussey, Vice-Chairman of the Division, be authorized to act in the election of a representative from the Public Health Service on the Division of Medical Sciences, subject to the approval of the Council of the National Academy of Sciences. (*Adopted.*)

The Chairman of the Council stated that Dr. Christian would report for duty as Chairman of the Division of Medical Sciences on October 1, 1919.

The Chairman of the Council presented the following report from the Executive Committee of the Division of Biology and Agriculture:

1. That Miss Isabel Bevier, Head of the Department of Domestic Science of the University of Illinois has been recommended for appointment on the Committee on Human Nutrition, and Mr. W. H. Jordan of the New York Agricultural Station at Geneva and President Raymond A. Pearson of the State Agricultural College of Iowa, Ames, Iowa, for appointment on the Committee on Animal Nutrition.

2. That Messrs. L. O. Howard, W. E. Osgood, C. E. Allen, K. F. Kellerman, G. M. Duggar have been appointed as new members of the Committee on Coöperation and Coördination.

3. That the Organization Committee on Forestry has been made a standing committee of the Division. (*Approved.*)

The Chairman of the Council also presented the following recommendation of the Executive Committee of the Division of Biology and Agriculture:

That in connection with the matter of the needs of the Marine Biological Laboratory a sum of money (about $130,000 will be needed) be appropriated for the construction, equipment, and maintenance of a new laboratory as outlined in the report of the Committee on the Marine Biological Laboratory; and that the annual sum of $5000 for five years be also appropriated for use in connection with accessions for the Marine Biological Laboratory Library.

After considerable discussion of the purposes of the Council, it was

Moved: That further consideration of this matter be postponed until the next meeting of the Executive Board. (*Adopted.*)

But as an outcome of the general discussion it was further

Moved: That the National Research Council give its moral support to the work of the Marine Biological Laboratory at Woods Hole, approve the program of the Laboratory for the enlargement of its facilities for scientific research, with the statement that active steps for the encouragement of this work are under consideration. (*Adopted.*)

It was the understanding that a copy of this minute, together with descriptive material, would be mailed to the members of the Executive Board for their consideration, and that the matter would come up for further action at the next meeting.

The Chairman of the Council also presented, for the Executive Committee of the Division of Biology and Agriculture the recommendation that a publication fund of $20,000 per year for a period of five years be granted to that Division.

Moved: That the Finance Committee of the National Research Council be instructed to consider and if practicable secure a publication fund of $20,000 per year for a period of five years to be appropriated to the Division of Biology and Agriculture. (*Adopted.*)

It was understood that the Division of Biology and Agriculture would control the expenditure of this fund.

The Executive Board took a recess at 1.50 to meet again at 3.00 p.m.

The Executive Board resumed its meeting at 3 p.m.

The following nomination was presented by Mr. Barrows, Executive Secretary of the Division of States Relations for membership in that Division:

Representing the Association of American State Geologists—Mr. Frank W. DeWolf, State Geologist of Illinois, Urbana, Illinois.

Moved: That the nomination of Mr. Frank W. DeWolf be approved with the recommendation to the President of the National Academy of Sciences that he be appointed a member of the National Research Council and assigned to the Division of States Relations. (*Adopted.*)

The following nominations for the Divisions mentioned, as members in the Division of States Relations were approved:

Industrial Relations	Mr. H. E. Howe
Chemistry and Chemical Technology	Mr. W. D. Bancroft
Geology and Geography	Mr. E. B. Mathews
Educational Relations	Mr. Vernon Kellogg

The following motion was presented:

Moved: That, beginning July 1, 1919, following each regular meeting of the Executive Board, a bulletin be published which shall include at the discretion of the Secretary the minutes and records of action of the Executive Board, and of the Interim Committee, of Divisions and of their Executive Committees, and of other administrative committees of the Council, for the interval following the preceding meeting of the Executive Board.

Moved: That the motion be laid on the table. (*Adopted.*)

The following motion relating to a bulletin on the History of the Organization of the National Research Council was presented:

Moved: That a bulletin be published to record the actions taken to effect the permanent organization of the National Research Council, and of its several Divisions, which shall include at the discretion of the Secretary pertinent minutes and records of actions of committees, governing boards, and Divisions. (*Adopted.*)

Mr. Wood presented a mimeographed copy of the organization of the National Research Council as of June 24, 1919, which he had prepared, stating that the lists were not entirely complete.

Moved: That this organization be held until complete, and then included in the bulletin of the history of the organization. (*Adopted.*)

The Chairman stated that Mr. Leuschner would not be able to return to the Council to continue as Secretary, and that the Council appreciated the work he had done as Secretary and the sacrifice which he had made in leaving his scientific work in California in order to aid at a critical period.

The Chairman also stated that in the absence of Mr. Leuschner the work of the office of the Secretary had been carried on very ably by Mr. Wood, whose services were greatly appreciated by the Council.

Following the instructions of the Interim Committee at its meeting of July 15, 1919, Mr. H. E. Howe, as Chairman of the Committee on Organization of Administration, presented the following motions:

Moved: That beginning with this date all committees of the Council and its Divisions shall terminate June 30, 1920 and thereafter all committees except Executive Committees of Divisions shall unless previously discharged terminate on June 30 of each year. (*Adopted.*)

Moved: That the Executive Committees of Divisions shall be elected at the annual meetings for the succeeding year. (*Adopted.*)

Mr. Hussey suggested that two committees, one on Organization of Administration and the other the Budget Committee, be combined.

Moved: That the Committee on Organization of Administration and the Budget Committee be discharged, that the joint functions of the two committees be combined, and that the Chairman be authorized to appoint a new committee of three on Organization and Budget. (*Adopted.*)

The Chairman thereupon appointed the following to serve on this Committee: Mr. H. E. Howe, Chairman, Mr. F. L. Ransome, Mr. Vernon Kellogg.

Moved: That the Chairman be authorized to appoint a committee of three on publicity, with Mr. Vernon Kellogg as Chairman. (*Adopted.*)

The Chairman thereupon appointed the following to serve with Mr. Kellogg on this committee: Mr. R. M. Yerkes, Mr. H. E. Howe.

The Chairman announced to the Executive Board the death of Mr. Andrew Carnegie, philanthropist, and it was the unanimous opinion of the Board that resolutions should be adopted showing the appreciation of what Mr. Carnegie had done for the advancement of scientific research.

Moved: That the Chairman be authorized to appoint a committee to draft resolutions regarding the life and work of Mr. Carnegie, and to report at the next meeting of the Executive Board. (*Adopted.*)

The Chairman thereupon appointed the following to serve on this committee: Mr. R. S. Woodward, Chairman, Mr. C. D. Walcott.

The Chairman, Mr. Angell, presented a communication from Mr. Benjamin S. Hanchett, Regent of the University of Michigan, addressed to and enclosed with one from Mr. M. E. Cooley, Dean of the Colleges of Engineering and Architecture of Michigan concerning a proposed industrial research laboratory at the University of Michigan, which is to be established through gifts from a Manufacturers Association of the state of Michigan, asking an opinion as to the advisability of the University of Michigan entering into this new field. These communications are filed in the office of the Chairman of the Council.

Moved: That if invited by the responsible authorities of the University of Michigan the National Research Council is ready to express an opinion in the matter. (*Adopted.*)

Moved: That the Executive Board adjourn to October 14, 1919, subject to the call of the Chair. (*Adopted.*)

The meeting adjourned at 4.50 p.m.

PAUL BROCKETT, *Assistant Secretary.*

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES

Volume 5 DECEMBER 22, 1919 Number 12

THE CHANGE OF MOLECULAR KINETIC ENERGY INTO MOLECULAR POTENTIAL ENERGY: THE ENTROPY PRINCIPLE AND MOLECULAR ASSOCIATION

By William D. Harkins

Kent Chemical Laboratory, University of Chicago

Communicated by J. Stieglitz, September 16, 1919

My work on the orientation of molecules in the surfaces of liquids (Langmuir has also worked on orientation) has led to the recognition of a remarkable new principle or law concerning the change of molecular kinetic energy into molecular potential energy. None of the kinetic relations already found in the very simple case where gases alone are involved, have been found to be as exact as the laws of thermodynamics, since by their very nature such relations are affected by many extraneous complicating factors. It need not then be surprising if similar relations involving the much more complicated and hitherto obscure kinetic phenomena of the liquid and the solid states should prove to be approximate rather than exact. Thus it is well known that Raoult's law concerning the kinetics of the vaporization of the various components in a solution, is exact only in an extremely limited range, that is when the components are practically alike with respect to cohesion[1] or with respect to the electromagnetic fields surrounding their molecules.[2] It is, therefore, somewhat startling to find that my new relation or law is, in one of its forms, much more generally applicable than Raoult's law, at least if the data at present accepted in this connection are as exact as they are supposed to be.

The new principle or law will first be stated in one of its special forms as follows: *Whenever a molecule moves from the interior of a liquid into the surface* in such a way as to form a new surface, *the average amount of its*

kinetic energy which is converted into potential energy is equal to 144% of the mean translational kinetic energy of a gas molecule at the same temperature. This indicates that in general only the faster moving molecules possess sufficient kinetic energy to carry them into the surface. All known plane surfaces have a positive free surface energy, that is the molecular potential energy in a plane surface in which the above principle holds is always greater than 144% of the mean kinetic energy of its molecules.

Application to the theory of surfaces.—As has been indicated in the last paragraph the molecules in a surface possess potential energy by virtue of their position. When a new surface is formed the principle given above indicates that a definite portion of this potential energy results from a tranformation of the kinetic energy of molecular motion into the potential form, and that the amount of energy supplied in this form depends only upon the temperature; and is proportional to the temperature. *The free energy of the surface is simply the difference between the total energy, which depends upon the structure of the surface, and the latent heat of the surface which is conditioned by the above law.* It is easy to see why, on this basis, the surface tension or the free surface energy decreases with the temperature. The total surface energy is approximately constant while the temperature is varied, provided the critical temperature is not too closely approached, which is the condition also for the application of the new principle since the surface film thickens as the critical temperature is neared. Since the contribution of the kinetic energy of molecular motion to this total energy is proportional to the absolute temperature, the free surface energy must decrease with the temperature.

The entropy law.—According to Lunn[3] heat has two measurable aspects, energy and entropy. The first law of thermodynamics relates to heat changes in which energy is, but entropy is not, conserved. The second law considers heat changes in which energy in the form of heat alone is not, but entropy is, conserved. The principle discovered by me has been stated above in terms of energy but it may be much more simply stated in terms of entropy as follows: *Whenever a molecule moves from the interior of a liquid into its surface in such a way as to form a part of a new surface, the entropy of surface formation is not only independent of the nature of the molecule, but is also independent of the temperature.* The numerical value of this entropy is 2.96×10^{-16} ergs per degree per molecule.

This entropy is not the ordinary thermodynamic entropy but is about

15% less than the thermodynamic entropy calculated for the area occupied by one molecule. The entropy which corresponds to the value given above is that which is calculated for the area of surface occupied by one molecule on the supposition that the arrangement of the molecules is such as to give approximately the same number of molecules in the surface as if it were built up on the plan of a cubic lattice, and the area for which the entropy is calculated is always that occupied by one molecule no matter what the temperature may be. The difference between these two entropies will be discussed later, but it is not essential *since either of them shows the constancy required by the relation under discussion.* When the extent of the molecular orientation in the surface is known it should be taken into account.

Relation of the entropy law to molecular association.—Although the entropy principle presented in this paper was discovered by the writer not more than a few years ago, a number of empirical relations which involve its validity without its recognition by their discoverers, have been well known for many years. The empirical relation directly related to the form of the entropy principle already given, is known as the law of Eötvös,[4] Ramsay, and Shields.[5] While a part of this relation was developed by Eötvös from a special form of the theorem of corresponding states, in its final form, as given by Ramsay and Shields, it may be considered as purely empirical, and it is this latter form which is given by the entropy principle. Indeed, though it does not seem to have been recognized by them, Ramsay and Shields relation gives the entropy of a surface, though in very peculiar units, and also the entropy expressed by it is not the ordinary thermodynamic entropy.

The thermodynamic entropy may be expressed in ergs per degree per square centimeter, or in other units. The Ramsay and Shields relation may be expressed in terms of entropy, if a somewhat unusual unit is used for the measurement of the area, in the following terms: *The entropy of the surface of a liquid is 2.12 ergs per degree for an area which is equal to the area of one face of a cube which contains one gram molecule of the liquid.* Since the volume of such a cube varies with the temperature, the area to which the entropy is referred also varies with the temperature.

The value in the use of such a peculiar system does not lie in the use of the same number of molecules in the box in every case, but in the use of the area of one face of the box, the inherent idea being that in this way the same number of molecules in the surface are obtained in every case when the molecules in the surface are counted by considering that their complexity in the surface to be the same as that inside the liquid.

It is obvious that this depends either upon the approximation to a cubic lattice arrangement, or to a similarity in the surface arrangement in every case. It is therefore to be expected that the relation would not hold in any case where the molecular orientation in the surface is such as to cause a deviation from such similarity.

While the normal value for the fraction of the molecular kinetic energy which is converted into the potential form is 144% of the mean molecular kinetic energy, which corresponds to a surface entropy of 2.96×10^{-16} ergs per molecule, and while what have been usually termed normal or unassociated liquids give these values very closely, certain liquids have been found for which the entropy value is lower. Thus the alcohols, organic acids, and water, give at ordinary temperatures not far from one-half the normal values (one-third in the case of water), but as the temperature increases, even for such liquids the entropy increases and approaches the normal value. Such results as these are easily explained on the basis of the assumption made by Ramsay and Shields and their followers in the association school, who consider that at low temperatures the molecules in such liquids are associated into larger groups than correspond to the formula weight, but that the association decreases as the temperature increases.

Criticisms of the Ramsay-Shields method of determining molecular association.—Certain sweeping criticisms of the Ramsay-Shields method of calculating the degree of molecular association which have been made, I consider to have no validity. On the other hand objections might be raised which seem not to have occurred to the workers in this field. The most sweeping criticism of the method, which has been expressed many times, is that since it is a "surface tension method it gives no true indication of the molecular state of the liquid as a whole." From this view-point it is considered that it is not the association in the liquid, but the association in the surface, which is calculated by this method. With this criticism I disagree most strongly, for if Ramsay and Shields have calculated any association at all it is not that in the surface, but that which exists in the body of the liquid, as seems to me apparent from the entropy principle, for the kinetic energy which is converted into the potential energy of the surface is the molecular kinetic energy of the molecules just before they move into the surface.

A second criticism which has been made is that very complex molecules, particularly those which contain several, paraffin chains such as tripalmatin and tristearin, give much too large values of the Ramsay-Shields constant. I have found, however, that a very great reduction

in the value of the constant is obtained if the molecular orientation in the surface is taken into consideration, which was not done by Ramsay and Shields. While the values which are thus obtained are still somewhat larger than the normal, it must be remembered that such complex molecules may vibrate in parts, which might very well give larger amounts of kinetic energy available for transformation into potential energy.

While Ramsay and Shields evidently did not see at all the important theoretical basis for their method it is nevertheless true that their equation involved the entropy of formation of the surface, and therefore kept closer to this basis than the very great number of supposedly improved equations which have been developed from it.

In so far as the experimental results for the temperature coefficients of free surface energy as determined by Ramsay and Shields can be trusted, our calculations show that the normal value of the surface entropy per degree per molecule is very close to the normal value (2.96×10^{-16} ergs) for the following substances over a wide temperature range: benzene, carbon tetrachloride, ethyl acetate, ethyl ether, methyl formate, and chlorobenzene. The results of Baly and Donnan give 2.8×10^{-16} for liquid nitrogen, and 2.75×10^{-16} for liquid oxygen.

On the other hand the values for argon (1.6×10^{-16}) and for mercury (1.5×10^{-16}), though not so well established experimentally, are not very much above 50% of the normal values, and the work of Jaeger indicates that molten salts also give low values. Unfortunately the data for argon were obtained over such a small temperature range that the entropy obtained may be very much in error. In the case of mercury and the molten salts, those who belong to the extreme association school would assume that the entire deviation is due to molecular association, but I am not at all convinced that the association is at all definite, especially since our present knowledge of the structure of metals and salts indicates that there may not be such a thing as a definite molecular weight in any case. This subject will be treated more comprehensively in a later paper.

A general form of the entropy principle.—In the preceding paragraph I have presented the entropy principle in its relation to the formation of surfaces, in which case it seems to hold remarkably well. Possibly it is only in the formation of surfaces that such a relation will be found to hold with exactness, as the following reasoning will indicate. In the formation of a surface the *total* energy converted into molecular potential energy must be of such a magnitude as to overcome the

cohesive forces to the extent which is necessary in the formation of the surface. However, as has already been stated, all of this energy does not come from the molecular motion, but a part of it is introduced by the action of an external force, which does the work necessary to produce what is called the free energy of the surface. The energy which is not supplied by the molecular motion, must be supplied in the form of work, so the amount of energy supplied by the molecular motion is not uniquely determined by the cohesive forces. When a liquid vaporizes in an ordinary experiment, the *whole* of the energy necessary for the separation of the molecules against the cohesive forces, is supplied by the energy of molecular motion, except for the amount which may be supplied by changes in the potential energy of the molecules themselves, and the latter is probably not a very important factor. It is obvious that the contribution of the molecular motion to the formation of a surface may be largely independent of the cohesive forces involved, and this is made probable by the validity of the entropy principle in this case. The cohesion enters so directly into the heat of vaporization that it would seem doubtful if for it the entropy principle would hold. It would seem surprising, too, if the principle should hold for melting, for sublimation, or for sublimation combined with dissociation.

Nevertheless, a study of the literature shows that while the entropy principle itself has hitherto remained unrecognized, there are a number of empirical relations, which if they hold, involve the validity of a much more general entropy principle. Without discussing to what extent I believe such a principle is valid, I will state it in two different forms as the theoretical basis of all of these empirical relations. In order to give this statement I will define a *region* as a phase, surface, or interface. *When in a system consisting of one component, a molecule moves from one region into another, the average molecular kinetic energy which is converted into the form of molecular potential energy depends in general only on the change of state, that is on the region from which the molecule comes and the one into which it goes, provided* that when a vapor phase is involved one condition which enters is the state of the vapor with reference to one variable. According to Trouton this condition is that the pressure of the vapor shall be the same in all cases, and this has been modified by Hildebrand, with an increase in accuracy, to the condition that the molecular concentration of the vapor phase shall be the same in all cases. In the second form, the principle states that the *entropy* of the change depends only on the change of state, and not on the individual nature of the molecules. In the application of this

principle the surface of a metal is not considered as a region of the same class as the surface of water or an organic liquid, since the characteristics of the two regions are quite unlike.

Corresponding to the various empirical relations the following 'normal' values of the entropy may be given:

	Entropy in ergs per degree per molecule $\times 10^{16}$
1. Liquid to surface	2.96
2. Liquid to vapor at the special concentration of:	
c = 0.00507 mols per liter	18.8
c = 0.0127 mols per liter	16.7
c = 0.0201 mols per liter	15.7
(These values become less accurate as the concentration of the vapor increases.)	
3. Solid to vapor at the melting point	21.0
4. Solid to liquid	9.0
5. Solids dissociate to 760 mm. vapor pressure	22.0

Of these the first is the most exact, the second holds moderately well under the conditions imposed, and the fourth, as might be expected, is one of the least accurate. Walden's rule is that the molar heat of fusion divided by the temperature is equal to 13.5 calories per degree for normal substances, or the molar entropy of fusion has the given value. This rule Walden[7] found to hold for a large number of organic substances. When the data did not correspond with what should be obtained according to the rule, Walden assumed that it still remains valid, but that the molecular weight is different from that given by the formula. However it is evident that this explanation is not sufficient to account for all of the deviations which exist.

At my request Mr. L. E. Roberts has studied practically all of the available data on the entropy of melting, and has found the general relations which hold. One of the greatest obstacles in this connection is that the data are in many cases extremely inaccurate. They indicate that the *latent heat of melting* of a metallic element which crystallizes in the regular system, *increases as the melting point rises*, and the entropy averages about 2.2 calories per gram atom per degree.[8] The salts show a somewhat similar relation, and at the same time there seems to be a general increase in the entropy of fusion with the number of gram atoms in the formula weight of the salt. There is a great deal of irregularity, but for the halogen salts the entropy is of the order of 2.2 calories per degree per gram atom, or about the same value as is found for the metals.[9] Hydrogen, hydroxides and water of crystallization are represented by lower values. Thus the entropy of fusion of the hydroxides of sodium, potassium, rubidium, and caesium, is about 2.8 calories per

degree per gram molecule. On turning to molecular compounds such as ammonia and carbon dioxide it is found that the entropy of fusion is 9.3 for the former and 8.9 for the latter, while for benzene the value is 8.3. Corresponding to Walden's rule a large number of organic compounds have entropies of fusion between 12 and 14 calories per degree, while many of the substances with smaller entropies of fusion possess other properties characteristic of associated liquids when they are in the liquid state. As might be expected substances with very complex formulae give high values, the increase with molecular complexity being very distinct. Thus with stearic acid ($C_1H_{36}O_2$) the value rises to 40, while the acid with two carbon atoms has an entropy of only 9.5, with nine carbon atoms of 10.5, while in the case of the 12 carbon atom acid the value rises to 27.

It is thus to be seen that the entropy is a very important function in a study of the transfer of molecules from one region to another, and that *in general the price which a molecule has to pay in terms of energy in order to undergo any certain change, increases with the temperature* or the molecules pay in proportion to their wealth with respect to energy. There is also an increase in this energy price whenever the complexity of the molecules increases sufficiently. The price in terms of entropy is much more constant than the price in terms of energy, and in this sense there is an analogy to the action of a Carnot engine.

It is believed that the point of view presented in abstract in this short paper will greatly change the present attitude in regard to the determination of molecular association in liquids, and it is possible that it may be of importance in a study of the general subject of the partition of energy, as well as in changes of kinetic into potential energy.[10]

The complete paper of which this is a part will be presented to the *Journal of the American Chemical Society* by Mr. L. E. Roberts and the writer, for publication in a later issue.

[1] *J. Amer. Chem. Soc., Easton, Pa.*, 38, 1916, (1452).

[2] *Ibid.*, 41, 1919, (970–92). PROCEEDINGS, May, 1919.

[3] *Physic. Rev.*, July, 1919.

[4] *Leipzig, Ann. Physik*, 27, 1886, (452).

[5] *London, Phil. Trans. Roy. Soc.*, 184A, 1893, (647), *Zs. Physik. Chem., Leipsig*, 12, 1893, (647).

[6] *J. Amer. Chem. Soc.*, 37, 1915, (975).

[7] *Zs. Elektrochem.*, 14, 1908, (715).

[8] Crompton, *J. Chem. Soc.*, 67, 1895, (315–327). T. W. Richards, *J. Franklin Institute*, 1902.

[9] Wayling, *Phil. Mag.*, 37, 1919, (495).

[10] Since this paper was submitted I have received a copy of a paper presented to the Société Française de Physique by M. J. Duclaux on June 6, 1919. He believes that there are quantities of energy of magnitude 6.6×10^{-16} ergs.

THE DISPLACEMENT OF THE GRAVITATING NEEDLE IN ITS DEPENDENCE ON ATMOSPHERIC TEMPERATURES

By Carl Barus

Department of Physics, Brown University

Communicated October 8, 1919

1. *Introductory.*—In *Science* (**50,** pp. 214, 279, 1919) I communicated some of the early results, showing that the deflection of the needle of a gravitation apparatus varies in marked degree with the temperature on the outside of the building. I have since carried these experiments on for another month and the evidence has become more definitely interpretable. The work was done in a semi-subterranean room, in which the thermostat shows temperature variations which do not usually exceed a fraction of a degree. The room is large and so damp that all electrical excitation is excluded. Tests with radium fully confirmed this. Moreover the room is kept dark. The apparatus (Proceedings, **4,** p. 338, 1918) placed on the north-south wall of the pier confronts an eastern 30-inch wall, at a distance of about 4 meters and the outside of this is illuminated by sunlight, if present, in the morning, only.

2. *Observations.*—The observations during July and August are given at the bottom of the figure, the two curves being mean results of the a.m. and p.m. readings, respectively. The telescopic reading of the scale is y, so that Δy denotes the mean (static) excursion or double amplitude, when the attracting mass, $M = 1$ kgm. is passed from one side to the other of the attracted shot ($m = 0.6$ gram), at the end of a needle suspended by a quartz fiber. The actual excursion of the shot is $\Delta x = 0.01455\ \Delta y$, so that the magnification is about 70. The figure shows that even these mean excursions vary enormously, from values much below $\Delta y = 2$ to values above 7, easily five times. If individual excursions were taken, ratios as high as 10 might be found, in spite of the practically constant room temperature. On the upper part of the chart I have inserted the temperature observations θ in degrees F., made at Providence by the United States Weather Bureau, as well as the temperature variations $\Delta\theta$ (high minus low) of the successive days of the months, the same abscissas holding for all curves.

In the earlier data there seemed to be a close association between the Δy and θ curves. In the present data the regions a, b, c, d, e, belong

together, though the Δy curve follows the θ curve with a lag of one or more days. A far better agreement in sense, not quantitatively always, now appears between the Δy and $\Delta\theta$ curves, and here in the given time scale, practically without a lag. To bring this to the eye more clearly, I have indicated the corresponding successive cusps in both curves with the same numbers 1 to 23. The agreement is in fact as close as it can possibly be, remembering that $\Delta\theta$ holds for twenty-four hours of the day and Δy only for the daylight interval of observation. In the same way the a.m. and p.m. curves differ enormously when there is sunlight, and very little in damp cloudy rainy weather (R in curve). In general

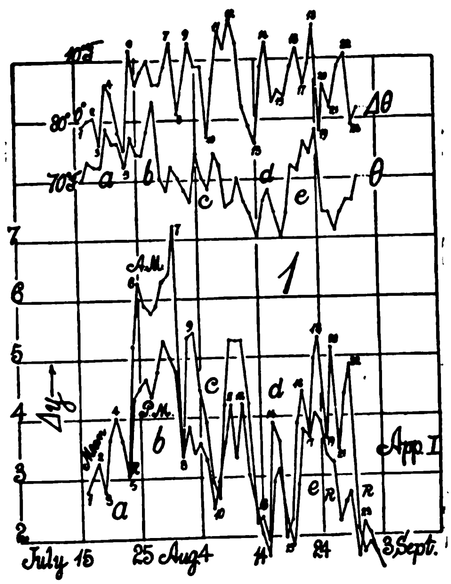


FIG. 1

and apart from details, the a.m. excursions reappeared in a subdued form in the p.m. results.

3. *The needle in vacuum.*—In *Science* I also communicated a series of results since much amplified, showing that for a case of two glass plates spaced by an impregnated wood frame, the initial attractions could be diminished to about one-third of their value by exhausting the case. The excursions diminished with the pressure, at a mean rate of 1%, per mercury centimeter of pressure. The glass plates in this case were about 1.8 cm. apart, inside. In case of the plenum the general character of the a.m. and p.m. excursion did not essentially differ from the graphs for apparatus I.

With the object of gaining some insight into the remarkable behavior at low pressure a new apparatus (no. III) was constructed with the glass plates spaced by a rectangular frame made of square brass tubing. The inside distance between plates was here 1.3 cm.; but in other respects it closely resembled the wood frame specified. The results with this metal case, however, differed totally from those of the other. In the morning of a bright day, there was usually marked repulsion between *M* and *m*, which changed gradually into an attraction at the close of the day. The repulsion was often so strong that the ends of the needle were pushed up into contact with the glass plates, to which position they returned whenever removed by tapping.

It was found, however, that the needle could be immediately freed by exhaustion of the case. In other words the repulsions passed continuously into attractions which were here at their maximum at the highest

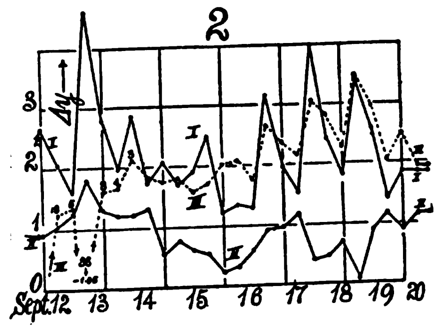


FIG. 2

exhaustions. The behavior of the metal case was thus the reverse of that of the wood case. In the former exhaustion removed a repulsion; in the latter, an attraction. It is difficult to assign a reason for this as there are three forces in contention: viz., gravitation and the radiant forces of the case (static) and of the external mass *M*. One is tempted to contrast the non-conducting wood with the conducting metal. The greater narrowness of the frame of the latter, however, gives the forces due to temperature distributions an advantage. In one respect the exhausted metal case has shown marked superiority; at a definite high vacuum, the excursions of the needle on any day are without drift; they are nevertheless variable on successive days. It is thus also improbable that this vacuum excursion corresponds to the gravitational attraction, so that an adequately trustworthy excursion is yet in arrears.

4. *Record of the vacuum needle.*—To exhibit these relations more clearly I have constructed figure 2, which contains a record of mean

results for three different apparatus, observations on the same vertical being made at the same time. The individual observations were taken thirty minutes apart Three or four means were deduced for the day. Apparatus I on the N.S. face of the pier fronting East has already been referred to in connection with figure I. Apparatus II was placed in a niche on the EW. wall of the pier fronting north, surrounding *on all sides* within 1 meter by the interior brick walls of the building. It thus receives secondary radiation only, and the graph in its details, departs utterly from curve I, particularly on clear days. If curves I and II were smoothed, however, they would show some resemblance.

Curve III are the results for the metal case (on the E.W. wall fronting south) with the needle kept in the partial vacuum, pressure p, marked on the curve. On the morning of September 12 and 13 at 39 and 36 cm. the two bodies M and m repelled each other. Even at lower pressures ($p = 8, 5, 3$, etc.) the results seemed fluctuating. Hence after September 14, I observed for $p < 1.5$ c.m., only (numeral omitted), there being a slight leak in the apparatus so that a higher vacuum could not be held for a half hour. The astonishing feature of these high vacuum (III) results is that they agree very closely with the observations (I) made in a plenum; whereas if III had also been observed in a plenum, the results would necessarily be in total opposition to I, as the repulsions at the beginning of curve III indicate.

Now it may be shown by direct tests (*Science*, l.c.) that the radiant forces of a hot body, M are repulsions for $p < 4$ cm. and attraction for higher pressures in the case. The exact pressure of radiant equilibrium resulting from this inversion is for incidental reasons difficult to specify; but one may estimate that in high vacua, y varies about 5 mm. per cm. of pressure p.

It follows from this that in the plenum apparatus (I) with eastern exposure, the attracting body M must be relatively warm in the morning and cold in the afternoon; while in the vacuum apparatus (III) with southern exposure, the body M is relatively cold in the morning and warm in the afternoon; for in such a case the radiant forces have the same sign. Hence the agreement in kind of the plenum graph, I, and the vacuum graph, III, in figure 2, after September 14, is a demonstration in a dark room, at practically constant temperature, of the rotation of the sun.

[1] Advance note from a Report to the Carnegie Institute of Washington, D. C.

CONDITIONS NECESSARY AND SUFFICIENT FOR THE EXISTENCE OF A STIELTJES INTEGRAL

By R. D. Carmichael

Department of Mathematics, University of Illinois

Communicated by E. H. Moore, October 8, 1919

The purpose of this note is to suggest a method for deriving a necessary and sufficient condition for the existence of the Stieltjes integral of $f(x)$ as to $u(x)$ from a to b in each of several forms generalizing those frequently employed (see *Encyclopédie des sciences mathématiques*, II_1, pp. 171–174) in the special case of Cauchy-Riemann integration—where $u(x)$ is of bounded variation and $f(x)$ is bounded on the interval (ab), $M \geqq f(x) \geqq m$. Bliss (Proceedings 3, 1917, pp. 633–637) has obtained one of these forms, perhaps the most satisfying of any; this note closes with his theorem, of which we give a new demonstration. Some of the other theorems stated (though here derived otherwise) are immediate consequences of the one due to Bliss or are otherwise intimately related to it, as the reader will readily see. Since it is believed that the present treatment will be found useful in connection with that of Bliss, our notation has been made to conform to his; moreover, reference to his paper is given for such isolated steps in the proof as may readily be supplied from it.

For a given partition π of (ab) due to the points $x_0 = a, x_1,x_2, \ldots, x_{n-1}, x_n = b, 0 < x_i - x_{i-1} < \delta$, form the sums

$$s_\pi = \sum_{i=1}^{n} M_i \Delta_i u, \quad S_\pi = \sum_{i=1}^{n} f(X_i)\,\Delta_i u, \quad \sigma_\pi = \sum_{i=1}^{n} m_i \Delta_i u,$$

where $\Delta_i u = u(x_i) - u(x_{i-1})$, X_i is any point of the interval (x_{i-1}, x_i), and $M_i[m_i]$ is the least upper bound [the greatest lower bound] of $f(x)$ on (x_{i-1}, x_i). If the limit of S_π exists as δ approaches zero, this limit is the Stieltjes integral of $f(x)$ as to $u(x)$ from a to b. In case $u(x)$ is a monotonic non-decreasing function, the limit, if existing, of $s_\pi[\sigma_\pi]$, as δ approaches zero, is here called the upper [lower] Stieltjes integral of $f(x)$ as to $u(x)$ from a to b.

Let us determine conditions under which the upper integral shall certainly exist when $u(x)$ is monotonic non-decreasing. If we form a repartition π' of (ab) as to π by taking the points forming π and certain additional points, it is clear that $S_{\pi'} \leqq S_\pi$. Moreover, there is ob-

viously a lower bound to $S_{\pi'}$. Hence if the number of divisions of (ab) is increased by repartitions in such wise that δ approaches zero, the sum $S_{\pi'}$ approaches a definite finite limit. Let us ask under what conditions we shall be led to a contradiction by supposing that two convergent sequences of sums σ_π for sequences of partitions π (whether formed by repartitions or not) with norms δ approaching 0 lead to different limits N_1 and N_2 where $N_1 < N_2$. Let ξ and η be two arbitrarily small positive numbers such that $N_1 + \xi + \eta < N_2$. Let π_1 be a partition of (ab) into s intervals belonging to the sequence by which N_1 is defined and let s be so great that $S_{\pi_1} < N_1 + \xi$. Let π_2 be a partition of (ab) into t intervals where t is an integer greater than s and subject to being made as large as one pleases. Let π_3 be a partition formed by the points of π_1 and π_2, so that π_3 is a repartition of both π_1 and π_2. Then we have

$$S_{\pi_2} - S_{\pi_3} \leqq \sum_{\alpha=1}^{s-1} (M_{i_\alpha} - m_{i_\alpha})\, \Delta_{i_\alpha} u,$$

where the intervals $(x_{i_\alpha - 1}, x_{i_\alpha})$, obviously at most $s - 1$ in number, are all the intervals of π_2 which are separated into parts in forming π_3. Since t may be made large at our choice and since the sum in the second member of the foregoing relation never has more than $s - 1$ terms, whatever the value of t, it is clear that t may be chosen so large that this sum is less than η provided that $u(x)$ is continuous at the discontinuities of $f(x)$. Then we have $S_{\pi_2} \leqq S_{\pi_3} + \eta$. But we have seen that $S_{\pi_3} \leqq S_{\pi_1} < N_1 + \xi$. Hence we have $S_{\pi_2} < N_1 + \xi + \eta$. But, as t increases, S_{π_2} approaches N_2. Hence we have $N_2 \leqq N_1 + \xi + \eta$, contrary to the hypothesis $N_1 + \xi + \eta < N_2$. Hence the upper integral of $f(x)$ as to a monotonic non-decreasing function $u(x)$ exists provided that $u(x)$ is continuous at the points of discontinuity of $f(x)$. The corresponding result may likewise be proved for the case of a non-decreasing function $u(x)$; and also for the case of the lower integral. Hence, since every function of bounded variation $u(x)$ may be expressed as the difference of two monotonic non-decreasing functions which are continuous at the points of continuity of $u(x)$, we have the following theorem:

THEOREM I. *If $u(x)$ is of bounded variation and $f(x)$ is bounded on (ab) and if $f(x)$ is continuous at the points of discontinuity of $u(x)$, then the limits as δ approaches 0 of the sums s_π, σ_π of the preceding paragraph both exist.*

Let us now write the function $u(x)$ in the form

$$u(x) = u(a) + P(x) - N(x),$$

where $P(x)$ and $N(x)$ are respectively the positive and the negative variation of $u(x)$ on (ax); and by $U(x)$ denote the total variation $P(x) + N(x)$. Then $P(x)$, $N(x)$, $U(x)$ are continuous at the points of continuity of $u(x)$.

A sufficient condition for the existence of the integral of $f(x)$ as to $u(x)$ from a to b is the existence of the integral of $f(x)$ as to $P(x)$ and as to $N(x)$. Sufficient to this is the existence and equality of the upper and lower integrals of $f(x)$ as to $P(x)$ and as to $N(x)$. In case $u(x)$ is continuous or $f(x)$ is continuous at the discontinuities of $u(x)$, these upper and lower integrals surely exist and a sufficient condition for their equality is that the sums

$$\sum_{i=1}^{n}(M_i-m_i)\{P(x_i)-P(x_{i-1})\} \text{ and } \sum_{i=1}^{n}(M_i-m_i)\{N(x_i)-N(x_{i-1})\}$$

shall have the limit zero as δ approaches zero. Hence a sufficient condition for the existence of the integral of $f(x)$ as to $u(x)$ from a to b is that

$$\lim_{\delta=0}\sum_{i=1}^{n}(M_i-m_i)\{U(x_i)-U(x_{i-1})\} = 0. \tag{1}$$

Bliss (l. c., p. 634, ll. 1–10) has shown that a necessary condition for the existence of the integral is that

$$\lim_{\delta=0}\sum_{i=1}^{n}(M_i-m_i)\,|u(x_i)-u(x_{i-1})| = 0. \tag{2}$$

From this necessary condition if follows readily that $f(x)$ must be continuous at the points of discontinuity of $u(x)$ (see Bliss, l. c., p. 636, ll. 7–17). If we write $u(x) = v(x) + j(x)$ (Bliss, p. 636, ll. 1–7), where $j(x)$ is the function of 'jumps' of $u(x)$, it may be shown (Bliss, p. 636, ll. 18–36) that the integral of $f(x)$ as to $j(x)$ exists whenever that as to $u(x)$ exists. In the same way it may be shown that the integral of $f(x)$ as to $J(x)$ must also exist, where $J(x)$ is the total variation of $j(x)$ on the interval (ax). Hence a necessary condition for the existence of the integral of $f(x)$ as to $u(x)$ is the existence of the integral of $f(x)$ as to $v(x)$.

We propose to show next that the existence of the integral of $f(x)$ as to $u(x)$ implies that of $f(x)$ as to $U(x)$. In view of the results of the

preceding paragraph and of the fact that $U(x)$ is obviously equal to $V(x) + J(x)$, where $V(x)$ is the total variation of $v(x)$ on (ax), it is obviously sufficient to prove this for the case when $u(x)$ is a continuous function. Now, when $u(x)$ is continuous, we have (Vallée Poussin, *Cours d'Analyse*, vol. 1, 3rd edn, p. 73)

$$\lim_{\delta=0} \sum_{i=1}^{n} | u(x_i) - u(x_{i-1}) | = U(b), \tag{3}$$

with a like relation when b is replaced by x and the interval (ab) by the interval (ax). Since no $| u(x_i) - u(x_{i-1}) |$ is greater than the corresponding difference $U(x_i) - U(x_{i-1})$ and since the sum of the latter differences, for $i = 1, 2, \ldots, n$, is $U(b)$, we see that

$$\lim_{\delta=0} \sum_{i=1}^{n} [U(x_i) - U(x_{i-1}) - | u(x_i) - u(x_{i-1}) |] = 0 \tag{4}$$

and that no bracketed term here is negative. Hence for every ϵ there exists a δ_1 such that the i^{th} term ($i = 1, 2, \ldots, n$) of the sum in (4) is less than ϵ when $\delta < \delta_1$. Hence, for such δ, we have

$$0 \leqq \sum_{i=1}^{n} (M_i - m_i)[U(x_i) - U(x_{i-1}) - | u(x_i) - u(x_{i-1}) |] < (M - m)\epsilon \tag{5}$$

Hence, as δ approaches zero the sum in (5) approaches zero as a limit. This result and relation (2) imply (1). But the latter is sufficient to the existence of the integral of $f(x)$ as to $U(x)$, and indeed as to $u(x)$.

We are thus led to the following theorem:

THEOREM II. *If $u(x)$ is of bounded variation and $f(x)$ is bounded on (ab), then a necessary and sufficient condition for the existence of the integral of $f(x)$ as to $u(x)$ is the existence of the integral of $f(x)$ as to $U(x)$.*

Since (1) and (2) are identical when $u(x) = U(x)$ we now have readily the following theorems:

THEOREM III. *A necessary and sufficient condition for the existence of the integral of the bounded function $f(x)$ as to the function $u(x)$ of bounded variation is that the upper and lower integrals of $f(x)$ as to the total variation function $U(x)$ of $u(x)$ shall exist and be equal.*

THEOREM IV. *A necessary and sufficient condition for the existence of the integral from a to b of the bounded function $f(x)$ as to the function $u(x)$ of bounded variation is that the total oscillation of $f(x)$ as to $u(x)$ from a to b shall be zero, that is, that*

$$\lim_{\delta=0}\sum_{i=1}^{n}(M_i - m_i)[U(x_i) - U(x_{i-1})] = 0.$$

THEOREM V. *In order that a bounded function $f(x)$ shall be integrable from a to b as to a function $u(x)$ of bounded variation, it is necessary and sufficient that the interval (ab) may be divided into partial intervals so that the total variation of $u(x)$ in those in which the oscillation of $f(x)$ is greater than an arbitrarily preassigned positive number ω shall also be as small as one wishes.*

THEOREM VI. *If $u(x)$ is of bounded variation and $f(x)$ is bounded on the interval (ab), then a necessary and sufficient condition for the existence of the integral of $f(x)$ as to $u(x)$ from a to b is that the total variation of $u(x)$ on the set D of discontinuities of $f(x)$ shall be zero (Theorem of Bliss).*

Of the four preceding theorems the only one needing further proof is the last. [For the definition of the total variation of $u(x)$ on a set of points, see Bliss, l. c., p. 633, ll. 12–19.] Let $\epsilon_1, \epsilon_2, \epsilon_3, \ldots$ be a sequence of positive numbers decreasing monotonically toward zero, and let $D_1, D_2, D_3, \ldots$ be the closed set of points at which the oscillation of $f(x)$ is $\geqq \epsilon_1$, $\geqq \epsilon_2$, $\geqq \epsilon_3, \ldots$ Then the set D of discontinuities of $f(x)$ is the limit of the set D_n when n is indefinitely increased. Now if $f(x)$ is integrable as to $u(x)$ we have seen that the interval (ab) may be divided into partial intervals so that the total variation of $u(x)$ on those in which the oscillation of $f(x)$ is greater than an arbitrarily preassigned positive number shall be as small as one pleases; and this implies that the total variation of $u(x)$ on D_n, and hence on D, is zero. Again, if the total variation of $u(x)$ on D is zero so is it on D_n for every n; and hence $f(x)$ is integrable as to $u(x)$ since it is such that the interval (ab) may be divided into partial intervals so that the sum of those in which the oscillation is greater than an arbitrarily preassigned positive number shall be as small as one pleases.

TRANSFORMATIONS OF CYCLIC SYSTEMS OF CIRCLES

By L. P. Eisenhart

Department of Mathematics, Princeton University

Communicated by E. H. Moore, October 9, 1919

When two surfaces, S and $\bar{S}$, are applicable, there is a unique conjugate system on S which corresponds to a conjugate system on $\bar{S}$. Denote these conjugate systems, or *nets*, by N and $\bar{N}$ respectively.

The cartesian coördinates x, y, z, of N and $\bar{x}$, $\bar{y}$, $\bar{z}$ of $\overline{N}$ are solutions of an equation of the form

$$\frac{\partial^2\theta}{\partial u\partial v} = a\frac{\partial\theta}{\partial u} + b\frac{\partial\theta}{\partial v}, \tag{1}$$

which we call the common *point equation* of N and $\overline{N}$.

Let M and $\overline{M}$ be corresponding points of N and $\overline{N}$. With M as center describe a sphere whose radius is the distance from $\overline{M}$ to the origin. Let Σ_1 and Σ_2 be the sheets of the envelope of the spheres as M moves over N, and let μ_1 and μ_2 be the points of contact with Σ_1 and Σ_2 of the sphere with center at M. The null spheres with centers at μ_1 and μ_2 meet the tangent plane of N in a circle C. These ∞^2 circles from a cyclic system, that is they are orthogonal to ∞^1 surfaces.[1]

If h and l are any pair of solutions of the system

$$\frac{\partial h}{\partial v} = (l-h)\,a, \qquad \frac{\partial l}{\partial u} = (h-l)\,b, \tag{2}$$

the functions x', y', z', defined by the quadratures

$$\frac{\partial x'}{\partial u} = h\frac{\partial x}{\partial u}, \frac{\partial x'}{\partial v} = l\frac{\partial x}{\partial v}; \frac{\partial y'}{\partial u} = h\frac{\partial y}{\partial u}, \frac{\partial y'}{\partial v} = l\frac{\partial y}{\partial v}; \frac{\partial z'}{\partial u} = h\frac{\partial z}{\partial u}, \frac{\partial z'}{\partial v} = l\frac{\partial z}{\partial v}, \tag{3}$$

are the coördinates of a net N' parallel to N, and the functions $\bar{x}'$, $\bar{y}'$, $\bar{z}'$, defined by

$$\frac{\partial \bar{x}'}{\partial u} = h\frac{\partial \bar{x}}{\partial u}, \frac{\partial \bar{x}'}{\partial v} = l\frac{\partial \bar{x}}{\partial v}; \frac{\partial \bar{y}'}{\partial u} = h\frac{\partial \bar{y}}{\partial u}, \frac{\partial \bar{y}'}{\partial v} = l\frac{\partial \bar{y}}{\partial v}; \frac{\partial \bar{z}'}{\partial u} = h\frac{\partial \bar{z}}{\partial u}, \frac{\partial \bar{z}'}{\partial v} = l\frac{\partial \bar{z}}{\partial v}, \tag{4}$$

are the coördinates of a net $\overline{N}'$ parallel to $\overline{N}$. Moreover, the nets N^2 and $\overline{N}'$ are applicable, and consequently the function $\theta' = \Sigma x'^2 - \Sigma\bar{x}''$ is a solution of the point equation of N' and $\overline{N}'$. By the quadratures

$$\frac{\partial\theta}{\partial u} = \frac{1}{h}\frac{\partial\theta'}{\partial u}, \quad \frac{\partial\theta}{\partial v} = \frac{1}{l}\frac{\partial\theta'}{\partial v}$$

we obtain a solution θ of (1).

The functions x_1, y_1, z_1, and $\bar{x}_1$, $\bar{y}_1$, $\bar{z}_1$, defined by equations of the form

$$x_1 = x - \frac{\theta}{\theta'}x', \qquad \bar{x}_1 = \bar{x} - \frac{\theta}{\theta'}\bar{x}' \tag{5}$$

are the cartesian coördinates of two applicable nets N_1 and $\overline{N}_1$, which are T transforms of N and $\overline{N}_1$ respectively; a net and a T transform

are such that the developables of the congruence of lines joining corresponding points of the two nets meet the surfaces on which the nets lie in these nets.[2]

Since N_1 and $\overline{N}_1$ are applicable nets, we can obtain a cyclic system of circles C_1, in the manner described in the second paragraph. Hence each pair of solutions of (2) determines a transformation of the cyclic system of circles C into a cyclic system of circles C_1, such that the nets enveloped by the planes of the circles C and C_1 are in the relation of a transformation T. Moreover, it can be shown that corresponding circles lie on a sphere. We say that two such cyclic systems are in relation T.

Darboux[3] has stated the results of the second paragraph in the following form: If a surface $\overline{S}$ rolls over an applicable surface S, and Q is a point invariably fixed to S, the isotropic generators of the null sphere with center Q meet the plane of contact of S and $\overline{S}$ in points of a circle C which generate the surfaces orthogonal to the cyclic system of circles C. Making use of these ideas, we give the following interpretation of the above transformations of cyclic systems:

If N and $\overline{N}$ are applicable nets, and N_1 and $\overline{N}_1$ are respective T transforms by means of (5), *where $\theta' = \Sigma x'^2 - \Sigma \overline{x}'^2$, the cyclic systems in which a point sphere invariably bound to $\overline{N}$ and $\overline{N}_1$ meets the planes of contact, as $\overline{N}$ rolls on N and $\overline{N}_1$ on N_1, are in relation T.*

It can be shown that the two surfaces orthogonal to these respective cyclic systems which are generated by the points where an isotropic generator of the null sphere meets the plane of contact are in the relation of a transformation of Ribaucour, that is, these surfaces are the sheets of the envelope of a two parameter family of spheres and the lines of curvature on the two sheets correspond.

[1] Guichard, *Ann. Sci. Ec. norm., Paris,* (Ser. 3), 20, 1903, (202).

[2] Eisenhart, these PROCEEDINGS, 3, 1917, (637).

[3] *Leçons sur la théorie générale des surfaces,* vol. 4, 123.

NEW MEASUREMENTS OF THE VAPOR PRESSURE OF MERCURY

By Alan W. C. Menzies

Department of Chemistry, Princeton University

Communicated by W. A. Noyes, October 14, 1919

Writing in 1908, Laby[1] remarked "The vapor pressure of mercury is intrinsically important; it has been determined for a wider range of temperatures than that of any other substance. . . . Yet the greatest—and, it should be added, unnecessary—disagreement is to be found in the current values for this vapor pressure, nor is there any table combining all the existing observations." Laby thereupon proceeded to collate the best observations available at that time and published a weighted average table.

In 1910 Smith and I published[2] the results of 43 direct observations of the vapor pressure of mercury by a static method over the temperature range 250° to 435°. The pressure measurement was immensely facilitated by the use of the newly-devised 'static isoteniscope;'[3] and a good form of platinum resistance thermometry was available. The average divergence in temperature of an individual observation from the smooth curve given by the equation

$$\log p = 9.9073436 - 3276.628/\theta - 0.6519904 \log \theta, \qquad \text{(R)}$$

whose constants were chosen to fit our results, was 0.050°. Because of the rather high degree of consistency thus attained, we were encouraged to exterpolate the curve in both directions and found that the values so obtained agreed remarkably well with the average experimental findings of those who had worked either above or below our temperature region.[4] A critical discussion of our own and of the older work on this subject, with tabular comparisons, may be found in the paper referred to.[2]

In 1909, Knudsen developed a relationship[5] connecting the weight of gas passing through tubes containing pierced diaphragms with the difference of gas pressure at the ends, the density of the gas, the resistance of the tube and diaphragm and the time of flow. He applied this to measure the vapor pressure of mercury[6] from 0° to 50°, above which temperature his relationship failed to hold. From the lower to the upper end of this range, his results run from 10.9 to 6.2% respectively in pressure lower than the values given by extrapolation of our 1910 results

by the above equation. This corresponds to temperature discrepancies of 1° at 0° and 0.8° at 50°. One regrets that such an elegant method is not more direct. Later, Knudsen[7] applied his admirable 'absolute manometer,' under certain difficulties, to measure this vapor pressure from $-10°$ to $+24.4°$, and obtained results ranging from about 5% higher to about 10% lower in pressure, at the respective ends of this range, than the values furnished by our equation.

In 1913 Villiers[8] obtained a series of values from 60° to 100° which ran higher than ours by 2.7% of our calculated pressure at 60° to 7.5% at 100°.

In 1914, Haber and Kirschbaum,[9] adopting a suggestion of Langmuir's,[10] measured the damping of a quartz fiber vibrating in mercury vapor at 20°, and obtained a single value by an independent method. This value lies 3.8% in pressure below that given by our equation, or, otherwise, differs by 0.41° for the same pressure.

In glancing over the results obtained by these various workers, here stated baldly without criticism, one is inclined to believe that our equation of 1910 continues to stand the test of time by averaging their scattered results as well as can be hoped for. At the same time one is impressed by the pre-war activity in this field, especially in the lower temperature ranges. It appeared, therefore, to be of interest to obtain, by a method as direct as possible, new measurements below 250°, the lowest point of the range studied in 1910; and, in the work here described I have extended that range from 250° to 120°.

As the static isoteniscope is not especially well suited for the measurement of the rather low pressures with which we are here concerned, an entirely new plan was adopted. Two McLeod gauges of suitable capacities were calibrated and sealed to a central pressure reservoir. The gauges, which were operated by gas pressure and without rubber connexions, contained purified mercury. The reservoir and connected gauges could be charged with dry hydrogen at any pressure desired, and sealed off by mercury. The smaller of the two gauges was completely immersed in a riotously stirred oil bath whose temperature was measured by a mercurial thermometer whose thread was all submerged. The corrections of this thermometer were known[11] to tenths of a degree. The other, larger, gauge and the reservoir were maintained in baths at room temperature. On operating the two gauges simultaneously, different pressure readings were obtained, due to the condensation of the mercury vapor that contributed part of the total pressure in the hot gauge; and from this difference in reading the vapor pressure of mercury at the

temperature of the hot gauge could be computed in an obvious manner, if Dalton's law were assumed.

For details of the experiments and mode of reduction of the results, as well as for tabular comparison with the heretofore existing data, a paper may be consulted that will appear elsewhere.[12] It should, however, perhaps be stated here that, at the pressures and compressions used, the neglect of the known divergence of hydrogen from the simple gas laws would cause an error very much smaller than other errors of the experiment. Equlibrium in the hot gauge was, apparently, always established sooner than a measurement could be made. A small correction was applied to correct for the vapor pressure of mercury in the cold gauge. A possible systematic error may arise from the fact that the apparent vapor pressure of a liquid in contact with a gas is not in all cases identical with the vapor pressure at the same temperature of the same liquid in a vacuum. Although this fact was observed by Regnault,[13] it has apparently escaped the notice of many users of the gas-current method for determining small vapor pressures. The error is the greater the more soluble the gas in the liquid, although it is not conditioned solely by this solubility as such. With hydrogen gas at atmospheric pressure, the order of this error,[14] in the case of liquids in which hydrogen shows marked solubility such as ether, carbon disulphide and acetone, is about 1% at 30°; with mercury, in which no evidence of solution was found, and at the temperatures here in question, the order of the error might be expected to be much smaller, were it not that low gas pressures may be especially prejudicial. Further study of this possible error is planned, and one can only say, with T. W. Richards, that these results are of but a preliminary character.

The brief table below summarizes the results obtained. The 'calculated' values in the third column are those derived from the Kirchoff-Rankine-Dupré equation (R), whose constants were given above; the values in the fifth column are likewise derived from this equation. In the last column, P stands for the pressure, in millimeters of mercury, of hydrogen in the reservoir.

Vapor pressures of mercury

TEMPERATURE	VAPOR PRESSURE IN MILLIMETERS OF MERCURY AT 0°		$\frac{\text{FOUND}}{\text{CALCULATED}} \times 100$	PERCENTAGE CHANGE OF PRESSURE PER 0.1°	P
	Found	Calc.			
191.5°	13.02	13.02	100.00	0.34	14.28
150.0°	2.802	2.811	99.67	0.39	3.564
121.8°	0.829	0.823	100.75	0.47	1.738

Since the values thus found for the vapor pressures of mercury are obtained, broadly, by subtracting the reading of the smaller and less accurate hot gauge from those of the larger and more accurate cold gauge, it will be seen that the greatest precision (so far as gauge readings are concerned) is to be expected when the vapor pressure most closely[15] approaches P. This is borne out by the results as tabulated.

It is evident that the results found agree with those calculated almost as closely as can be expected, even having regard to the temperature error (0.1°) alone. For this reason one may state, meantime, that the equation (R) may be applied, without alteration of its constants, to give the vapor pressure of mercury over the range 120° to 250° as well as from 250° to 435°, as previously found. A table of the values given by this equation for every two degrees from 0° to 458° has already been published.[2] Further experimental extension of the temperature range, as well as greater precision, may be attempted later, should facilities be available, either by this or else by another method shortly to be described.

It is, perhaps, of interest to consider the upper temperature limit of this vapor pressure curve, namely the critical temperature of mercury. This has been the subject of much speculation.[16] Recently, in 1916 and 1917, J. J. van Laar, employing certain theoretical considerations, has computed 969°C.[17] and later 899°C.[18] for the critical temperature; while, in 1918, E. Ariès,[19] from other considerations, arrives at 1077°C. In such cases, a little experiment is worth much theory. At the Minneapolis meeting of the American Chemical Society in 1910, I described[20] some experiments in which I had heated mercury in a thick-walled capillary tube of quartz glass of 0.2 mm. bore, and continued to see, through a telescope, liquid and gas phases distinct up to 1275°C. At this temperature the meniscus disappeared; but this was found to be due, not to the attainment of the critical temperature, but to the slight blowing out of the quartz capillary with consequent enlargement of its volume. The tube was never ruptured, but the very viscous liquid quartz glass had yielded somewhat to relieve the rather high pressure of several hundred atmospheres. In 1912, Koenigsberger,[21] using a quartz capillary of dimensions very similar to mine, reported that he had reached the critical temperature about 1270°C; but I was able to point out[22] the cause of Koenigsberger's mistake. Owing to lack of facilities, I have not been able to develop this work further; but this has now been done, by the same capillary quartz tube and telescope method, by Bender[23] at Freiburg, who finds that the critical temperature lies between 1500° and 1600°C.

[1] Laby, *London, Phil. Mag.*, (Ser. 6), 16, 1908, (789).

[2] Smith and Menzies, *J. Amer. Chem. Soc.*, 32, 1910, (1434).

[3] Smith and Menzies, *Ibid.*, 32, 1910, (1412).

[4] Pfaundler, *Leipsig, Ann. Physik*, (Ser. 3), 63, 1897, (36); Morley, *Phil. Mag.*, (Ser. 6), 7, 1904, (662); Hertz, *Ann. Physik.*, (Ser. 3), 17, 1882, (193); Regnault, *Paris, Mém. Acad. Sci.*, 21, 1847, (30, 502); 26, 1862, (506); Ramsay and Young, *J. Chem. Soc.*, 49, 1886, (37); Young, *Ibid.*, 59, 1891, (629); Cailletet, Colardeau and Rivière, *Paris, C. R. Acad. Sci.*, 130, 1900, (1585), etc.

[5] Knudsen, *Ann. Physik*, 28, 1909, (1002).

[6] Knudsen, *Ibid.*, 29, 1909, (184).

[7] Knudsen, *Ibid.*, 32, 1910, (809).

[8] Villiers, *Ann. Chim. Phys.*, (Ser. 8), 30, 1913, (588).

[9] Haber and Kirschbaum, *Zr. Elektrochemie.* 20, 1914, (301).

[10] Langmuir, *J. Amer. Chem. Soc.*, 35, 1913, (105).

[11] Reichsanstalt certificate dated 1914.

[12] Menzies, *J. Amer. Chem. Soc.*, probably November, 1919.

[13] Regnault, *Ann. phys chim.*, (Ser. 3), 15, 1845, (129); *Mém. Acad. Sci.*, 26, 1862, (679).

[14] Campbell, *Trans. Faraday Soc.*, 10, 1914, (197).

[15] If these two pressures were very close, and if the gauge tubes were too narrow, the total pressure within the hot gauge might be appreciably greater than the pressure within the reservoir.

[16] Happel, *Ann. Physik*, (Ser. 4) 13, 1904, (340), etc.

[17] Van Laar, *Verslag. Akad. Wetenschappen*, 24, 1916, (1635).

[18] Van Laar, *Proc. Acad. Sci. Amsterdam*, 20, 1917, (138).

[19] Ariès, *Paris, C. R. Acad. Sci.*, 166, 1918, (334).

[20] "A Lower Limit for the Critical Temperature of Mercury," presented before the Division of Inorganic and Physical Chemistry, 1910.

[21] Koenigsberger, *Chem. Ztg.*, 135, 1912, (1321).

[22] Menzies, *J. Amer. Chem. Soc.*, 35, 1913, (1065).

[23] Bender, *Physik. Zr.*, 16, 1915, (246); 19, 1918, (410).

COHESION, INTERNAL PRESSURE, ADHESION, TENSILE STRENGTH, TENSILE ENERGY, NEGATIVE SURFACE ENERGY, AND MOLECULAR ATTRACTION

By William D. Harkins

Kent Chemical Laboratory, University of Chicago

Communicated by W. A. Noyes, October 14, 1919

Work of adhesion.—For the past seven years I have been engaged in the study of molecular attraction from a new point of view, as represented by a thermodynamic equation which does not seem to have been developed before, in spite of the fact that it is exceedingly simple. It gives what may be called the *total adhesional energy* (E_A), or the total energy involved in the approach of two unlike surfaces. The equation developed by Dupré[1] in 1869 gives the *adhesional work* (W_A), which is

the amount of work involved in the process, which is equal to the decrease of free energy during the approach:

$$W_A = -\Delta\gamma = \gamma_1 + \gamma_2 - \gamma_{1,2} \qquad (1)$$

where γ_1 and γ_2 give the free energy of the two unlike surfaces before their approach, and $\gamma_{1,2}$ is the free energy of the interface.

It has been shown by Hardy,[2] and by Harkins, Brown, and Davies,[3] that the adhesional work between organic liquids and water is represented by values which are characteristic for each class of organic compounds. Our own work indicated also that at the interface between water and an organic liquid, any groups in the latter which contain oxygen, nitrogen, or either triple or double bonds, are oriented toward the aqueous phase, while the hydrocarbon radical turns toward the organic liquid. This orientation is of great importance in connection with either the adhesional work or the adhesional energy, since its result is to increase the adhesion between such an organic liquid and water. Further facts which are in accord with this theory of orientation, also developed by Langmuir,[4] are presented later in this paper, in the section on negative surface energy.

Work of surface cohesion.—Since γ is zero when the two liquids become identical, the Dupré equation reduces to

$$w_{SC} = -\Delta\gamma = 2\,\gamma \qquad (2)$$

where W_{SC} represents the work of the surface cohesion. The free surface energy (γ) is usually measured in ergs per square centimeter. It is evident that W_{SC} is the work necessary to break a bar of liquid or solid with a cross section of 1 sq. cm., in such a way as to give two plane surfaces of 1 sq. cm. each, that is the break must occur perpendicular to the long axis of the bar. This amount of work is clearly that which would be done in the determination of the tensile strength of a steel or other metal bar, if the test could be carried out under *ideal* conditions, which are that the bar during the test must remain of uniform cross section, and the break must occur in such a way as to give two surfaces which are as closely plane as the surface of a liquid, though in the actual tests this condition is not met, since the bar is much distorted. The work of surface cohesion may therefore be called the *tensile work* (W_T). This is equal to the *tensile force* integrated through the distance necessary to pull the two surfaces completely apart, or

$$W_T = \int_{S_0}^{\infty} F_T\,ds = 2\gamma$$

Energy of adhesion.—The equation developed by me gives the *total adhesional energy* (E_A), and is as follows:

$$E_A = -\Delta E_S = (\gamma_1 + l_1) + (\gamma_2 + l_2) - (\gamma_{1,2} + l_{1,2}) \qquad (4)$$

The energy of adhesion is equal to the *total surface attraction*, (π'') due to molecular attraction, integrated through the distance necessary to pull the two surfaces completely apart, or

$$E_A = \int_{S_0}^{\infty} \pi'' \, ds = E_1 + E_2 - E_{1.2} \qquad (5)$$

The only experimental results which have been obtained in connection with equation 4 are those of experiments by Dr. E. C. H. Davies and Mr. Y. C. Cheng as carried out under my direction. This seems to be the first work aside from that on the heat of adsorption, which bears very directly on the molecular attraction between *unlike* substances.

Energy of surface cohesion.—When the two liquids become identical equation 4 reduces to the form

$$E_{SC} = -\Delta E_S = 2\,(\gamma + l) = 2\,E_S \qquad (6)$$

It is evident that the total energy of surface cohesion is equal to the total tensile energy (E_T), or

$$E_T = -\Delta E_S = 2E_S \qquad (7)$$

Thus the *total energy* used in pulling a bar of unit cross section apart in such a way as to form two unit plane surfaces, is equal to twice the total surface energy. It is also equal to the force of surface cohesion (F_{SC}) integrated through the distance necessary to pull the two surfaces completely apart, or

$$E_T = \int_{S_0}^{\infty} F_{SC} \, ds$$

Energy of cohesion.—The *cohesional energy* of a solid or a liquid may be defined as the energy which would be liberated in the formation of the solid or liquid from its individual molecules, the molecules in the initial state being placed so far apart that they do not attract each other appreciably. This is equal to the latent heat of vaporization plus the heat absorbed in the expansion of the vapor until it becomes very dilute, minus the external work of vaporization, or it is equal to the internal latent heat of vaporization when the liquid is vaporized at a low pressure.

Internal pressure or cohesion.—The internal pressure of a liquid or a solid has been defined as the rate of transfer of momentum across a unit plane area inside the liquid or solid; and the average force of attraction across this unit area, which is numerically equal to the internal pressure, is the cohesional force, or the *cohesion*.

While the work and total energy of adhesion and of surface cohesion, and the energy of cohesion, may all be obtained from experimental results by the use of simple and exact thermodynamic equations, this is not true of the internal pressure or cohesion. As a matter of fact, there is at present no known means of calculating the cohesion, but there are many methods, which do not agree among themselves, of calculating from inexact equations, values which for various liquids are supposed, when arranged in order of magnitude, to lie in the same order in general as the cohesions themselves. In fact, the cohesion is often defined as equal to a/v,[2] the pressure correction term in van der Waals equation. However, since this equation is far from exact in its application to liquids, it is obvious that the cohesion calculated cannot represent at all accurately the internal pressure.

Molecular attraction.—All of the phenomena thus far considered in this paper may be considered as due to the attraction between the molecules in a liquid or a solid. It is customary to consider the molecules as spheres or as points, with the attractive forces dependent upon the distance between the molecules alone, when they are all alike. It has been shown by Harkins, Brown, and Davis,[3] by a measurement of the amounts of energy involved, and by Langmuir[4] by a less direct method, that the forces around different parts of a molecule may be very different in magnitude. Thus in the case of organic compounds the forces are very much higher around any groups containing oxygen, nitrogen, triple, or double bonds, than they are around the hydrocarbon chains. The investigations of Harkins, Grafton and Ewing (these PROCEEDINGS, **5**, 1919, 571) show that if organic substances are arranged according to the magnitude of their adhesional surface work toward mercury, they are not so arranged with respect to water. In this respect the adhesional forces seem to have something of the specific nature which indicates chemical action, and it is well known that the recent work on crystal structure demonstrates that such crystals as those of diamond and of graphite are held together by primary valence bonds. Langmuir[4] considers all cohesional and adhesional forces as chemical, while van Laar[8] has recently published the results of an extensive series of calculations which show that the square root of van der Waals' con-

stant of attraction (a) is additive, and therefore comes to the conclusion that all such forces are physical. The calculations of Einstein,[5] Kleemann,[6] and of Harkins and Clark,[7] have also given coefficients of atomic attraction which are moderately exact constants. Since all of these facts when considered together make it probable that cohesional forces are often less specific than those involved in ordinary chemical reactions, while in many cases they are the same valence forces, it seems to me preferable to use neither of the two words, physical or chemical, and to consider that cohesion is due to electrical and magnetic, or electromagnetic forces, which are probably largely electrical. In a paper [3c] on "An Electromagnetic Hypothesis of the Kinetics of Heterogeneous Equilibrium, the Structure of Liquids, and Cohesion" I have already traced a connection between cohesion and the completeness of the outer or valence shell, of electrons in the atom or the molecule. The cohesion decreases as the completeness of the outer shell of electrons in the molecule increases.

On the relation between cohesion and cohesional and adhesional work and energy.—A number of attempts have been made to calculate the *cohesion*, which is the cohesional force per unit area of the cohesional pressure, from the *cohesional surface work*, and this attempt has been more or less justified by the fact that the values thus obtained, while not good in numerical agreement with those given by a/v^3, lie on the whole in the same relative order. Such calculations have been made by Mathews,[9] and by Hildebrand,[10] but neither of them have shown to what extent the cohesion and the cohesional surface work are related. The term a/v^2 may be said to represent, more or less imperfectly it is true, the total effect of the molecular attraction in decreasing the external pressure, which decrease is the cohesion. The cohesional surface work, on the other hand does not represent the total effect of the molecular attraction, even as it acts in a surface, since the molecules move into the surface not only by means of the energy which is contributed in the form of work, but also by means of the utilization of the kinetic energy of molecular motion, or the latent heat of the surface. Thus the formation of the surface of a pure liquid, with the exception of a few liquids in which liquid crystal formation is involved, is always accompanied by cooling. It is, therefore, the *total cohesional surface energy* and *not the related work*, which represents the total effect of the molecular attraction, or the cohesional effect.

It may seem remarkable, from the point of view of the last paragraph, that the calculation of even the relative cohesion of liquids[10] from a

very simple equation, $\gamma/v^{1/3}$, where V is the molecular volume, should give results which lie in somewhat the correct order. This is undoubtedly because, as shown by Harkins, the contribution of the kinetic energy of a molecule to the total energy of the surface, is on the whole, independent of the nature of the molecule,—at least for such substances as have been used in the calculation of cohesion,—and is dependent on the temperature alone. Therefore, so long as the molecular volume is nearly the same, and the orientation of the molecules in the surface is not an important factor, at any definite temperature the latent heat of the surface is nearly independent of the nature of the substance, so that when substances are arranged in the order of their cohesional surface work or their free surface energy, they are also arranged in the general order of their total surface energy.

A second method of calculating the cohesion from the cohesional work or free surface energy, is based on the well-known assumption of Stefan,[11] that the average work involved in bringing a molecule to the surface of a liquid, has one-half of the energy value involved on the average in its complete vaporization. It is obvious from Stefan's paper, that his principle does not involve the *work* but the *total surface energy*, which is supposed to be one-half of the latent heat of vaporization. That this rule is far from true is indicated by the results of extensive calculations by Mr. L. E. Roberts and myself, which show that the fractional contribution of the surface energy toward the complete vaporization increases with the temperature, with a normal range of from one-third at lower temperatures to 0.8 or more as the critical temperature is approached, though the higher values are uncertain. Thus a molecule which at a high corresponding temperature passes from the body of the liquid into the surface, goes, in a *fractional sense with reference to energy*, much more nearly into the vapor state than when the corresponding temperature is low.

Negative surface energy.—The phenomenon of *negative surface energy* was first discovered two years ago by Dr. E. C. H. Davies and me, but has not been announced previous to this time in print. Not only Donnan, but also Tolman and Wolfgang Ostwald, have *assumed* the existence of a negative surface tension or free surface energy. My own investigations have convinced me that the discovery of a negative free surface energy for a plane, uncharged surface is improbable, though it is quite likely that there is such a phenomenon in the case of highly curved phase boundaries. What we have to announce here is the discovery of *a negative total surface energy for a plane, uncharged sur-*

face. Thus, contrary to the rule found in the past, the surface or interface between octyl alcohol and water ***gives off energy when it is extended***, but, nevertheless, the ***surface cannot be formed without the expenditure of work***. The apparent contradiction is due to the fact that while the molecular motion aids in the formation of an ordinary surface, in the case of the interface under discussion the molecular motion hinders the extension of the surface. This is in accord with the theory presented in our earlier papers, and by Langmuir, that at such an interface there is an orientation of the molecules, since the molecular motion reduces the extent of the orientation.

When the interface between octyl alcohol and water is pulled out adiabatically there is thus a heating of the surface, while an ordinary surface is cooled, so that the potential energy of the molecules is decreased by passing into the alcohol-water interface. The negative surface energy, is, it is true, very small, with a numerical value of two ergs. per sq. cm., while the free surface energy is 8.33, and the latent heat is -10.3 ergs. In contrast with this, it is found that the total surface energy of the hexane-water interface is not only positive but large, with a value of 66.5 ergs. These relations are of considerable interest, and their bearing on interfacial structure, which is of great importance in physiology, will be discussed in a later paper in the *Journal of the American Chemical Society*.

[1] Dupré, *Theorie Mécanique de la Chaleur*, Paris, 1869, p. 69; Lord Rayleigh, *London, Phil. Mag.*, (5) 30, 1890, (461).

[2] Hardy, *London, Proc. Roy. Soc.*, **86B**, 1911, (634).

[3] (a). Harkins, Brown, and Davies, *J. Amer. Chem. Soc.*, 39, 1917, (354–64).
(b). Harkins, Davies, and Clark, *ibid.*, 541–96.
(c). Harkins and King, *ibid.*, 41, 970–92, (1919), these PROCEEDINGS, 5, 1919, (152–9).

[4] Langmuir, *J. Amer. Chem. Soc.*, 39, 1917, (1848–1906), these PROCEEDINGS, 3, 1917, (251–7); abstract in Met. Chem. Eng., 15, 1916, (468). Fraenkel, *Phil. Mag.*, 33, 1917, (297–322).

[5] Einstein, *Leipsig, Ann. Physik.*, 4, 1901, (513).

[6] Kleemann, *Phil. Mag.*, 18, 1909, (39, 491, 901).

[7] Harkins and Clark, unpublished calculations.

[8] van Laar, *Zs. anorg. Chemie*, 104, 1916, (57–156).

[9] Mathews, *J. Physic. Chem.*, 17, (603–28).

[10] Hildebrand, *J. Amer. Chem. Soc.*, 41, 1919, (1067–80).

[11] *Leipsig, Ann. Physik*, 29, 1886, (655).

THE ADHESION BETWEEN MERCURY, WATER, AND ORGANIC SUBSTANCES, AND THE FUNDAMENTAL PRINCIPLES OF FLOTATION

By William D. Harkins

Kent Chemical Laboratory, University of Chicago

Communicated by W. A. Noyes, October 14, 1919

The primary purpose of this investigation was to determine the effects of the molecular attraction at the surface of a metal, and to compare these effects with those of the surface of an oxygen compound such as water. Since the flotation process depends upon the preferential wetting and adhesion of gas films on metals, including the heavy sulphides, etc., on the one hand and silica and similar substances on the other, the general principles learned in connection with such a study, should be fundamental for the study of the process. I have been informed by Dr. E. C. Bingham that the adhesion between organic substances and metals is also fundamental with respect to the characteristics of lubricants.

The equation of Dupré[1] and that of Harkins, both of which are based on pure thermodynamics, give us the most accurate means for the study of the effects of molecular attraction at surfaces. The equation of Dupré gives the *adhesional work* done during the approach of 1 sq. cm. of one surface to meet the same area of the other. This is numerically equal to the work necessary to pull the two surfaces apart. The work of approach is also equal to the decrease of free surface energy $(-\Delta\gamma)$ during the process which is given by the equation

$$W_A = -\Delta\gamma = \gamma_1 + \gamma_2 - \gamma_{1,2}$$

where γ_1 and γ_2 give the free energy of the two unlike surfaces before their approach, and $\gamma_{1,2}$ is the free energy of the interface.

The equation of Harkins gives the *total energy of approach* (E_A) and this may be called the *total adhesional energy*, as follows:

$$E_A = -\Delta E_S = (\gamma_1 + l_1) + (\gamma_2 + l_2) - (\gamma_{1,2} + l_{1,2})$$

where l represents the latent heat of the surface or interface in ergs per square centimeter. The total adhesional energy is closely related to the molecular surface attraction, while the adhesional work is the tensile force necessary to pull the two surfaces apart, integrated through the distance which they move during separation, but given a negative sign.

The data obtained in this laboratory indicate that the adhesional work between a mercury surface and the surface of an organic liquid is always greater, at least for all substances investigated, than that between the same organic liquid and water, and also greater than that between the organic liquid and itself. This last might be more properly described by the term *cohesional surface work*. A second point of interest is that for about half the organic substances investigated, the *difference* between the adhesional work against mercury and that against water, is nearly constant, with a value between 80 and 92 ergs. This is true for such liquids as the paraffin hydrocarbons, benzene, toluene, the xylenes, carbon tetrachloride, chloroform, and nitrobenzene, so *the work is by no means entirely specific*. On the other hand the adhesional work toward water is specifically high in the case of the alcohols, water and ether, while that toward mercury is very high in the case of the compounds of sulphur, iodine, and bromine, and also oleic acid, which indicates that the specific effects are very marked.

At the interface between organic compounds and water, groups which contain oxygen or nitrogen, double or triple bonds, orient toward the water; at a mercury surface the sulphur, bromine, or iodine, should turn toward the mercury.

In order to illustrate these relations a few of the data obtained in the laboratory are presented in table 1. The experimental work on mercury was done under my direction by Dr. E. H. Grafton and Warren W. Ewing, while that on water was selected from an extensive set of data obtained by Drs. F. E. Brown, G. L. Clark, E. C. H. Davies, Mr. L. E. Roberts, and Mr. Y. C. Cheng.

The well known rule of Antonow[2] states that the interfacial tension between two liquids is equal to the surface tension of the liquid which has the higher surface tension minus that which has the lower surface tension. Where this rule does not hold for an interface organic liquid-water, the discrepancy is explained by Antonow as due to the fact that the interface is between two solutions and not two pure liquids, so the surface tensions used should be those of the solutions or phases, instead of the pure liquids. The Antonow rule sprang from the idea that an interface at the two individual surfaces still exists, but that one acts in a negative sense on the other. This rule is in direct discordance with the theory developed independently by Langmuir and by Harkins, that the molecules in surfaces and interfaces are oriented. This theory need not be discussed here, since the above table clearly proves the Antonow rule to be incorrect for immiscible liquids. Thus his rule if it were applica-

TABLE 1

THE ADHESIONAL WORK BETWEEN MERCURY, WATER, AND ORGANIC LIQUIDS, WITH THE COHESIONAL SURFACE WORK IN ORGANIC SUBSTANCES FOR COMPARISON. (LIQUIDS ARRANGED IN ORDER OF ADHESIONAL WORK TOWARD MERCURY)

(IN ERGS PER SQUARE CENTIMETER)

(1) LIQUID	(2) INTERFACIAL TENSION AGAINST MERCURY	(3) ADHESIONAL WORK AGAINST MERCURY	(4) COHESIONAL WORK AGAINST ITSELF	(5) DIFFERENCE (3) MINUS (4)	(6) ADHESIONAL WORK AGAINST WATER	(7) DIFFERENCE MERCURY MINUS WATER (3–6)
(Air)	465					
Hexane	378	120	36.9	83	40	80
Ethyl ether	379	123	43.6	79	73	50
Octane	375	127	43.5	83	44	83
Carbon tetrachloride	362	150	53.3	97	56	94
Chloroform	357	155	54.3	101	67	88
Benzene	363	146	57.6	88	67	79
Toluene	359	151	58.0	93	67	84
m-Xylene	357	152	58.0	94	64	88
o-Xylene	359	153	58.0	95	67	86
p-Xylene	361	155	54.0	101	64	91
Iso-butyl alcohol	348	155	45.6	109	94	61
Secondary octyl alcohol	348	159				
Octyl alcohol	352	161	55.1	106	92	69
Methylene chloride	341	169	53.0	116	71	98
Ethylidene chloride	337	174	49.2	125		
Nitrobenzene	350	173	86.8	86	91	82
Carbon bisulfide	336	175	62.8	113	56	119
Aniline	341	181	85.2	96	110	71
Water	375	182	145.6	36	145.6	36
Oleic acid	322	191	65.0	136	89.6	101
Ethyl iodide	322	195	56.4	139	63	132
Ethylene bromide	326	197	77.4	120	75	122
Methyl iodide	304	211				
Acetylene tetrabromide	293	230	99.3	131	84	146
Mercury		960	960.0		182.6	(778)

ble to the pure liquids would state that the interfacial tension between mercury and acetylene tetrabromide is 430, while the experimental value is 293, an extremely great deviation. Taking another example at random it is found that the Antonow rule in this form gives the mercury-octyl alcohol interfacial tension as 452, while the experi-

mental value is 352. As a matter of fact, the deviation between the result given by rule and by experiment seldom falls below 90 ergs, and is often higher, which seems to disprove the rule.

In spite of the above objections it must be admitted that Antonow's rule has been of considerable use in connection with phase boundaries between water and organic liquids. It should be realized, too, that the above objections to it involve a certain hypothesis, that is that the amount of the organic liquid dissolved in the mercury is too small to reduce the surface tension by 90 ergs or more when the mercury phase is split apart in such a way as to form surfaces from the interior part of the mercury phase.

Preliminary experiments on the adsorption of benzene and other vapors on a mercury surface show that their positive adsorption is very much greater than on a water surface at the same concentration of vapor. Water present either in the air or in such organic liquids as hexane, benzene, ether, etc., is also adsorbed positively, and the adsorption is very marked even when only a very small amount of water is present. Mercury salts, such as mecurous iodide or bromide, when dissolved in an organic iodide or bromide, are strongly adsorbed. The high adsorption of such substances on mercury as compared with that on water is in accord with my theory that the adsorption increases in general with the rate of drop in the intensity of the electromagnetic intermolecular stray field at the surface of the liquid, since this drop is very much more rapid for mercury than for water. According to the results obtained by F. Schmidt[3] caesium, rubidium, and potassium, when dissolved in mercury are strongly, and sodium, tin, lead, and gold, are slightly positively adsorbed. Negative adsorption is most marked with barium as a solute, and to a lesser extent with strontium, calcium, cadmium, zinc, thallium, and lithium, in decreasing order of effect.

The experimental difficulties in connection with the investigation of the surface of a liquid metal are considerable, and they have been solved thus far by Dr. Grafton and Mr. W. W. Ewing. The result which is used in table 1 for the surface tension of mercury in a vacuum, is not our own, and may be too high, but this would not affect the value of the results, since it introduces only a small constant difference in all of the numbers which represent the work of adsorption. The liquids were purified with great care. Many supposedly pure organic substances which are not sulphur compounds, and which have the correct boiling points, contain enough organic sulphides to make determinations made upon them entirely useless. This was especially true of the xylenes, which were boiled with mercury for several days to remove the sulphur.

Experiments have been made on the adsorption of organic acids from aqueous solution, and the results finally obtained, together with those on soaps, all of which have a high positive adsorption, will be reported in the complete papers which will be published in the *Journal of the American Chemical Society*. A recent paper by Hardy[4] gives valuable information on the application of the principles of surface action to the problem of lubrication. Hardy was the first worker to investigate the application of the Dupré equation to interfaces between water and organic liquids. Our complete papers will consider the relation between adhesion and lubrication. Experiments are now in progress on the heat of adsorption of liquids on solids, both on dispersiod and plane surfaces, and calculations are in progress which will relate the results given in this paper to the atomic and molecular distances.

[1] Dupre, *Theorie Mécanique de la Chaleur*, Paris, 1869, p. 69; Lord Rayleigh, Phil. Mag. (5) **30**, (1890), (461), Hardy, *Proc. Roy. Soc., London*, (A) **88**, 1913, (303–33), Harkins, Brown, and Davies, *J. Amer. Chem. Soc.*, **39**, 1917, (354–64).

[2] Antonow, *J. Chem. Phys.*, **5**, 1907, (384).

[3] F. Schmidt, *Leipzig, Ann. Physik*, **39**, 1912, (1108).

[4] Hardy, *Phil. Mag.*, May, 1919.

A BENIGN TUMOR THAT IS HEREDITARY IN DROSOPHILA

By Mary B. Stark

Zoological Laboratory, Indiana University

Communicated by T. H. Morgan, October 16, 1919

1. *A non-lethal hereditary tumor.*—In a strain of flies with a lethal tumor, i.e. a tumor occurring in one-half of the males and causing their death (*J. Canc. Res.*, July, 1918; *J. Exper. Zool.*, February, 1919) another tumor has appeared as a mutation. The new tumor differs from the lethal one in that it is not sex-linked, i.e., it appears in females as well as males, and further in that it does not cause the death of the flies in which it occurs.

After several generations of inbreeding of males and females with tumors, a stock was obtained which breeds true to the tumor—the tumor appearing in all the flies.

Since the new tumor is not sex-linked its gene is not located in the X chromosome. To locate the gene in one of the other chromosomes, females with tumors were mated to star dichaete males. The gene for

star is located in the second chromosome and that for dichaete in the third chromosome. In the F_2 generation tumors reappeared as frequently among the star as among non-star flies. On the other hand none of the dichaete flies had tumors. This result indicates that one at least of the genes essential for tumor development is in the third chromosome closely linked to dichaete. To locate the gene at a definite locus in the third chromosome, flies with tumors were mated to dichaete 'hairless.' In the back crosses, no crossing over between the tumor and dichaete was obtained, while the crossing over between the tumor and hairless was the normal amount, 25%. The gene of the tumor is thus shown to be very close to that of dichaete which is at about 11.7 (table 1). The expected number of tumor bearing flies in the back cross is 50%. As seen in table 1 not more than 5% developed the tumor. This indicates

TABLE 1

DATE	WILD TYPE		HAIRLESS DICHAETE		HAIRLESS		DICHAETE		HAIRLESS		TUMOR	
	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀	♂
July 10, 1918	13	13	26	24	7	5	10	5	0	2	1	0
	28	25	58	54	9	7	14	9	2	3	3	0
	37	27	28	20	4	3	5	5	4	3	1	1
	17	16	19	17	3	4	9	5	2	2	0	1
	16	14	26	20	4	2	4	4	2	1	1	0
	32	38	30	26	14	16	20	18	4	5	2	0
Total	143	133	187	161	41	37	62	46	14	16	8	2

Crossing over between tumor and hairless = 10/40 = 25%.

that more than one gene is concerned with the inheritance of the tumor. Further investigations of this point are being made.

2. *Position of tumor in larva.*—To determine the time of appearance of the tumor and its position in the body 208 larvae were isolated and examined for tumors. Tumors were found in 180 of these (table 2). They seem to occur more often in the twelfth and thirteenth segments, occurring in the twelfth 97 times and in the thirteenth 72 times. Two tumors were observed in each of thirty-eight larvae. Twenty-seven of these cases had the tumors in different segments and eleven in the same segment. One larva had three tumors. Twenty-five larvae with tumors had in addition smaller tumors which were regarded as metastases. These when very small are often carried into the heart with the blood and there develop into narrow elongated tumors as shown in figure 1.

The tumor may develop in the early or in the late larval stages. Fully developed tumors have been found in two-day old larvae and very young tumors in four-day old larvae. Of the 208 larvae isolated, only 32 were without tumors. Tumors must have developed later in these individuals since all of the flies that emerged from them had tumors.

One hundred and seventy-six flies emerged from the 208 larvae isolated, approximately the same percentage as in the control experiment in which 200 larvae from a normal stock were isolated, of which 170 emerged as flies.

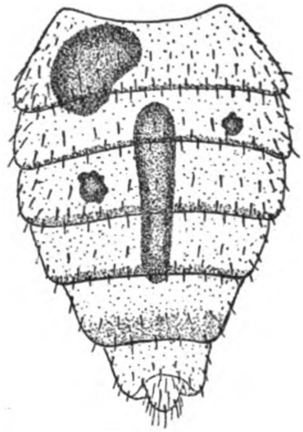
TABLE 2

POSITION OF TUMORS IN SEGMENTS	NUMBER OF LARVAE WITH TUMORS	NUMBER WITH ONE TUMOR	NUMBER WITH TWO TUMORS	NUMBER WITH THREE TUMORS	NUMBER WITH METASTASES
14	5	5			
14–13–12	1			1	
13	49	44	5		6
13–12	16		16		4
13–11	2		2		
13–10	1		1		
13–8	3		3		
12	75	65	10		12
12–11	4		4		1
12–8	1		1		1
11	8	6	2		
9	9	9			
8	5	5			
6	1	1			1
Total............	180	135	88	3	25

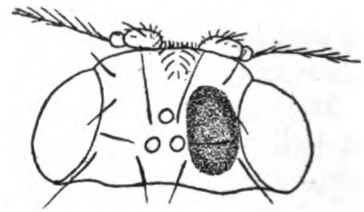
Total number of larvae, 180. Total number of tumors, 226. Total number of metastases, 36.

3. Development of tumor.—The structure of the tumor in an early stage in its development in the larva is shown in figure 2. The cells are rounded or polygonal in shape and contain pigment. As the tumor grows older the amount of pigment increases, the cells filled with it become crowded towards the periphery and flattened as shown in figure 3.

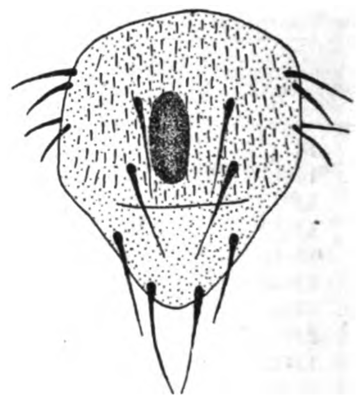
The tumor takes its origin in groups of cells similar in structure to the cells found just inside of the hypodermis of the larva. These cells are originally derived from the hypodermal cells. In the fly the tumor when fully matured is entirely permeated by pigment and is black in color. The cells have stopped increasing in number. As the fly grows older the tumor tends to shrink in size but does not disappear.



1

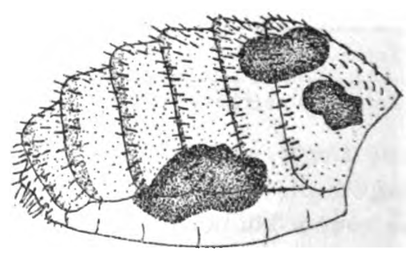


6



7

4



8

All figures were drawn with the aid of a camera lucida, using a 16 mm. objective and ocular no. 5 with tube length of 165 mm. excepting figures 2, 3, and 5 for which the 4 mm. objective was used.

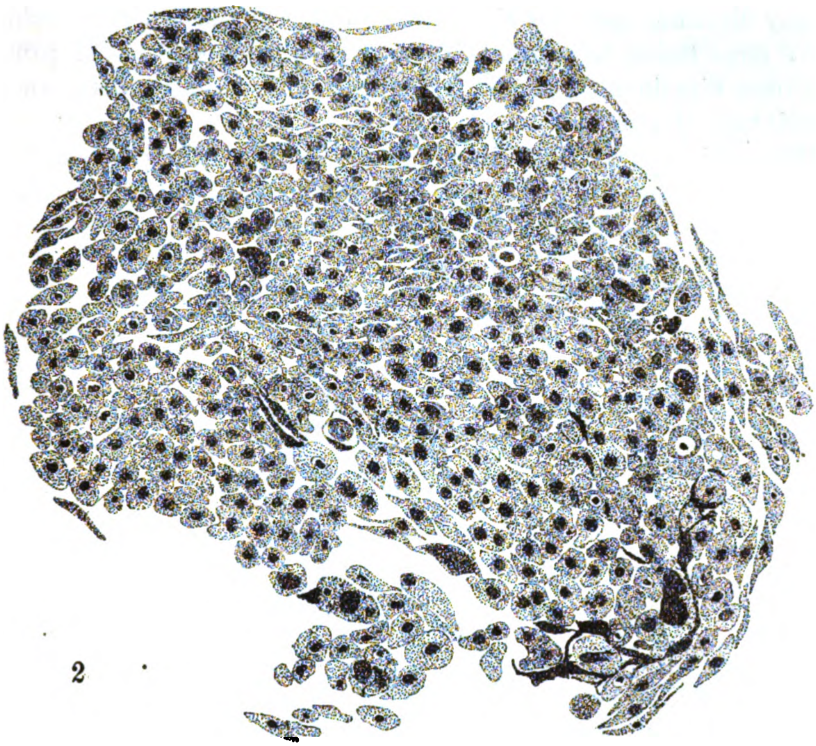
FIG. 1. Dorsal view of abdomen showing a tumor in dorsal artery.

FIG. 4. Dorsal view of head, thorax and first segment of abdomen showing a tumor in the place of the left wing.

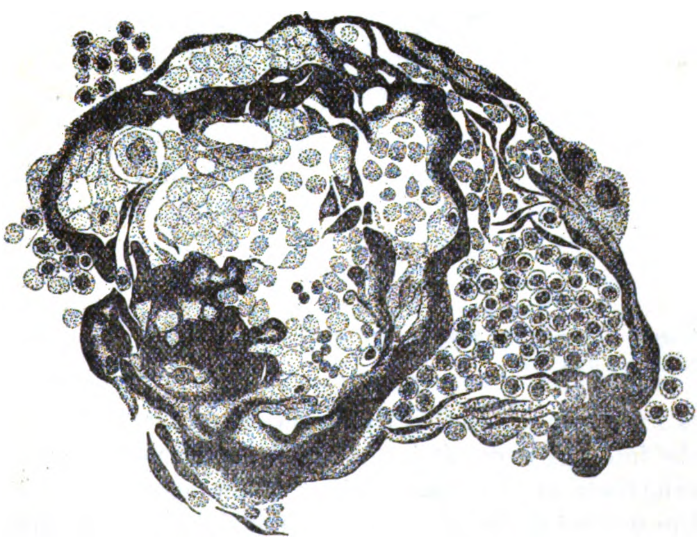
FIG. 6. Dorsal view of head with a tumor.

FIG. 7. Dorsal view of thorax with one tumor.

FIG. 8. Side view of abdomen with three tumors.



2

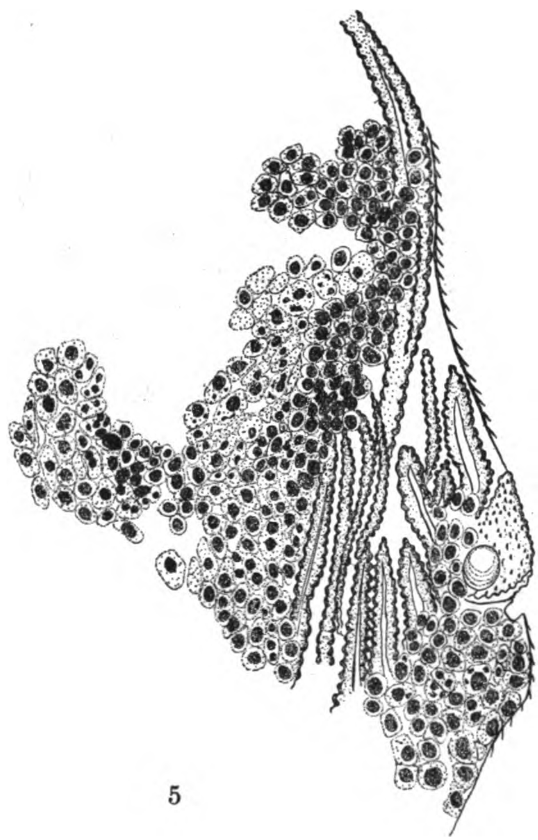


3

FIG. 2. A section through the center of a tumor in an early stage of development.
FIG. 3. A section through the center of a tumor in a late stage of development.

Many flies have appeared with only one wing. In place of the missing wing a large tumor has developed as shown in figure 4. The offspring from these flies do not throw a greater number of one-winged flies than do their normal sisters.

Microscopic examination of sections of the wing tumor reveals the fact that the wing disc had begun to develop but an ingrowth of the tumor cells had checked the development (figure 5).



5

FIG. 5. A section through the wing tumor (fig. 2) showing ingrowth of tumor cells into the partly developed wing disc.

Tumors have also appeared in place of the other appendages. In several, the fore-legs were missing, a tumor having prevented the complete development of the imaginal discs of the legs. These flies had difficulty in balancing the body and did not live long. In many flies only one leg or only a portion of the leg would be displaced by a tumor.

The tumor may appear in the head, see figure 6. Many flies with head-tumors seemed to have no power of coordination and lived only a few days, while other flies did not seem to be disturbed in any way whatever.

When the tumor developed in the abdomen or in the thorax (figs. 1, 7 and 8) and did not interfere with the development of the appendages the length of life of the fly seemed to be normal. Table 3, part 'a,' gives the length of life of the females of five pairs of flies, also the number of offspring from each pair. The length of life of the female and the number of offspring correspond closely with same of control as indicated in part 'b' of table 3.

TABLE 3

NUMBER OF PAIRS	LENGTH OF LIFE OF ♀	NUMBER OF DAUGHTERS	NUMBER OF SONS
a. Pairs of flies having tumors (February 30, 1919)			
1	18 days	296	257
2	18 days	264	271
3	19 days	198	110
4	36 days	154	139
5	16 days	154	163
b. Control pairs of normal flies			
1	20	304	264
2	35	263	232
3	22	198	207

It is evident that the presence of the tumor in the abdomen and in the thorax of the body is not sufficiently injurious to decrease seriously the length of life of the fly.

4. *Inoculation of tumors.*—Under aseptic conditions young tumors were removed from larvae and inserted into larvae of a different strain normally free from tumors. Forty larvae were thus inoculated. Only five percent (two) survived the operation, but these continued to live, completed metamorphosis and carried the inserted tumor into the adult fly. Both these flies were females and were sterile. Matings were attempted with several males but without success. The tumor was found to resemble exactly the tumor that occurs normally in the adult fly in the stock from which it was originally removed.

A few of the larvae from which the tumors were removed recovered from the operation. Some of these completed metamorphosis. All of the offspring from these had tumors showing that the tumor is due to something in the germ-plasm which is handed over from one generation

to another even though the parents are deprived of the tumors by an operation.

I am gratefully indebted to Professor Morgan and Doctor Bridges for helpful suggestions.

Conclusions.—1. A non-lethal tumor appeared as a mutation in the lethal tumor strain.

2. The locus of the gene of the new tumor is close to that of the dichaete in the third chromosome.

3. The tumor may occur in any segment of the larva but seems to occur more often in the twelfth and thirteenth segments.

4. The cells of the tumor are rounded or polygonal in shape and show the presence of pigment.

5. Ingrowth of tumor cells into the imaginal discs of the appendages checks the development of the parts.

6. Young tumors were inserted into larvae of normal strains. Five per cent survived the operation, completed metamorphosis and carried the inserted tumor into the adult fly.

METALLIC SALTS OF PYRROL, INDOL AND CARBAZOL

By Edward C. Franklin

Department of Chemistry, Leland Stanford University

Communicated, October 20, 1919

The ammono acids, that is to say, the acids of the ammonia system of acids, bases and salts,[1] are derivatives of ammonia in which one or two hydrogen atoms of the ammonia molecule are replaced by negative groups. A number of examples of compounds so related to ammonia are nitramide or nitrosyl amide, NO_2NH_2; acetamide or acetyl amide, CH_3CONH_2; phthalimide or phthalylimide, $C_6H_4\left\langle{CO \atop CO}\right\rangle NH$; benzenesulfonnitramide or nitrosyl benzenesulfonyl imide, $C_6H_5SO_2NHNO_2$; methylnitramine or methyl nitrosyl imide; acetanilide or phenyl acetyl imide; trinitraniline, $C_6H_2(NO_2)_3NH_2$; cyanamide, $CNNH_2$; urea, $CO(NH_2)_2$ etc., etc.

These substances are true acids ranging in acidity from benzenesulfonnitramide, which approaches the ordinary mineral acids in strength, through phthalimide and methyl nitramine, which are well known to possess weak acid properties, to acetamide and urea which are not

ordinarily recognized as acids at all. The acid properties of acetamide, urea and other very weak ammono acids, however, show themselves distinctly when in solution in liquid ammonia as has been shown by the writer and his students in earlier papers.

The fact that acids too weak to be recognized as such in aqueous solution are still capable of showing acid properties when in solution in liquid ammonia is undoubtedly due to the lower solvolytic[2] action of ammonia as compared with water which in turn results from the very slight autoionization of the former solvent. To what extent liquid ammonia is dissociated has not been accurately determined. Since, however, it is a comparatively easy matter to obtain liquid ammonia with a specific conductance less than one-eighth that of water[3] it must be considerably less ionized than is the latter solvent.

It happens therefore that salts of the weaker ammono acids, which, because of the strong hydrolytic action of water are incapable of existence in the presence of this solvent, have been easily prepared from liquid ammonia solutions:

Pyrrol, $\begin{matrix} CH = CH\diagdown \\ | \qquad\qquad NH, \\ CH = CH\diagup \end{matrix}$ indol, $C_6H_4\begin{matrix} \diagup CH \diagdown \\ \\ \diagdown NH \diagup \end{matrix}CH$ and carbazol, $C_6H_4\underset{NH}{\diagdown\!\!\diagup}C_6H_4$, are obviously derivatives of ammonia in which ammonia hydrogen is replaced by distinctly negative groups. They are therefore to be classed among the ammono acids and as such should react, in liquid ammonia solution, with the more electro positive metals and with their amides, the ammono bases, to form salts.

Experiments will be described in a paper to be printed in the *Journal of Physical Chemistry* showing that pyrrol, indol and carbazol react in liquid ammonia solution with metallic potassium, sodium, calcium and magnesium and with the amides of potassium, sodium, calcium and silver to form the following well defined salts:

Sodium pyrrol, C_4H_4NNa and $C_4H_4NNa.NH_3$.
Calcium pyrrol, $(C_4H_4N)_2Ca$ and $(C_4H_4N)_2Ca.4NH_3$.
Magnesium pyrrol, $(C_4H_4N)_2Mg.2NH_3$.
Silver pyrrol, $C_4H_4NAg.NH_3$.
Sodium indol, $C_8H_6NNa.xNH_3$.
Potassium indol, $C_8H_6NK.xNH_3$.
Calcium indol, $(C_8H_6N)_2Ca.4NH_3$.
Magnesium indol, $(C_8H_6N)_2Mg.4NH_3$.

Silver indol, $C_8H_6NAg.NH_3$.
Potassium carbazol, $C_{12}H_8NK.2NH_3$ and $C_{12}H_8NK.NH_3$.
Calcium carbazol, $(C_{12}H_8N)_2Ca.7NH_3$ and $(C_{12}H_8N)_2Ca.4NH_3$.
Silver carbazol, $C_{12}H_8NAg.2NH_3$ and $C_{12}H_8NAg.NH_3$.

[1] *Amer. Chem. J.*, **28**, 1902, (83); **47**, 1912, (285); *Eighth Int. Cong. App. Chem.* **6**, 1912, (119) and *J. Amer. Chem. Soc.*, **37**, 1915, (2279).

[2] Solvolysis is used as a general term to include hydrolysis, ammonolysis, amidolysis, aminolysis, alcoholysis, etc.

[3] Franklin and Kraus, *J. Amer. Chem. Soc.*, **27**, 1905, (191).

GROWTH AND REPRODUCTION IN FOWLS IN THE ABSENCE OF CAROTINOIDS AND THE PHYSIOLOGICAL RELATION OF YELLOW PIGMENTATION TO EGG LAYING

By Leroy S. Palmer

Division of Agricultural Biochemistry, University of Minnesota

Communicated by W. A. Noyes, November 1, 1919

The chemical identification of each of the recognized vitamines as individual substances or chemical groups is greatly to be desired. As the result of certain studies[1] which I made on the physiological relation between the yellow carotin and xanthophyll pigments of plants and the yellow lipochromes of animal tissues and fluids, I became impressed with the fact that there seemed to be more than a casual relation between the simultaneous presence of the plant carotinoids and fat-soluble vitamine in butter fat and egg yolk and in the leafy parts of green plants, and the simultaneous absence of carotinoids and fat-soluble vitamine from lard.

That phase of my carotinoid studies showing the physiological identity of egg yolk lipochrome with plant xanthophyll suggested that the fowl should be a suitable animal upon which to test the relation of plant carotinoids to growth and reproduction. It was decided to approach the question by attempting to raise a flock of chickens from hatching to maturity on a ration devoid of carotin and xanthophylls. The problem which was presented was therefore mainly one of selecting a ration devoid of yellow plant pigments but which was presumably adequate otherwise for the normal growth of chickens.

Three experiments were undertaken. The first was a preliminary experiment, during the winter of 1916–17, to test the efficacy of a ration of white corn, white corn bran, bleached flour, skim-milk and

bone meal, beginning with birds weighing about one-half pound each. Several birds reached the period of fecundity on this ration devoid of carotinoids, and the yolks of the eggs laid were remarkably deficient in color.

In the second experiment, carried out during the season of 1917–18, 75 White Leghorn chicks were placed on a practically identical carotinoid-free ration immediately after hatching. None of these birds reached maturity, partly, it is believed, because of nutritional difficulties, and partly, it is admitted, because of a number of unfortunate accidents in caring for the birds, which caused the death of the larger share of them.

The nutritional difficulties of the second experiment were overcome in the third trial begun in April, 1918, by introducing into the ration both pork liver, which is rich in vitamines,[2] but devoid of carotinoids,[3] and a roughage of paper pulp.[4] A flock of 50 vigorous, normal White Leghorn chickens were raised from hatching to maturity on this modified white corn and skim-milk ration. The mature birds were free from yellow pigmentation.

Not only was normal growth secured on this ration, but the hens in the flock exhibited normal fecundity. Seventeen of the hens whose egg records were kept averaged 52 eggs each during a period of 233 days. Several had considerably higher records. One hen laid 44 eggs during a period of 59 days.

Especially interesting was the character of the pigmentation of the egg yolks. The hard-boiled yolks were colorless but the raw yolks had a faintly yellow color. The pigment, however, was neither carotin nor xanthophyll. Acetone readily extracted the coloring matter from the raw yolks but the pigmented fat which could be obtained from this extract failed to give the reduction test with ferric chloride, which I have shown[5] to be characteristic of carotinoids. Attempts to identify the pigment with Barbieri's[6] ovochromine or with bilirubin, were not successful.

It was felt that the negative relation between carotinoids and fat-soluble vitamine as exhibited by normal growth and fecundity in chickens on rations devoid of carotinoids could not be regarded as established unless the carotinoid-free eggs should prove fertile and normal chicks be hatched from them. Inasmuch as the cocks and hens of the carotinoid-free flock were kept together throughout the experiment the eggs were presumably fertile. About 90 eggs, in all, were incubated at various times. Forty-one livable chicks were hatched

from these eggs. The young chicks appeared normal in every way except for the complete absence of yellow pigmentation from the shanks, beaks, and other skin parts.

The newly hatched chicks were immediately placed on a carotinoid-free ration and were cared for as nearly as possible in the same manner as the young chicks of the preceding generation. By the end of three months, however, all had died. Although it is probable that this unfortunate result may be explained on the ground that the chicks were hatched very late in the season and therefore had to combat a period of extreme heat, as well as a very restricted diet during the most precarious period of their growth, nevertheless the question remains open as to whether it is possible to continue the carotinoid-free condition into more than one generation.

Physiological relation of pigmentation to egg laying.—Practical poultry men have recognized for several years that a relation exists between the amount of yellow pigment visible in the shanks, ear lobes, beak and vent of hens of the Leghorn, Plymouth Rock, Wyandotte and Rhode Island Red breeds and their previous egg laying activity. Extensive biometric analyses have been made by Blakeslee and Warner[7] and by these authors with Harris and Kirkpatrick[8] of data collected at the Storrs Agricultural Experiment Station egg laying competitions in order to establish the character of this relation. The results show a positive correlation between pale colored shanks, ear lobes, beak, etc., and a recent more or less heavy egg production.

The hypothesis which has been adopted by these investigators to explain the physiological relationship which has been observed between fecundity and pigmentation is that the growth of the egg abstracts the pigment from the body tissues. The idea that the relationship could be explained also on the basis that the egg yolk abstracts fat-soluble pigment from the food, thus precluding its localization in the body tissues, was advanced by Harris, Blakeslee and Warner[9] in an earlier paper, but was apparently abandoned. The high percentage of fat in the blood of laying hens, as compared with non-laying hens, as shown by Warner and Edmonds,[10] and by Riddle and Harris,[11] is believed by the former authors to support the hypothesis that the tissue fat is being transferred to the egg yolk during laying with a consequent subtraction of pigment.

The success attained in raising a flock of White Leghorn fowls entirely lacking in pigmentation in both adipose tissue and visible skin parts presented the opportunity for ascertaining the true physiological relation

between fecundity and the fading of the yellow pigmentation of the shanks, ear lobes, etc. The fact that the carotinoid-free hens exhibited normal fecundity enhanced greatly their value for the investigation.

The question was attacked in two ways, first, by observing the histological changes in the shank skin when carotinoid-free food was fed to non-laying pigmented birds, and second, by observing the effects on the tissue and skin pigmentation of feeding carotinoid-rich food to laying carotinoid-free hens. The birds used for the histological studies comprise several yellow shanked White Leghorn cockerels, the specific source of whose pigmentation was not known, and one cockerel from the carotinoid-free flock whose feed was changed from the carotinoid-free ration to one composed principally of yellow corn. The visible skin parts of the latter bird took on a yellow color very rapidly after the introduction of the yellow corn until at the end of 42 days his plumage had a rich creamy appearance and the shanks, beak, ear lobes and vent a deep yellow color. Each of the pigmented birds was placed on a carotinoid-free ration and histological studies made on vertical frozen sections of the shank skin of individuals from time to time as the pigment gradually faded.

As the result of these studies the observation of Barrows[12] was confirmed that the yellow pigment of the shank skin is confined chiefly to the Malphigian layer of the epidermis, with some pigment in the corium. Especially instructive were the sections after staining with Nile blue. The sample of this dye which was used was found to be dichromatic with respect to fat and pigment, fat staining red and carotinoid pigment deep blue. By this means it was determined that carotinoid pigment exists free in granular condition in the shank epidermis, which is contrary to the results reported by Barrows, who concluded that the lipochrome of the shank skin is dissolved in fat. The failure of Sudan III to color the visible skin parts of fowls, as observed by Blakeslee,[13] and confirmed by me, is explained readily by the observation that the Malphigian layer of both the pigmented and non-pigmented skin lacks appreciable amounts of stainable fat.

The histological studies of the shank skin as the xanthophyll gradually faded on the carotinoid-free ration showed first a disappearance of pigment from the corium, then a disappearance from the outer layer of the corium which gradually extended to the rete of Malphigi, the last pigment to disappear being the xanthophyll at the base of the Malphigian layer. These observations are interpreted to mean that when

the supply of xanthophyll for the skin is cut off by reason of its removal from the food, or for any other reason, any xanthophyll present in the corium layer of the skin of the shank, ear lobes, etc., is deposited in the rete of Malphigi. At the same time the xanthophyll deposits in the outer layer of the epidermis either wear off by reason of the normal replacement of the outer cells by those lower down, or are oxidized because of closer contact with the air. The xanthophyll deposits in the rete of Malphigi in time become a part of the outer layers of epidermis and are lost also. The skin thus finally becomes free from visible yellow pigment.

The significance of this interpretation at once becomes apparent in the light of the results secured when xanthophyll-rich rations were fed to the laying carotinoid-free hens.. After a month on rations containing an abundance of green feed or yellow corn not a trace of xanthophyll had appeared in the ear lobes, shank or vent, and the adipose tissue had taken up such a small amount of yellow color that a very careful examination of the rendered, melted, body fat was necessary to detect the increase in color in comparison with the fat from birds which had received no carotinoids in their ration from birth. The blood serum and the yolks of the eggs laid during the feeding of the xanthophyll-rich rations, however, contained an abundance of yellow pigment.

As the result of the histological studies and feeding trials the author believes that the correct explanation of the physiological relation between egg laying and the fading of visible yellow pigmentation from the bodies of fowls of certain breeds is that in cockerels and non-laying females the visible skin parts represent a normal path of excretion of the xanthophyll pigment derived from the food. Egg laying deflects the excretion entirely to the ovaries, and even prevents the incorporation of xanthophyll with the tissue fat, and this continues as long as the ovaries function with regularity, whether the egg production be at the rate of one egg a day or one egg a week. The result is that the pigment found in the skin at the onset of fecundity is gradually excreted toward the epidermis where it either wears away as the result of the normal structural changes in the epidermis, or becomes oxidized, and thereby decolorized. The movement of yellow skin pigment during fecundity is thus outward and not inward toward the ovaries.

Influence of various feeds and certain dyes on the color of the egg yolk and body fat.—A critical study was made of the effect of certain coloring matters on the pigmentation of adipose tissue, egg yolk and visible skin parts, and also of the relative xanthophyll content of various

materials commonly used as chicken feed, using the carotinoid-free flock.

Carotin alone, fed as naturally highly colored butter fat, and the orange-yellow pigment of the annatto seed, were found to be without influence on the color of the adipose tissue or visible skin parts. Sudan III colored only the adipose tissue of non-laying birds, the visible skin parts being unaffected by this dye. With laying birds, the egg yolk as well as the adipose tissue was colored by Sudan III, but not the visible skin parts.

Yellow corn and green feed only were found to be rich in xanthophyll when various plant and animal materials were tested for their xanthophyll content by their effect on the color of the egg yolks when fed to hens laying carotinoid-free eggs. A little of the pigment was found in hempseed, barley, gluten feed and red corn. Wheat, wheat bran, oats, cottonseed meal, rape seed, meat scraps, blood meal, skim-milk and butter-milk were found to contain negligible quantities of xanthophyll.

The experiments here reviewed were conducted at the Missouri Agricultural Experiment Station with the coöperation of Prof. H. L. Kempster of the Department of Poultry Husbandry. The complete data appeared in a recent issue of the *Journal of Biological Chemistry*.

[1] Palmer, L. S. and Eckles, C. H., *J. Biol. Chem.*, 17, 1914, (191–249); *Missouri Agric. Exp. Sta. Bull.*, Nos. 9, 10, 11, 12, 1914, (313–450). Palmer, L. S., *J. Biol. Chem.*, 23, 1915, (261–279); 27, 1916, (27–32).

[2] Osborne, T. B. and Mendel, L. B., *J. Biol. Chem.*, 32, 1917, (309–323); 34, 1918, (17–27).

[3] Palmer, L. S., *Ibid.*, 27, 1916, (27–32).

[4] Osborne, T. B. and Mendel, L. B., *Ibid.*, 33, 1918, (433–438).

[5] Palmer, L. S. and Thrun, W. E., *J. Ind. Eng. Chem.*, 8, 1916, (614–618).

[6] Barbieri, N. A., *Paris, C. R. Acad. Sci.*, 154, 1912, (1726–1730).

[7] Blakeslee, A. F. and Warner, D. E., *Science*, 41, 1915, (432–434); *Amer. Nat.*, 49, 1915, (360–368).

[8] Blakeslee, A. F., Harris, J. A., Warner, D. E. and Kirkpatrick, W. F., *Storrs Agric. Exp. Sta. Bull.* No. 92, 1917, (95–194).

[9] Harris, J. A., Blakeslee, A. F. and Warner, D. E., These PROCEEDINGS, 3, 1917, (237).

[10] Warner, D. E. and Edmond, H. D., *J. Biol. Chem.*, 31, 1917, (281–294).

[11] Riddle, O. and Harris, J. A., *Ibid.*, 34, 1918, (161–174).

[12] Barrows, H. R., *Maine Agric. Exp. Sta. Bull.*, No. 232, 1914, (237–252).

[13] Blakeslee, A. F., *Storrs Agric. Exp. Sta. Bull.*, No. 92, 1917, (152).

RADIATIONLESS ORBITS

By Edwin Bidwell Wilson

Department of Physics, Massachusetts Institute of Technology

Read before the Academy, November 10, 1919

The reaction on an electric charge due to non-uniform motion has been calculated (by Lorentz, for example) and found to be

$$\mathbf{F} = \frac{e^2}{6\pi c^3}\frac{d^2\mathbf{v}}{dt^2} \tag{1}$$

this term $\mathbf{F}$ is merely the first of a series which converges with extreme rapidity provided the dimensions of the charge are small compared with the distance light would travel in the time required for a sensible charge in the velocity.[1] The activity of the force $\mathbf{F}$ determines the rate R of radiation of mechanical energy as

$$-R = \mathbf{F}\cdot\mathbf{v} = \frac{e^2}{6\pi c^3}\mathbf{v}\cdot\frac{d^2\mathbf{v}}{dt^2} \tag{2}$$

What plane orbits are such as to make the radiation as determined by (2) identically zero? The answer to this question is contained in the general integral of the quadratic differential equation of the third order

$$\mathbf{v}\cdot\frac{d^2\mathbf{v}}{dt^2} = \frac{dx}{dt}\frac{d^3x}{dt^3} + \frac{dy}{dt}\frac{d^3y}{dt^3} = 0. \tag{3}$$

The integration may be obtained easily by considering $\mathbf{v}$ as the radius vector in the hydrograph. Then $d^2\mathbf{v}/dt^2$ is the acceleration of the moving point in the hydrograph and its radial and tangential components are

$$\frac{d^2v}{dt^2} - v\left(\frac{d\varphi}{dt}\right)^2 \quad \text{and} \quad \frac{1}{v}\frac{d}{dt}\left(v^2\frac{d\varphi}{dt}\right)$$

of φ be the inclination of $\mathbf{v}$ to a fixed direction. The condition of perpendicularity (3) may thus be written

$$\frac{d^2v}{dt^2} - v\left(\frac{d\varphi}{dt}\right)^2 = 0, \qquad \text{or} \qquad \frac{d\varphi}{dt} = \sqrt{\frac{v''}{v}}.$$

Then

$$x = \int [v\cos\int\sqrt{v''/v}\,dt]\,dt, \tag{4}$$

$$y = \int [v\sin\int\sqrt{v''/v}\,dt]\,dt$$

are the parametric equations of the path, v being any function of the time. The equations contain three constants of integration, which allow an arbitrary choice of axes, and one arbitrary function v. The accelerations along and perpendicular to the path are v' and $\sqrt{vv''}$; the total acceleration is $\sqrt{v'^2 + vv''}$, and its inclination to the path, $\tan^{-1}(\sqrt{vv''}/v')$.

A simple case may be had by taking $v = e^{-at}$, which represents a particle combing to rest. Equations (4) may be written

$$x = \int_\infty^t e^{-at} \cos at\, dt, \qquad y = \int_\infty^t e^{-at} \sin at\, dt.$$

Or

$$x = \frac{1}{\sqrt{2}\, a} e^{-at} \cos (at - \tfrac{3}{4}\pi), \qquad y = \frac{1}{\sqrt{2}\, a} e^{-at} \sin (at - \tfrac{3}{4}\pi).$$

And if

$$\theta = at - 3\pi/4, \qquad r = \frac{1}{\sqrt{2}\, a} e^{-\theta - 3\pi/4}$$

is the polar equation of the locus—showing an equiangular spiral with 45° between radius and tangent. If $z = x + iy$,

$$z = \frac{1}{\sqrt{2}\, a} e^{-a(1-i)t - 3\pi i/4} \tag{5}$$

The vector velocity dz/dt is 135° ahead of the radius z, and the acceleration is 270° ahead, which means a retarding acceleration decreasing the areal velocity and perpendicular to the radius. This type of acceleration is found in the straight-bore centrifugal gun.

In (5) the magnitudes of the radius $\mathbf{r}$, velocity $\mathbf{v}$, acceleration $\mathbf{v}'$, and its rate $\mathbf{v}''$ are in geometric progression with the ratios $\sqrt{2}\, a$, the value of $\mathbf{v}'$ being $\sqrt{2}\, ae^{-at}$. If this acceleration be resolved along the radius and along the normal to the path, the respective components are $2\sqrt{2}ae^{-at}$ outward and $2\, ae^{-at}$ inward. A constant magnetic field perpendicular to the plane of motion is competent to furnish a component acceleration along the normal to the path and proportional to the velocity. The electric field in a plane perpendicular to the line joining two like charges at its middle point will act radially from the point of equilibrium upon a like charge with a force proportional to the distance. A proper slight adjustment of the intensity of the magnetic field will take care of the force (1). It is therefore possible easily to set up an electrical problem which is satisfied by the path (5). The general dynamical equation for the motion thus adjusted is

$$\frac{d^2z}{dt^2} - 2a\,(1 + aA)\,i\,\frac{dz}{dt} - 2a^2z = A\,\frac{d^3z}{dt^3}, \tag{6}$$

and has the roots $a(1 - i)$ and $1/A$ in addition to $-a(1 - i)$; two of the roots therefore give increasing motions. The root $1/A$ is the large extraneous root that comes in when the force (1) is used;[2] the root $a(1 - i)$ gives a non-radiating orbit. Any orbit of the whole set

$$z = C_1\, e^{-a(1-i)t} + C_2\, e^{a(1-i)t} \tag{7}$$

is also non-radiating.

The orbits (4) have been described as non-radiating because the force (1), being normal to the path, does no work. It is only in this sense that they are free from radiation: the equation of energy holds, the sum of the kinetic and potential energies is constant. (As the motions are supposed to be small, quasi-stationary, relativity effects have been ignored.) From the point of view of electro-magnetic theory the rate of radiation varies as the square of the acceleration and can never vanish in accelerated motion; and there is a coördinate radiation of momentum. The relation between the two rates of radiation is shown by the equation

$$\frac{d}{dt}\left(\mathbf{v}\cdot\frac{d\mathbf{v}}{dt}\right) = \frac{d\mathbf{v}}{dt}\cdot\frac{d\mathbf{v}}{dt} + \mathbf{v}\cdot\frac{d^2\mathbf{v}}{dt^2}$$

The total radiation is the same according to both points of view whenever estimated between two instants for which $\mathbf{v}.d\mathbf{v}/dt$ has the same values; but the instantaneous rates cannot be indentically equal except for orbits in which $\mathbf{v}.d\mathbf{v}/dt$ is always the same constant independent of the time. I have discussed elsewhere the effect of replacing the law (1) by its alternative in the case of the rectilinear oscillation,[3]—which then ceases to be an oscillation.

From the point of view of relativity the rate of radiation of energy (and momentum) has been treated by several authors and has been shown to vary with the square of the generalized curvature of the path when regarded as a space-time locus in four dimensions.[4] For any real motion, where the velocity is less than that of light, the radiation must be positive and radiationless orbits, other than straight lines uniformly described, are impossible. If, however, velocities greater than light were possible and if the formula for the rate of radiation still held, the orbits would be those for which the curvature vector was a minimum line. Although this state of affairs may have no present

physical interest, it does have a purely mathematical interest, and I shall therefore determine also these orbits.

The expression which must be integrated is[4]

$$\frac{d\mathbf{v}}{dt}\cdot\frac{d\mathbf{v}}{dt} - \left(\mathbf{v}\times\frac{d\mathbf{v}}{dt}\right)\cdot\left(\mathbf{v}\times\frac{d\mathbf{v}}{dt}\right), \tag{8}$$

provided the units are so chosen that the velocity of light is 1. Let R be the radius of curvature of the orbit (in ordinary space). The tangential and normal resolution of acceleration throws (8) into

$$\left(\frac{dv}{dt}\right)^2 + \frac{v^4}{R^2} = \frac{v^6}{R^2} \quad \text{or} \quad \left(\frac{dv}{ds}\right)^2 = \frac{v^4 - v^2}{R^2}.$$

Hence

$$v = \sec \int \frac{ds}{R}. \tag{9}$$

If then any space curve be given intrinsically by the equation $R = f(s)$, equation (9) determines the velocity at which the curve must be traced if there is to be no radiation as estimated by the usual electro-magnetic formula.

[1] See H. A. Lorentz, *Theory of Electrons*, Art. 37, B. G. Teubner, Leipzig, 1916.
[2] See M. Planck, *Theorie der Wärmestrahlung*, p. 110, J. A. Barth, Leipzig, 1906.
[3] Wilson, E. B., *Boston*, *Proc. Amer. Acad. Arts Sci.*, 50, 1914, (105–128).
[4] See Wilson and Lewis, *Ibid.*, 48, 1912, (387–507), especially p. 481.

THE EFFECT UPON THE ATOM OF THE PASSAGE OF AN ALPHA RAY THROUGH IT[1]

By R. A. Millikan, V. H. Gottschalk and M. J. Kelly

Ryerson Physical Laboratory, University of Chicago

Read before the Academy, November 11, 1919

In 1910, by catching at the instant of ionization the positive residues of atoms ionized by X-rays, and by beta and gamma rays of radium, it was conclusively shown[2] that the act of ionization by these agencies uniformly consists in the detachment of a single negative electron from a neutral atom.

The method consisted in balancing the force of gravity acting upon a minute oil-drop by a strong vertical electrical field, holding the oil-drop under observation in a telescope with the aid of a powerful beam of light,

passing a sharply limited beam of X-rays, beta rays or γ rays immediately underneath the drop, catching upon the drop the positive ion, formed by the ionization of a neutral molecule by the rays under investigation, and finally measuring the charge communicated to the drop by the advent of the ion upon it through observing the speed imparted to the drop by its new increment in charge.

Just before the war, Millikan attacked the more difficult and the more interesting problem of catching by the same general method, the ions formed by the passage of an alpha particle through an atom, expecting in this case to find that this relatively huge and powerful ionizing agent would often detach more than one negative electron from a single atom. When he was called to other duties by the war, the experimental work already begun was continued and completed by Gottschalk and Kelly. The results are as follows:

1. Alpha rays have been shot through atoms of the most diverse sorts (H, C, O, N, Cl, I, Hg) and of atomic weights from 1 to 200, without bringing to light in any case evidence of the formation of multiply-valent ions.

2. Twenty-nine hundred ions formed by the passage of α rays through neutral molecules have been caught on oil drops at the instant of ionization and the charges carried by each of these ions individually measured. Of these 2900 captures, 5 might possibly have corresponded to double charges, though even these were in all probability due to the nearly simultaneous advent upon the drop of two singly charged ions.

3. In no single case has an α particle been observed to form an ion carrying three or more charges, even though mercury, from which octivalent ions had been expected, was one of the substances tested.

4. *Alpha ray ionization consists, then at least 99 times out of a 100, in the case of all the gases and vapors studied, in the detachment of a single negative electron from a neutral molecule.*

[1] A detailed report of these experiments will shortly be published in the *Physical Review*.

[2] Millikan, R. A., and Fletcher, H., *London Phil. Mag.*, (Ser. 6) 21, 1911, (753).

ON THE EMBRYOLOGICAL BASIS OF HUMAN MORTALITY[1]

By Raymond Pearl

Department of Biometry and Vital Statistics, Johns Hopkins University

Read before the Academy, November 11, 1919

1. In order to get a clearer idea of the underlying biological factors in human mortality I have rearranged the 'causes of death' listed in the International Classification of the Causes of Death, which is the code used generally by vital statisticians, into a new classification on a biological basis. It is not possible with our present statistical material to make a completely and precisely logical classification, but I have endeavored to come as close to it as is possible. The underlying idea of this new classification is, as the first operation, to group all causes of death under the heads of the several organ systems of the body, the functional breakdown of which is the immediate or predominant cause of the cessation of life. All except a few of the statistically recognized causes of death in the International Classification can be assigned places in such a biologically grouped list. It has a sound logical foundation in the fact that, biologically considered, death results because some organ system, or group of organ systems, fails to continue its functions. Practically, the plan involves the reassignment of all of the several causes of death now grouped by vital statisticians under heading 'I. General diseases.' It also involves the re-distributing of causes of death now listed under the puerperal state, malformations, early infancy, and certain of those under external causes.

The headings finally decided upon for the new classification are as follows:

I. Circulatory system, blood, and blood-forming organs
II. Respiratory system
III. Primary and secondary sex organs
IV. Kidneys and related excretory organs
V. Skeletal and muscular systems
VI. Alimentary tract and associated organs concerned in metabolism
VII. Nervous system and sense organs
VIII. Skin
IX. Endocrinal system
X. All other causes of death

It should be emphasized that the underlying idea of this rearrangement of the causes of death is to put all those lethal entities together which bring about death because of the functional organic breakdown of the same general organ system. The cause of this functional breakdown may be anything whatever in the range of pathology. It may be due to bacterial infection; it may be due to trophic disturbances; it may be due to mechanical disturbances which prevent the continuation of normal function; or to any other cause whatsoever. In other words, the basis of the present classification is not that of pathological causation, but it is rather that of organological breakdown. We are now looking at the

TABLE 1

SHOWING THE RELATIVE IMPORTANCE OF DIFFERENT ORGAN SYSTEMS IN HUMAN MORTALITY

GROUP NUMBER	ORGAN SYSTEM	DEATH RATES PER 100,000			
		Registration area, U. S. A.		England and Wales, 1914	Sao Paulo, 1917
		1906–10	1901–05		
II	Respiratory system	395.7	460.5	420.2	417.5
VI	Alimentary tract and associated organs	334.9	340.4	274.1	613.8
I	Circulatory system, blood	209.8	196.8	208.6	254.8
VII	Nervous system and sense organs	175.6	192.9	151.9	124.3
IV	Kidneys and related excretory organs	107.2	107.4	19.4	83.4
III	Primary and secondary sex organs	88.1	77.4	95.4	103.2
V	Skeletal and muscular system	12.6	13.7	18.2	6.8
VIII	Skin	10.1	13.3	12.0	7.9
IX	Endocrinal system	1.5	1.2	1.9	1.1
	Total death rate classifiable on a biological basis	1335.5	1403.6	1231.7	1612.8
X	All other causes of death	171.3	211.9	141.4	109.8

question of death from the standpoint of the pure biologist, who concerns himself not with what causes a cessation of function, but rather with what part of the organism ceases of function, and therefore causes death.

2. In table 1 the death rates per 100,000 are arranged in descending order of importance (for the United States Registration Area 1906–10) by organ systems. Four sets of data are used: (a) the United States Registration Area for the five years, 1906–10 inclusive; (b) the same, 1901–05; (c) England and Wales, 1914; and (d) Sao Paulo, Brazil, 1917.

3. The data show that in the United States, during the decade covered, more deaths resulted from the breakdown of the respiratory system than from the failure of any other organ system of the body. The same thing

is true of England and Wales. In Sao Paulo the alimentary tract takes first position, with the respiratory system a rather close second. The tremendous death rate in Sao Paulo chargeable to the alimentary tract is chiefly due to the relatively enormous number of deaths of infants under two from diarrhea and enteritis. Nothing approaching such a rate for this category as Sao Paulo shows is known in this country or England.

In all three localities studied the respiratory system and the alimentary tract together account for rather more than half of all the deaths biologically classifiable. These are the two organ systems which, while physically internal, come in contact directly at their surfaces with environmental entities (water, food, and air) with all their bacterial contamination. The only other organ system directly exposed to the environment is the skin. The alimentary canal and the lungs are, of course, in effect invaginated *surfaces* of the body. The mucous membranes which line them are far less resistant to environmental stresses, both physical and chemical, than is the skin with its protecting layers of stratified epithelium.

The organs concerned with the blood and its circulation stand third in importance in the mortality list. Biologically the blood, through its immunological mechanism constitutes the second line of defense which the body has against noxious invaders. The first line is the resistance of the outer cells of the skin and the lining epithelium of alimentary tract, lungs, and sexual and excretory organs. When invading organisms pass or break down these first two lines of defense the battle is then with the home guard, the cells of the organ systems which, like the industrial workers of a commonwealth, keep the body going as a whole functioning mechanism. Naturally it would be expected that the casualties would be far heavier in the first two defense lines (respiratory and alimentary systems, and blood and circulation) than in the home guard. Death rates when biologically classified bear out this expectation.

It is at first thought somewhat surprising that the breakdown of the nervous system is responsible for more deaths than that of the excretory system. When one bears in mind, however, the relative complexity of the two pieces of machinery, it is perceived that the relative position of the two in responsibility for mortality is what might be reasonably be expected.

In the United States the kidneys and related excretory organs are responsible for more deaths than the sex organs. This relation is reversed in England and Wales and in Sao Paulo. The difference is mainly

due, in the case of England, to two factors, premature birth and cancer. In Sao Paulo it is due to premature birth and syphilis.

In a broad sense the efforts of public health and hygiene have been directed against the affections comprised in the first two items in the table, respiratory system and alimentary tract. The figures in the first two columns for the two five year periods in the United States indicate roughly the rate of progress such measures are making, looking at the matter from a broad biological standpoint. In reference to the respiratory system there was a decline of 14% in the death rate between the two periods. This is substantial. It is practically all accounted for in phthisis, lobar pneumonia, and bronchitis. For the alimentary tract the case is not so good—indeed, far worse. Between the two periods the death rate from this cause group fell only 1.8%.

TABLE 2

Showing the Relative Influence of the Primary Germ Layers in Human Mortality

(Items 64 and 65 charged to ectoderm)

Locality	Death rate per 100,000 due to functional breakdown of organs embryologically developing from					
	Ectoderm	Per cent	Meso-derm	Per cent	Endo-derm	Per cent
United States Registration Area, 1906–10	191.1	14.3	425.2	31.8	719.6	53.9
United States Registration Area, 1901–05	210.6	15.0	407.1	29.0	786.2	56.0
England and Wales, 1914	177.1	14.4	374.0	30.3	681.5	55.3
Sao Paulo, 1917	134.9	8.4	468.0	29.0	1009.9	62.6

TABLE 3

Showing the Relative Influence of the Primary Germ Layers in Human Mortality

(Items 64 and 65 charged to mesoderm)

Locality	Death rate per 100,000 due to functional breakdown of organs embryologically developing from					
	Ectoderm	Per cent	Meso-derm	Per cent	Endo-derm	Per cent
United States Registration Area, 1906–10	116.9	8.7	499.4	37.4	719.6	53.9
United States Registration Area, 1901–05	137.3	9.8	480.4	34.2	786.2	56.0
England and Wales, 1914	107.9	6.7	443.2	36.0	681.5	55.3
Sao Paulo, 1917	101.3	6.3	501.6	31.1	1009.9	62.6

4. The next step in the investigation was to arrange all the organologically classifiable death rates under the primary germ layers (ectoderm, mesoderm, and endoderm) from which the organs concerned developed embryologically. The results are set forth in tables 2 and 3, and in

figure 1. Tables 2 and 3 give upper and lower limiting values to the death rates chargeable to ectoderm and mesoderm. The difference between the two depends upon the placing of deaths due to cerebral hemorrhage and apoplexy, and to 'softening of the brain'. The discussion of the embryological and pathological problems involved cannot be given here by reason of lack of space. The complete paper must be consulted.

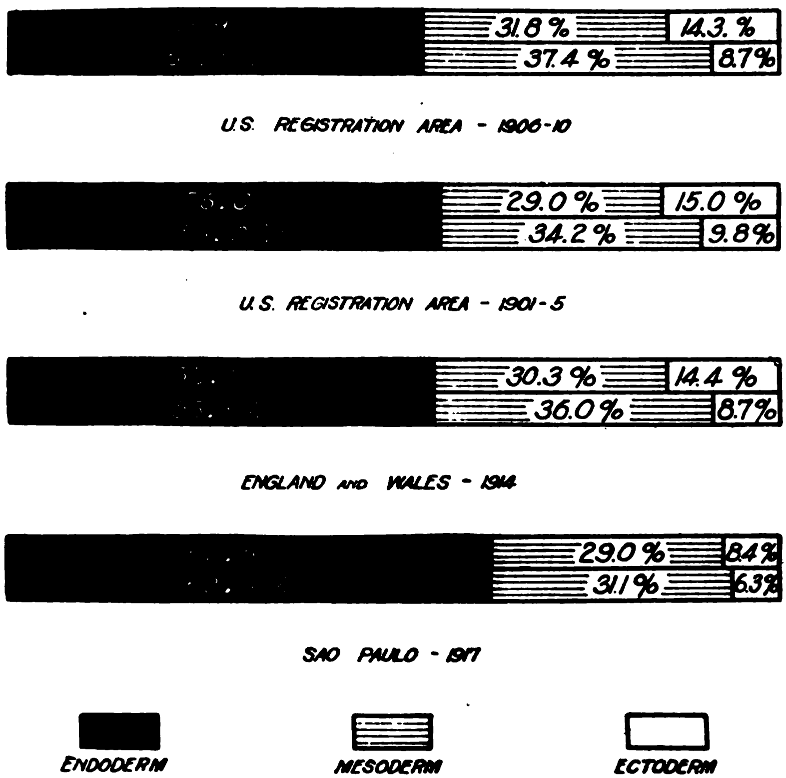


FIG. 1. DIAGRAM, SHOWING THE PERCENTAGES OF BIOLOGICALLY CLASSIFIABLE HUMAN MORTALITY RESULTING FROM BREAKDOWN OF ORGANS DEVELOPING FROM THE DIFFERENT GERM LAYERS

Upper bar of pair gives upper limit of mortality chargeable to ectoderm; lower bar gives lower limit of mortality chargeable to ectoderm.

The data of tables 2 and 3 are shown graphically in percentage form in figure 1.

The final results lead to a generalization of considerable interest and significance to the evolutionist. The figures show that in man, the highest product of organic evolution, about 57% of all the biologically classifiable deaths result from a breakdown and failure further to function

of organs arising from the endoderm in their embryological development while but from 8 to 13% can be regarded as a result of breakdown of organ systems arising from the ectoderm. The remaining 30 to 35% of the mortality results from failure of mesodermic organs. Taking a general view of comparative anatomy and embryology it is evident that in the evolutionary history through which man and the higher vertebrates have passed it is the ectoderm which has been most widely differentiated from its primitive condition, to the validity of which statement the central nervous system furnishes the most potent evidence. The endoderm has been least differentiated in the process of evolution, while the mesoderm occupies an intermediate position in this respect.

The results of this study add one more link to the already strong chain of evidence which indicates the highly important part played by innate constitutional biological factors as contrasted with environmental factors in the determination of the observed rates of human mortality. Here we have grouped human mortality into broad classes which rest upon a strictly biological basis. When this is done it is found that the proportionate subdivision of the mortality is strikingly similar in such widely dissimilar environments as the United States, England, and Southern Brazil. It is inconceivable that such congruent results would appear if the environment were the predominant factor in human mortality. This conclusion does not overlook the fact that in some diseases the environment, in a broad sense, is unquestionably the factor of greatest importance. Nor does it imply that every effort should not be used to measure in every case the precise relative influence of constitution or heredity as compared with environment in the natural history of particular diseases. This constitutes one of the most pressing and difficult problems of medical science.

[1] A complete account of this investigation will appear shortly in the *American Naturalist*. That paper must be consulted for detailed discussion of the intricate pathological and embryological points involved in the statistical treatment of the data in this investigation.

NATIONAL RESEARCH COUNCIL

EXTRACTS FROM THE MINUTES OF THE MEETING OF THE INTERIM COMMITTEE

At the Office of the National Research Council, August 19, 1919, at 10.00 a.m.

Mr. Angell in the chair.

Present: Messrs. Angell, Barrows, Howe, Ransome, Wood, and by invitation Mr. J. H. J. Yule.

The Chairman presented the resignation of Dr. W. F. Durand as Chairman of the Patent Office Committee of the National Research Council.

Moved: That the resignation of Mr. Durand as Chairman of the Patent Office Committee be accepted with an expression of thanks from the Council for the work which he has done as chairman of this committee. (*Adopted.*)

Moved: That Mr. L. H. Baekeland be elected Chairman of the Patent Office Committee of the National Research Council. (*Adopted.*)

The Chairman asked the Interim Committee to give further consideration to the request of Mr. Wade for authority to make purchases against appropriations and allotments already provided for.

Moved: That the Chief Clerk with the Bursar be authorized to make purchases against appropriated funds. (*Adopted.*)

Moved: That in view of the inability of Mr. Leuschner to return to his duties as Secretary, Mr. Wood be designated Acting Secretary until further action of the Committee. (*Adopted.*)

The Chairman asked informal consideration as to the advisability of the Chairman sending a letter, giving an outline of the present organization of the National Research Council without personnel, to the heads of the Departments of the Government, stating that the National Research Council had now, after completing its work under the war organization, reorganized under peace conditions and would be glad to consider any matters relating to research which they might desire to have advice upon. It was the sense of the meeting that the Chairman should prepare such a letter and send it to the heads of the Departments of the Government.

The meeting adjourned at 11.25 a.m.

Paul Brockett, *Assistant Secretary.*

EXTRACTS FROM THE MINUTES OF THE MEETING OF THE INTERIM COMMITTEE

At the Office of the National Research Council, October 2, 1919, at 9.15 a.m.

Mr. Angell in the chair.

Present: Messrs. Angell, Bancroft, Barrows, H. E. Howe, Kellogg, McClung, Mathews, Mendenhall, Ransome, Wood, Yerkes.

The minutes of the meeting of the Interim Committee of the National Research Council of August 19 were approved as circulated.

The Chairman, as chairman of the Finance Committee, rendered an informal report of a meeting of that committee held in New York on September 9.

Mr. Howe, Chairman of the Committee on Organization and Budget, reported that, after consultation with the Treasurer, he had taken out insurance to the amount of $5000 on that portion of the property contained in the offices of the Council which belongs to the National Research Council.

Moved: That the action taken with reference to insurance be approved. (*Adopted.*)

Mr. Kellogg, Chairman of the Publicity Committee, presented an informal report of the activities of that committee.

On recommendation of Mr. Yerkes, Chairman of the Committee on Publication, it was

Moved: That the Committee on Publication be authorized to publish Bulletins of the National Research Council in appropriate editions. (*Adopted.*)

Mr. Yerkes presented an informal report of the Committee on Publication concerning difficulties which had arisen in the publication of Bulletins of the Council, such as questions of postal rates, economy of printing, and the relation of the Bulletins of the Council to the PROCEEDINGS of the National Academy of Sciences. Action regarding the form of publication was postponed until the next meeting of the Executive Board.

The Vice Chairman of the Division of Industrial Relations made an informal report of the progress of the Division in its various activities.

On account of certain objections offered by one of the Divisions of Science and Technology to the change of name to Division of Industrial Research, the matter was again brought up for consideration. The Vice Chairman of the Division made a clarifying statement setting forth the actual functions of the Division. It was the sense of the meeting that the recommendation already made to the National Academy of Sciences, that the name be changed to Division of Industrial Research should be allowed to stand.

On recommendation of the Vice Chairman of the Division it was

Moved: That the exhibit to be put in by the Western Electric Company, all expenses of which they are to defray, be authorized. (*Adopted.*)

The Chairman of the Research Information Service drew attention to a recommendation made by the Committee on International Catalogue of Scientific Literature, which provides that the catalogue be changed to simply an author's list of publications arranged under the proper division of science, but not subdivided further, and that this list be published once in five years. In view of the thought given by the Council to the matter of abstracts and

bibliographies, and of its plans for the improvement of the status of scientific publications, and since it will be necessary for the Council to make individual response to this recommendation through the Foreign Secretary of the National Academy, it was

Moved: That a committee representing the several divisions of the Research Council be appointed by the Chairman, who shall serve as Chairman of the committee, to consider existing provisions for the abstracting and indexing of scientific literature in their relations to the proposed modification of the International Catalogue of Scientific Literature, and to formulate a plan for improving methods of preparing and issuing scientific abstracts and bibliographic lists. (*Adopted.*)

The Chairman of the Research Information Service requested reconsideration of the action of the Executive Board at its meeting on June 24, 1919, which disapproved the resolution of the Research Information Service requesting authorization to charge for its services for those exceptional cases which involve considerable time and labor in the search for the necessary information, or in special investigations.

Moved: That the Interim Committee ask the Executive Board to reconsider its action of June 24, relative to charges to be made by the Research Information Service for services in exceptional cases. (*Adopted.*)

The Chairman of the Division of Chemistry and Chemical Technology reported that in accordance with action of the Interim Committee on June 3, 1919, three trustees had been appointed for the business control of the publication of critical tables of physical and chemical constants, namely, Mr. Hugh K. Moore, appointed by the Chairman of the National Research Council; Mr. Julius Stieglitz, appointed by the President of the American Chemical Society; and Mr. A. L. Day, appointed by the President of the American Physical Society, who has not yet signified his acceptance of the appointment. A campaign has been proposed for securing the sum of $100,000 to carry on the work of publication, for the initial expenses of which a loan of $1500 has been requested by the Trustees.

Moved: That the Chairman of the Division of Physical Sciences and of the Division of Chemistry and Chemical Technology be authorized to transfer to this fund $500 each, with the understanding that this money shall be paid back to the account of these Divisions out of the subscriptions as soon as subscriptions come in. (*Adopted.*)

The Chairman of the Division reported that the Subcommittee on Ceramic Research has voted, with the approval of the Division, to secure funds for fellowships in ceramic research.

Moved: That the Committee on Ceramic Research be authorized to solicit funds for fellowships in ceramic research, as outlined in the minutes of the meeting of the Committee on Ceramic Research held on June 23, 1919. (*Adopted.*)

The Chairman of the Division of Educational Relations presented for the information of the Interim Committee a plan for a survey of research conditions in American Colleges and Universities.

The Chairman of the Division reported that a request for the appointment of Mr. Herbert Hoover as a Member at large of the Division of Educational Relations has been filed with the Secretary.

Moved: That the continuation of the appointment of Mr. Alfred D. Flinn, Secretary of the Engineering Foundation, as Assistant Secretary, by reciprocal arrangement, of the National Research Council, be confirmed. (*Adopted.*)

The Chairman presented a letter addressed to the Council by Mr. W. E. Mosher for the Commission of Congress dealing with the reclassification of salaries of scientific men, requesting the cooperation of the Council in securing data relative to salaries and duties of scientific men in universities and industrial institutions throughout the country.

Moved: That Mr. Mosher be invited to confer with Chairman of Divisions of the Council in regard to the information they desire. (*Adopted.*)

Moved: That the Chairman be authorized to appoint a Committee on By-Laws of the Council. (*Adopted.*)

The Chairman thereupon appointed Messrs. Angell, Mendenhall, and Wood.

On recommendation of the Acting Secretary it was

Moved: That the name, Editorial Committee, be changed to Committee on Publication. (*Adopted.*)

The Acting Secretary presented a letter of appreciation from Mr. Whitman Cross containing a request that the period of his services as Treasurer of the Council be shown as one year and four months instead of two and one-half years, in the resolutions of the Executive Board of August 12, 1919. (*Approved.*)

The Chairman reported that Mr. Vernon Kellogg had consented to assume conjointly the work of the secretarial office and the chairmanship of the Division of Educational Relations, Mr. Wood to continue in his present position.

The Chairman asked judgment of a plan proposed by Mr. Merriam for meetings at frequent intervals during several weeks to come in order that the Chairmen may keep closely in touch with the work of other Divisions. The plan was approved and the arrangement for such meetings left to the Chairman.

Regarding the regular meetings of the Interim Committee it was

Moved: That for the present meetings of the Interim Committee be held weekly on Tuesday at nine o'clock. (*Adopted.*)

The meeting adjourned at 12.45 p.m. until 9.00 a.m., Tuesday, October 7.

HARRY O. WOOD, *Acting Secretary.*

EXTRACTS FROM THE MINUTES OF THE MEETING OF THE INTERIM COMMITTEE

AT THE OFFICE OF THE NATIONAL RESEARCH COUNCIL, OCTOBER 7, 1919 AT 9.00 A.M.

Mr. Angell in the chair.

Present: Messrs. Angell, Bancroft, Barrows, Brockett, Christian, Kellogg, McClung, Mathews, Mendenhall, Ransome, Wood, and Yerkes.

Mr. Yerkes, Chairman of the Committee on Publication, reported that a conference had been arranged with Mr. Pearl for the afternoon of October 7, and asked the advice of members of the Interim Committee regarding questions to be discussed at the conference. This led to a discussion of the matter of publications at considerable length.

Mr. Kellogg, Chairman of the Publicity Committee, reported that arrangements had been made for the distribution of any information concerning the activities of the Council of sufficient news value to be published in newspapers. The Chairman of the Committee read a letter to be sent to Chairmen of Divisions requesting that all matter arising in the various Divisions which may be used in this connection be submitted to the Publicity Committee for distribution.

The Acting Chairman of the Division of Geology and Geography reported that the recommendations contained in the resolution of June 10, 1919, relative to coördination in field methods of map-making agencies of the Government had already been attended to by the Division of Geology and Geography, and that a complete report upon this would be furnished the Government Division after its organization has been completed.

The Chairman of the Council extended the welcome of the Interim Committee to Dr. Christian, and expressed the pleasure of its members at his arrival in Washington to participate in the activities of the Council.

The Chairman of the Council reported that six members of the Section on Psychology had already been selected and that a meeting of these is to be held in Washington on October 20. He reported further that six members of the Section on Anthropology had also been selected and it was hoped that a joint meeting of the two sections could be held on October 20.

The meeting adjourned at 10.45 a.m.

HARRY O. WOOD, *Acting Secretary.*

EXTRACTS FROM THE MINUTES OF THE MEETING OF THE EXECUTIVE BOARD

AT THE OFFICE OF THE NATIONAL RESEARCH COUNCIL, OCTOBER 14, 1919, AT 9.30 A.M.

Mr. Angell in the chair.

Present: Messrs. Abbot, C. A. Adams, Angell, Bancroft, Christian, Flinn, Johnston, Kellogg, McClung, Manning, Mathews, Mendenhall, Pearl, Pupin, Ransome, Townsend, Walcott, Woodward, Yerkes, and by invitation, Barrows, Brockett, Wood.

The minutes of the meetings of the Executive Board of the National Research Council on August 12 and of the Interim Committee on October 2 were approved as circulated with certain slight textual corrections.

Moved: That Mr. Vernon Kellogg, Chairman of the Division of Educational Relations, be appointed to serve also as Secretary of the National Research Council, to date from October 1, 1919. (*Adopted.*)

Moved: That Mr. Harry O. Wood, Acting Secretary of the National Research Council, be appointed Assistant Secretary. (*Adopted.*)

Confirming action taken by the Division of Engineering at its meeting on September 29, 1919, it was

Moved: That the Executive Board approve the appointment of Mr. Comfort A. Adams as Chairman of the Division of Engineering. (*Adopted.*)

Moved: That Mr. Henry M. Howe be appointed Honorary Chairman of the Division of Engineering for one year. (*Adopted.*)

Moved: That Mr. Galen H. Clevenger be appointed Vice-Chairman of the Division of Engineering without salary. (*Adopted.*)

The Treasurer presented a financial report for the period ending September 30, 1919.

The Chairman reported that Mr. Abraham Flexner, Secretary of the General Education Board, writing under date of September 6, 1919, had transmitted to him the following action of that Board taken on September 5, 1919:

Resolved, That, upon the request of the National Research Council under date of August 12, 1919, the sum of Ten Thousand Dollars ($10,000), or so much thereof as may be needed, be, and it hereby is, appropriated to the National Research Council, to enable the Division of Educational Relations to conduct certain educational investigations outlined in Dr. Angell's memorandum during the year ending September 30, 1920.

In view of the action of the General Education Board be it

Resolved, That the National Research Council accept with cordial appreciation the generous gift of the General Education Board of Ten Thousand Dollars ($10,000) for the special work of the Division of Educational Relations of the Council in making an investigation of the conditions affecting research in American educational institutions; and that the Council will exercise every effort to make the use of this gift yield results of advantage to American science and education. (*Approved.*)

Mr. Kellogg, Chairman of the Committee on Publicity, reported various steps taken to make the purposes and activities of the Council more widely known to the general public and the scientific men of the country through the newspaper press and technical journals.

The Chairman of the Division of Physicial Sciences, in response to a request from the Division of States Relations

Moved: That Professor Ernest Merritt be appointed to represent the Division of Physical Sciences in the Division of States Relations. (*Adopted.*)

The Chairman of the Division of Engineering reported the appointment of a committee for the investigation of high speed steel, of which Mr. J. V. Emmons has been named Chairman.

Moved: That the appointment of a committee for the investigation of high speed steel be approved. (*Adopted.*)

The Chairman of the Division of Medical Sciences reported that Professor Graham Lusk had been nominated by the Society of Biological Chemists as its representative in the Division of Medical Sciences.

Moved: That the nomination of Professor Graham Lusk be received and transmitted to the President of the National Academy of Sciences with the recommendation that he be appointed a member of the National Research Council and assigned to the Division of Medical Sciences. (*Adopted.*)

The Chairman of the Division of Biology and Agriculture presented for reconsideration a request that the National Research Council in raising funds for the support of Marine Biological Laboratory at Woods Hole. He emphasized the importance of the work carried on at this laboratory, which is handicapped at the present time by lack of funds. He asked that the Council take action.

After a short discussion consideration of the matter was postponed until the afternoon session.

The following nomination was presented by the Chairman of the Division for member at large of the Division of Educational Relations:

Mr. Herbert Hoover, Ex-United States Food Administrator; Trustee, Leland Stanford, Jr., University.

Moved: That the nomination of Mr. Herbert Hoover be approved with the recommendation to the President of the National Academy of Sciences that he be appointed a member of the National Research Council and assigned to the Division of Educational Relations. (*Adopted.*)

Upon recommendation of the Chairman of the Division it was

Moved: That Col. Robert I. Rees be appointed a representative from the Committee on Education and Special Training of the War Department as a member of the Division of Educational Relations to fill a vacancy in that position. (*Adopted.*)

The Chairman of the Council announced that the Division of Anthropology and Psychology is in process of organization, and that a meeting of the Division will be held on October 20.

The Chairman of the Research Information Service presented the following matters:

In response to a request from the Division of States Relations it was

Moved: That Mr. Edwin F. Gay be appointed to represent the Research Information Service in the Division of States Relations. (*Adopted.*)

The following nomination was presented by the Chairman of the Division for membership in that Division:

Mr. Frank Burke, Chief of the Bureau of Investigation of the Department of Justice.

Moved: That the name of Mr. Frank Burke, Chief of the Bureau of Investigation, as representative of the Department of Justice, be presented to the President of the United States in accordance with Article V, Section 3, of the Organization of the National Research Council, for designation by him for service with the National Research Council on the Research Information Service. (*Adopted.*)

The Chairman of the Division presented the following nominations for members at large of the Research Information Service:

Clement W. Andrews, Librarian, The John Crerar Library, Chicago, Illinois.

Milton C. Whitaker, Consulting Chemical Engineer, New York City.

Arthur D. Little, President, Arthur D. Little, Inc., Industrial Research Laboratories, 30 Charles River Road, Cambridge, Mass.

Fuller R. Callaway, LaGrange, Ga., Cotton Manufacturer.

F. S. Terry, Mgr., National Lamp Works of General Electric Co., Nela Park, Cleveland, Ohio.

W. R. De Field, President, W. R. De Field & Co., Chicago, Ill.

Charles F. Kettering, President, The Dayton Engineering Laboratories Co., Dayton, Ohio.

Moved: That the nominations of Messrs. Clement W. Andrews, Milton C. Whitaker, Arthur D. Little, Fuller R. Callaway, F. S. Terry, Charles F. Kettering, and W. R. DeField be approved with the recommendation to the President of the National Academy that they be appointed members of the National Research Council and assigned to the Research Information Service. (*Adopted.*)

The Chairman of the Research Information Service called attention to action taken by the Interim Committee at its meeting on October 2, 1919, recommending to the Executive Board reconsideration of its action on June 24, 1919, relative to the making of charges by the Research Information Service for services in exceptional cases. It appeared to be the sense of the meeting that such charges would be proper, but in view of the absence of a member of this Board, who had previously opposed the proposal, it was

Moved: That the matter be referred to the Interim Committee with power, and that an effort be made to consult absent members as to any grounds of opposition. (*Adopted.*)

The Chairman of the Council reported that, with the return of Mr. Walcott to the city, action is being taken in the organization of the Government Division, and it is hoped that it will be completed within a short time.

The Executive Secretary of the Division of States Relations reported the following nominations from the Divisions named to serve as members of the Division of States Relations:

For the Division of Industrial Relations: Mr. H. E. Howe.

For the Division of Educational Relations: Mr. Vernon Kellogg.

For the Division of Engineering: Mr. S. W. Stratton.

For the Division of Chemistry and Chemical Technology: Mr. W. D. Bancroft.

For the Division of Geology and Geography: Mr. E. B. Mathews.

Moved: That the nominations of Messrs. H. E. Howe, Vernon Kellogg, S. W. Stratton, W. D. Bancroft, and E. B. Mathews as members of the Division of States Relations be approved. (*Adopted.*)

The Executive Secretary of the Division of States Relations also reported the nomination from the Society of American Foresters of Mr. Hugh P. Baker to serve on the Division of States Relations.

Moved: That the nomination of Mr. Hugh P. Baker be approved, with the recommendation to the President of the National Academy of Sciences that he be appointed a member of the National Research Council and assigned to the Division of States Relations. (*Adopted.*)

The Executive Board took recess at 12.30 to meet again at 2.00 p.m.

The Executive Board resumed its meeting at 2.20 p.m.

The Chairman made reference to the matter of quorum and stated that consideration of this was referred to the Committee on By-Laws.

At the suggestion of the Chairman the legal relations of the National Research Council to the National Academy of Sciences were discussed, and it was

Moved: That the Chairman of the Council be authorized and requested to pursue this subject farther with a view to clarifying the status of the Council. (*Adopted.*)

The Chairman of the Council raised the question of policy in regard to research in the industries, especially as this relates to overlapping of interests in the various Divisions and, in particular, in the Divisions of Industrial Relations and of Engineering. A discussion at length followed.

Moved: That a committee of five on General Policy and Solicitation of Funds, of which the Chairman of the Council shall act as Chairman, be appointed. (*Adopted.*)

Appointed: Messrs. Angell (Chairman), C. A. Adams, H. E. Howe, Kellogg, and Walcott.

The question of securing financial aid for the Marine Biological Laboratory, which had been presented at the forenoon session, was brought up for further consideration.

Moved: That the matter of securing financial assistance for the Marine Biological Laboratory be referred for recommendation to the Committee on General Policy and Solicitation of Funds. (*Adopted.*)

The Chairman requested an expression of opinion by the Board concerning the policy of the Council in reference to matters of congressional legislation, with special reference to two bills at present pending,—one providing for a tax on scientific instruments, and the other the removal of exemption which has hitherto been enjoyed by educational institutions. It was the sense of the meeting that the Council should take no action.

The Chairman of the Council presented a recommendation submitted by Mr. Dunn, as Chairman of a committee appointed by the Executive Board on June 3, 1919, to provide for the continuation of retired Chairmen of the Council as members of the Executive Board.

Moved: That the Executive Board recommend to the Council of the National Academy of Sciences that Article IV, Section 2, of the Organization of the National Research Council be amended by the addition of the following sentence: "Chairmen, on their retirement, shall continue for three years as members of the Executive Board." (*Adopted.*)

The Chairman presented a recommendation from Mr. Alfred D. Flinn, that the action of the Interim Committee on October 2, 1919, providing for the continuation of the appointment of Mr. Flinn as Assistant Secretary of the National Research Council be amended to omit the name of Mr. Flinn, making reference only to the Secretary of the Engineering Foundation.

(*Approved.*)

On motion of the Treasurer various small items of appropriation and accounting were approved.

The Chairman stated that in the allotment of funds for the current year certain expenses had not been provided for, making it necessary for the Executive Board to appropriate small sums of money under various headings to cover such unauthorized items. He suggested that this matter should be referred to the Committee on Organization and Budget.

Moved: That the Committee on Organization and Budget be requested to make a redistribution of allotments of general expense temporarily and to prepare a motion to authorize action in future. (*Adopted.*)

The matter of the publication of minutes of Divisions, committees, etc., was discussed, and it was

Moved: That nothing be published by the National Research Council or any of its Divisions without the sanction of the Committee on Publication. (*Adopted.*)

The meeting adjourned at 4.20 p. m. to meet again at 9.30 a. m., November 18.

HARRY O. WOOD, *Assistant Secretary.*

INDEX